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BALANCING RELIABILITY AND MAINTENANCE COST RATE OF MULTI-STATE COMPONENTS WITH FAULT INTERVAL OMISSION

RÓWNOWAŻENIE WSKAŹNIKÓW NIEZAWODNOŚCI I KOSZTÓW UTRZYMANIA ELEMENTÓW WIELOSTANOWYCH Z POMINIĘCIEM PRZEDZIAŁU WYSTĄPIENIA USZKODZENIA

For the repairable multi-state component, reliability indexes are analyzed based on a homogenous Continuous Time Markov Chain (CTMC). If the component can work well when its repair time is sufficiently short, a threshold value for maintenance is introduced. When the fault interval is less than threshold time, the fault effect is considered neglected. In this paper, comparisons of availability show differences of the new model and the original model with or without fault interval omission. In addition, balancing the maintenance cost and lifetime of multi-state components is an important issue when threshold values are considered. Both constants and non-negative random variables are modeled respectively. Finally, numerical examples are presented to illustrate the results obtained in this paper.

Keywords: *multi-state systems (MSSs); fault effect omission; stochastic process; repairable model; maintenance cost rate.*

W przypadku naprawialnych elementów wielostanowych, wskaźniki niezawodności analizuje się w oparciu o łańcuch Markowa z czasem ciągłym. Jeśli element może działać prawidłowo, mimo uszkodzenia, dzięki wystarczająco krótkiemu czasowi naprawy, wprowadza się próg czasowy dla konserwacji. Gdy przedział czasu, w którym następuje uszkodzenie jest krótszy niż próg czasowy dla działań konserwacyjnych, wpływ uszkodzenia uważa się za nieistotny. Przeprowadzone w niniejszym artykule porównania gotowości wykazały różnice między nowym modelem a modelem oryginalnym z pominięciem lub bez pominięcia przedziału wystąpienia uszkodzenia. Ponadto, przy rozważaniu wartości progowych, ważną kwestią jest równoważenie kosztów utrzymania i żywotności elementów wielostanowych. W pracy próg wystąpienia uszkodzenia zamodelowano, odpowiednio, zarówno jako wartość stałą jak i nieujemną zmienną losową. Na koniec przedstawiono przykłady ilustrujące wyniki przedstawionych badań.

Słowa kluczowe: *systemy wielostanowe (MSS); pominięcie wpływu uszkodzenia; proces stochastyczny; model naprawialny; polityka utrzymania ruchu.*

1. Introduction

In practice many components and systems exhibit more than two output performances, these systems are called multi-state systems (MSSs) [6, 9, 10, 22]. Since the mid-1970s, numerous researches have been conducted which focus on MSS reliability [2]. Four commonly used approaches about MSS reliability have been formed gradually: the extension of Boolean model [26], stochastic process theory [18], universal generating function (UGF) technology [13,14] and Monte-Carlo simulation [23].

As to the stochastic process theory used in reliability analysis of MSSs, when the numbers of failures between arbitrary time intervals can be described as a Poisson process, Markov processes are often introduced to solve these questions [16, 20, 21]. When the operating time and repair time are non-exponentially distributed, a Semi-Markov process is often considered [7]. Besides Markov processes and Semi-Markov processes, the Wiener process [15, 19], the Gamma process [27] and the cumulative exposure process [10] are also considered in MSS reliability modelling. Research about MSS reliability has been a highlight topic in recent years and many new achievements are constantly emerging [17].

Studying on Markov repairable systems has always been an active branch in reliability theory. Jinhua Cao [11] studied the general model of Markov repairable systems, concluded the reliability analysis steps and deduced reliability indexes of voting systems, cold standby systems and warm standard systems. Cui et al. [5] proposed the definition of aggregated stochastic processes and applied into reliability analysis of repairable systems. Lisnianski [17] constructed a Markov reward model for reliability assessment of a multi-state system with variable demand. In his study, the process was assumed to be a homogenous Continuous Time Markov Chain (CTMC) with different possible states and corresponding transition possibility intensities. Other studies of reliability of multi-state systems using stochastic processes can be found in [25] and [29].

For a Markov repairable component containing N different output states, i.e., whose output performance is $G_1(t) > \cdots G_s(t) \geq w > G_{s+1}(t) > \cdots > G_N(t) = 0$, as $G_N(t) = 0$, so when output performance rate enters the $N-th$ state, fault occurs.

Maintenance is arranged immediately and after each repair, the component can return to the best state 1. When the component transfers from working states $1,2,\cdots,s$ to N and the repair time Y is less than a critical value τ, we consider the fault doesn't affect the output performance during the very short interval. For example, a daily water supply system exhibits plenty of output performances. If failure occurs and the system is repaired perfectly in a very short time interval, fault effect is neglected because the water reserved in the pipes is sufficient for the urban residents. That phenomenon is firstly noticed in [30] in 2006. Zheng et.al [30] studied a single-unit Markov repairable system with repair time omission and introduced a new stochastic process. By means of Ion-Channel theory [3, 4], she modeled the repairable system with repair time omission. Based on Zheng's research, other scholars expand her model to several components [1, 12, 18, 24, 28]. This paper tries to expand Zheng's conclusions to multiple states and will be more useful for actual multi-state repairable systems.

In this paper, we deduce reliability indexes according to the relationship between output performance and demand. Assuming that system performance keeps stationary when fault time is less than the threshold value, this paper builds a new stochastic process considering the neglected fault effect based on the original Markov process. Image in a power supply system, though output power capacity is lower than the demand, the system may not fail immediately due to some accumulators or external power sources. So we can think the system is still operating during the very short time. That is similar to the time-interval omission problem in Ion-Channel theory [3, 4].

In general, the organization of this paper is as follows. Section 2 deduces reliability indexes such as the instantaneous availability, steady-state availability, reliability and mean time to first failure (MTTFF) of the multi-state Markov repairable component. Section 3 builds a new stochastic process considering neglected fault effect and compares the change of reliability indexes. Both constants and nonnegative random variables of the fault threshold are modelled. In section 4, numerical examples are given to clarify the comparisons of two different stochastic processes. Finally, we get conclusions in this paper in section 5.

2. Multi-state repairable components

In a multi-state component containing $N(0<N<+\infty)$ states, output performance rate at time t is $\mathbf{G(t)}\in\{G_1(t),G_2(t),\cdots,G_N(t)\}$ and its corresponding state possibility is $\mathbf{P(t)}\in\{P_1(t),P_2(t),\cdots,P_N(t)\}$. When system demand $w(t)$ is a constant, i.e., $w(t)\equiv w$ [13]. Suppose $G_s(t)\ge w>G_{s+1}(t), N>s\ge 1$, Fig. 1 shows a possible behavior of MSS performance and demand as the realizations of a stochastic process. In this paper, the original system is referred as the old stochastic process while the new system considers neglected fault effect.

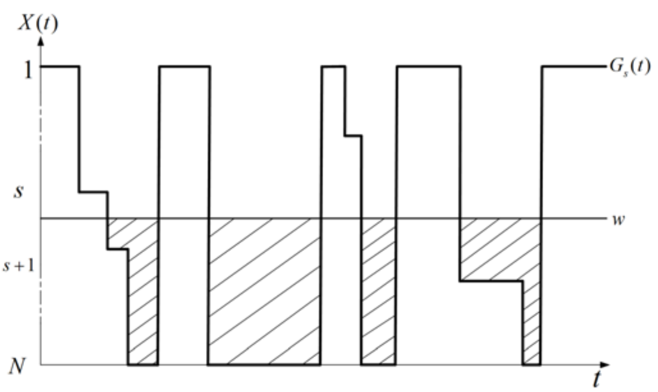

Fig. 1. A MSS behavior described with a stochastic process

2.1. Assumptions

Before we construct a Markov process for the multi-state repairable component, some proper assumptions should be given.

(1) Suppose $G_1(t)>\cdots G_s(t)>G_{s+1}(t)>\cdots>G_N(t)=0$, the component has two styles of failures including major failure and minor failure. When fault comes, repair is arranged immediately. The system can reach the best state 1 after each maintenance, which is shown in Fig. 2.

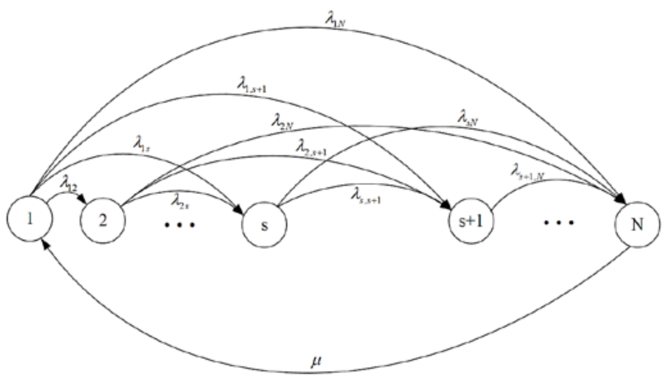

Fig. 2. State transmission process of the multi-state component

(2) The residence time at each state and the repair time have independent exponential distributions, i.e., $\lambda_{12},\lambda_{13},\cdots,\lambda_{N-1,N}$ and μ in Fig. 2 are constants and they are independent.

2.2. Reliability indexes

The possibility when the component is in state $j(j=1,2,\cdots,N)$ at time t is $P_j(t)$, let $P_j(t)=P(X(t)=j)$, then $\{X(t),t\ge 0\}$ is a homogenous Markov Chain based on the assumptions above and $P_{ij}(\Delta t)=P(X(t+\Delta t)=j|X(t)=i)$ is independent of t. Suppose the component is at its best output at $t=0$, that is, $P_1(0)=1,P_2(0)=0,\cdots,P_N(0)=0$. According to Chapman-Kolmogorov (C-K) equations [8], Eq.(1) is obtained:

$$\begin{cases}(P_1'(t),P_2'(t),\cdots,P_N'(t))=(P_1(t),P_2(t),\cdots,P_N(t))Q\\ P_1(0)=1,P_2(0)=0,\cdots,P_N(0)=0\end{cases} \tag{1}$$

In Eq. (1), Q is called the transition possibility intensity matrix and $Q=(q_{ij});i,j=1,2,\cdots,N$, where $q_{ij}=\lim\limits_{\Delta t\to 0}\dfrac{P_{ij}(\Delta t)}{\Delta t},i\ne j$. From Fig. 2, Q can be easily obtained:

$$Q=\begin{matrix}1\\2\\ \vdots\\ N-1\\ N\end{matrix}\begin{bmatrix}-\lambda_1 & \lambda_{12} & \cdots & \lambda_{1N}\\ 0 & -\lambda_2 & \cdots & \lambda_{2N}\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & \lambda_{N-1,N}\\ \mu & 0 & \cdots & -\mu\end{bmatrix}$$

where $\lambda_1=\sum\limits_{j=2}^{N}\lambda_{1j},\lambda_2=\sum\limits_{j=3}^{N}\lambda_{2j},\cdots,\lambda_{N-1}=\sum\limits_{j=N}^{N}\lambda_{N-1,j}=\lambda_{N-1,N}$.

Therefore, Eq.(1) becomes:

$$\begin{cases} P_1'(t) = -\sum_{j=2}^{N} \lambda_{1j} P_1(t) + \mu P_N(t) \\ P_2'(t) = \lambda_{12} P_1(t) - \sum_{j=3}^{N} \lambda_{2j} P_2(t) \\ \vdots \\ P_N'(t) = \lambda_{1N} P_1(t) + \cdots + \lambda_{N-1,N} P_{N-1}(t) - \mu P_N(t) \\ \sum_{j=1}^{N} P_j(t) = 1 \\ P_1(0) = 1, P_2(0) = 0, \cdots, P_N(0) = 0 \end{cases} \tag{2}$$

After Laplace transform and inverse Laplace transform, $P_1(t), P_2(t), \cdots, P_N(t)$ are obtained.

We use $A(t)$ to denote the instantaneous availability of the component at time t, then:

$$A(t) = \sum_{j=1}^{s} P_j(t) = \sum_{\mathbf{G(t)} \geq w} P_j(t) \tag{3}$$

Steady-state availability A represents the ratio whether output performance $\mathbf{G(t)}$ satisfies demand w after a long service time:

$$A = \lim_{t \to \infty} A(t) \tag{4}$$

Also, for the Markov process, we can get A from the following equations:

$$\begin{cases} (\pi_1, \pi_2, \cdots, \pi_N) Q = (0, 0, \cdots, 0) \\ \sum_{j=1}^{N} \pi_j = 1 \end{cases} \tag{5}$$

In which $\pi_1, \pi_2, \cdots, \pi_N$ is the limiting distribution (stationary distribution) of $P_1(t), P_2(t), \cdots, P_N(t)$. Thus:

$$A = \sum_{j=1}^{s} \pi_j \tag{6}$$

When reliability $R(t)$ of the multi-state component is required, we can introduce a new Markov process $\{\hat{X}(t), t \geq 0\}$. The state space $S = \{1, 2, \cdots, N\}$ can be divided into two parts: $S_a = \{1, 2, \cdots, s\}$, $S_b = \{s+1, s+2, \cdots, N\}$ and $S = S_a \cup S_b$. S_a is called an acceptable state subset while S_b an unacceptable state subset. Let $Q_j(t) = P(\hat{X}(t) = j), j = 1, 2, \cdots, s$, then we get a new C-K equation:

$$\begin{cases} (Q_1'(t), Q_2'(t), \cdots, Q_N'(t)) = (Q_1(t), Q_2(t), \cdots, Q_N(t)) \hat{Q} \\ Q_1(0) = 1, Q_2(0) = 0, \cdots, Q_N(0) = 0 \end{cases} \tag{7}$$

$\hat{Q}$ comes from Q and:

$$\hat{Q} = \begin{array}{c} 1 \\ \vdots \\ s \\ s+1 \\ \vdots \\ N \end{array} \left[\begin{array}{ccc|ccc} -\lambda_1 & \cdots & \lambda_{1s} & \lambda_{1,s+1} & \cdots & \lambda_{1N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & -\lambda_s & \lambda_{s,s+1} & \cdots & \lambda_{sN} \\ \hline & O & & & O & \end{array} \right]$$

It means states $s+1, s+2, \cdots, N$ are regarded as absorbing states, then we can get $Q_1(t), \cdots, Q_s(t)$ as:

$$\begin{cases} Q_1(t) = \exp(-\sum_{j=2}^{N} \lambda_{1j} t) \\ Q_2(t) = \dfrac{\lambda_{12} Q_1(t)}{\sum_{j=3}^{N} \lambda_{2j}} \left[1 - \exp(-\sum_{j=3}^{N} \lambda_{2j} t) \right] \\ \vdots \\ Q_s(t) = \dfrac{\lambda_{1s} Q_1(t) + \lambda_{2s} Q_2(t) + \cdots + \lambda_{s-1,s} Q_{s-1}(t)}{\sum_{j=s+1}^{N} \lambda_{sj}} \left[1 - \exp(-\sum_{j=s+1}^{N} \lambda_{sj} t) \right] \end{cases} \tag{8}$$

So $R(t)$ is obtained as:

$$R(t) = \sum_{j=1}^{s} Q_j(t) \tag{9}$$

Mean time to first failure (MTTFF) is:

$$MTTFF = \int_0^{\infty} R(t) dt = \sum_{j=1}^{s} \int_0^{\infty} Q_j(t) dt \tag{10}$$

3. Reliability with neglected fault effect

In this paper, two situations are considered for the threshold τ : τ is a constant and τ is a non-negative random variable with its distribution function $H(\tau)$.

3.1. The threshold is a constant

Here we introduce a new stochastic process $\{\tilde{X}(t), t \geq 0\}$ when considering neglected fault effect and:

$$\tilde{X}(t) = \begin{cases} 1, & when\ the\ system\ is\ up \\ 0, & when\ the\ system\ is\ down \end{cases} \tag{11}$$

Obviously, the new stochastic process $\{\tilde{X}(t), t \geq 0\}$ has a tight connection with $\{X(t), t \geq 0\}$ but it is not a Markov process any more. Fig. 3 shows the relationship between the two stochastic processes $\{X(t), t \geq 0\}$ and $\{X(t), t \geq 0\}$.

For $\{\tilde{X}(t), t \geq 0\}$, the instantaneous availability becomes:

$$\begin{aligned}\tilde{A}(t) &= P(the\ \ system\ \ is\ \ operating\ \ at\ \ time\ \ t)\\ &= P(\tilde{X}(t)=1)\\ &= P(\tilde{X}(t)=1, X(t)=1)+\cdots+P(\tilde{X}(t)=1, X(t)=N)\\ &= \sum_{i=1}^{N} P(\tilde{X}(t)=1, X(t)=i)\end{aligned} \tag{12}$$

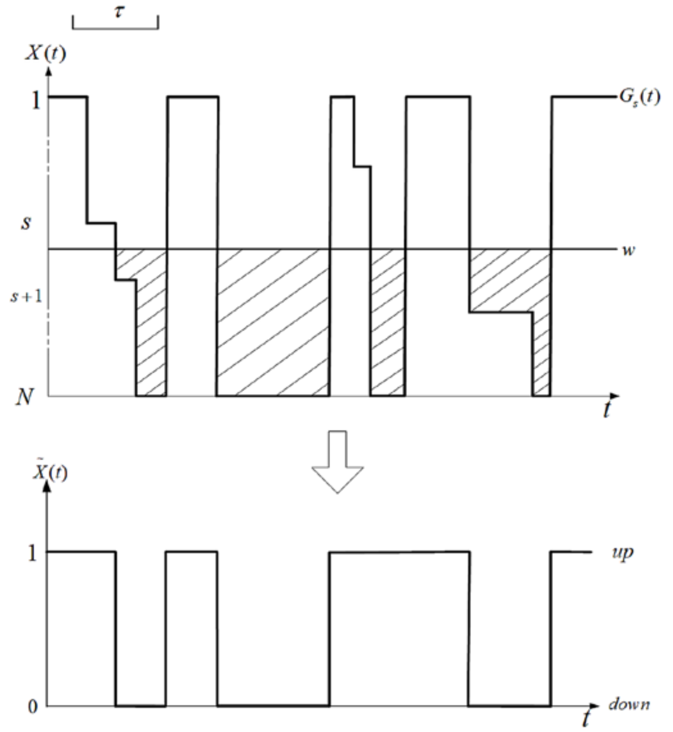

Fig. 3. Relationship between the two stochastic processes

From Fig. 3, when the original system is at state $i_1(i_1=1,2,\cdots,s)$, the new system is always operating. When the original system is at state $i_2(i_2=s+1,s+2,\cdots,N-1)$, the new system is always down. When the system transfers from state $i_3(i_3=1,2,\cdots,N-1)$ to state N, the system can works well as long as the repair time is less than fault threshold τ. Therefore, Eq.(12) becomes:

$$\begin{aligned}\tilde{A}(t) &= \sum_{i=1}^{N} P(\tilde{X}(t)=1, X(t)=i)\\ &= \sum_{i_1=1}^{s} P(\tilde{X}(t)=1, X(t)=i_1)+\sum_{i_2=s+1}^{N-1} P(\tilde{X}(t)=1, X(t)=i_2)+P(\tilde{X}(t)=1, X(t)=N)\\ &= \sum_{i_1=1}^{s}[P(\tilde{X}(t)=1 \mid X(t)=i_1)P(X(t)=i_1)]+\sum_{i_2=s+1}^{N-1}[P(\tilde{X}(t)=1 \mid X(t)=i_2)P(X(t)=i_2)]\\ &\quad +P(\tilde{X}(t)=1, X(t)=N)\\ &= \sum_{i_1=1}^{s} P(X(t)=i_1)+P(\tilde{X}(t)=1, X(t)=N)\end{aligned} \tag{13}$$

The 1st part of the last equation in Eq.(13) $\sum_{i_1=1}^{s} P(X(t)=i_1)$ is $A(t)$ in Eq.(2). So Eq.(13) becomes:

$$\tilde{A}(t) = A(t)+P(\tilde{X}(t)=1, X(t)=N) \tag{14}$$

As for $P(\tilde{X}(t)=1, X(t)=N)$, it represents that the original system is under repair while the repair time is less than the threshold τ, as shown in Fig.4. For the original system, $P(\tilde{X}(t)=1, X(t)=N)$ is associated tightly with the nearest state before N. Here we use $p_{j\to N}$ to represent the transmission possibility of state $j(j=1,2,\cdots,N-1)$ to state N.

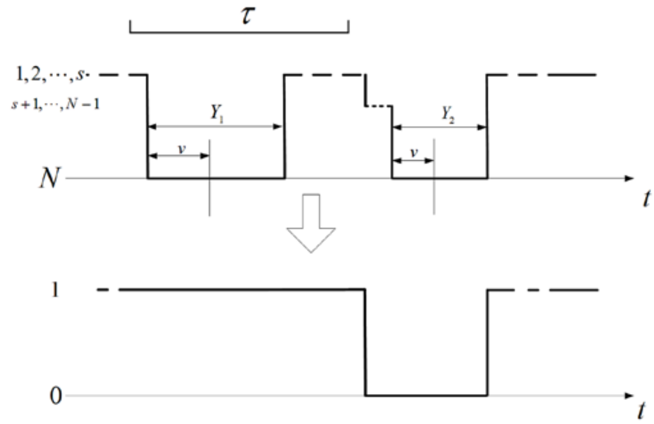

Fig. 4. The original system is under repair and the repair time is less than τ

From Fig. 4, if the nearest state before N is $i_1(i_1=1,2,\cdots,s)$, as $G_1(t)>G_2(t)>\cdots>G_s(t)\geq w$, so when the repair time is no longer than τ, the fault is neglected. If the nearest state before N is $i_2(i_2=s+1,s+2,\cdots,N-1)$, as $w>G_{s+1}(t)>G_{s+2}(t)>\cdots>G_{N-1}(t)$, so no matter how short τ is, the repair time cannot be neglected.

$$\begin{aligned}&P(\tilde{X}(t)=1, X(t)=N)\\ &= p_{1\to N}\int_0^{\min(t,\tau)} P(X(t-v)=1)P(v<Y<\tau)\lambda_{1N}dv+\cdots\\ &\quad + p_{N-1\to N}\int_0^{\min(t,\tau)} P(X(t-v)=N-1)P(v<Y<\tau)\lambda_{N-1,N}dv\\ &= \frac{\lambda_{1N}}{\lambda_{1N}+\cdots+\lambda_{N-1,N}}\int_0^{\min(t,\tau)} P(X(t-v)=1)P(v<Y<\tau)\lambda_{1N}dv+\cdots\\ &\quad + \frac{\lambda_{sN}}{\lambda_{1N}+\cdots+\lambda_{N-1,N}}\int_0^{\min(t,\tau)} P(X(t-v)=s)P(v<Y<\tau)\lambda_{sN}dv\\ &= \frac{1}{\sum_{j=1}^{N-1}\lambda_{jN}}\left[\sum_{i=1}^{s}\int_0^{\min(t,\tau)} P_i(t-v)P(v<Y<\tau)\lambda_{iN}^2 dv\right]\end{aligned} \tag{15}$$

In which Y is the repair time and it's exponentially distributed with $Y \sim G(t)=1-e^{-\mu t}$, so $P(v<Y<\tau)=e^{-\mu v}-e^{-\mu\tau}$. The instantaneous availability $\tilde{A}(t)$ becomes:

$$\tilde{A}(t) = A(t)+\frac{1}{\sum_{j=1}^{N-1}\lambda_{jN}}\left[\sum_{i=1}^{s}\int_0^{\min(t,\tau)} P_i(t-v)P(v<Y<\tau)\lambda_{iN}^2 dv\right] \tag{16}$$

The steady-state availability $\tilde{A}$ can be deduced when $t\to\infty$, and obviously $\min(t,\tau)=\tau$ at this time.

3.2. The threshold is a random variable

When τ is a nonnegative random variable with its distribution function is $H(\tau)$, Eq.(15) becomes:

$$
\begin{aligned}
&P(\tilde{X}(t)=1, X(t)=N)\\
&= p_{1\to N}\int_0^\infty\int_0^{\min(t,\tau)} P(X(t-v)=1)P(v<Y<\tau)\lambda_{1N}dvdH(\tau)+\cdots\\
&\quad + p_{N-1\to N}\int_0^\infty\int_0^{\min(t,\tau)} P(X(t-v)=N-1)P(v<Y<\tau)\lambda_{N-1,N}dvdH(\tau)\\
&= \frac{\lambda_{1N}}{\lambda_{1N}+\cdots+\lambda_{N-1,N}}\int_0^\infty\int_0^{\min(t,\tau)} P(X(t-v)=1)P(v<Y<\tau)\lambda_{1N}dvdH(\tau)+\cdots\\
&\quad + \frac{\lambda_{sN}}{\lambda_{1N}+\cdots+\lambda_{N-1,N}}\int_0^\infty\int_0^{\min(t,\tau)} P(X(t-v)=s)P(v<Y<\tau)\lambda_{sN}dvdH(\tau)\\
&= \frac{1}{\sum_{j=1}^{N-1}\lambda_{jN}}\left[\sum_{i=1}^{s}\int_0^\infty\int_0^{\min(t,\tau)} P_i(t-v)P(v<Y<\tau)\lambda_{iN}^2 dvdH(\tau)\right] \qquad (17)
\end{aligned}
$$

And $\tilde{A}(t)$ is:

$$
\tilde{A}(t) = A(t) + \frac{1}{\sum_{j=1}^{N-1}\lambda_{jN}}\left[\sum_{i=1}^{s}\int_0^\infty\int_0^{\min(t,\tau)} P_i(t-v)P(v<Y<\tau)\lambda_{iN}^2 dvdH(\tau)\right] \qquad (18)
$$

The steady-state availability becomes:

$$
\tilde{A} = \lim_{t\to\infty}\tilde{A}(t) \qquad (19)
$$

From above equations, when neglected fault effect is considered for a multi-state repairable component, availabilities (instantaneous availability and steady-state availability) are obviously higher than the ones in the original system. It is rational for certain conditions. Consider a flow transmission system which transfers liquid through pipes. For such a multi-state system, even when the system reaches a complete failure state, as long as the failure time is short enough to affect the output performance, the flow reserved in the pipes can satisfy the demand and at this very moment we regard the repair time ignorable or the fault effect neglected. So the system can work more hours than before when fault interval omission is considered.

3.3. Optimum maintenance cost rate

When fault interval omission of multi-state components is considered, different maintenance thresholds $\tau_1,\tau_2,\cdots,\tau_n$ are introduced. Under various $\tau_1,\tau_2,\cdots,\tau_n$, the lifetime of multi-state components is $T_1,T_2,\cdots,T_n$ respectively. As τ grows, though the lifetime T prolongs, the maintenance cost increases accordingly when fault incurs. Therefore, how to optimize the maintenance cost under different lifetime $T_1,T_2,\cdots,T_n$ with respect to thresholds $\tau_1,\tau_2,\cdots,\tau_n$ becomes significant in reliability engineering.

Suppose the total maintenance cost $C(t)$ of a multi-state component contains replacement cost $c_f(t)$ and preventive cost $c_p(t)$, depreciation rate of the component is $\alpha(\alpha>0)$. At the initial moment $c_f(0)=c_f$, then total maintenance cost $C(t)$ becomes:

$$
C(t) = c_f e^{-\alpha t} + c_p(e^{\alpha t}-1) \qquad (20)
$$

During the whole life cycle $(0,T]$ of the multi-state component, the tendency of maintenance cost $C(t)$ changes with time $t\in(0,T]$, as shown in Fig. 5.

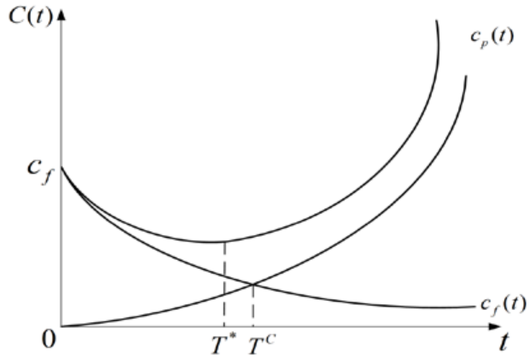

Fig. 5. Tendency of maintenance cost C(t) with t

The approach to get the best maintenance moment T^* is to differential $C(t)$ with respect to t, i.e.,

$$
T^* = (1/2\alpha)\ln(c_f/c_p) \qquad (21)
$$

Then according to the best T^*, the best maintenance threshold τ^* is obtained.

4. Illustrative examples

To illustrate the results obtained above in this paper, we consider a power generation system which contains four different output performance levels. Its output performance **G(t)** (s^{-1}) can be denoted as the generating capacity and $G_1(t)=100, G_2(t)=80, G_3(t)=60, G_4(t)=0$. Obviously, the power generator is a multi-state unit and state 1 shows the perfect output while state 4 is the complete failure state. After statistical analysis, the failure rates ($year^{-1}$) are:

$$
\lambda_{12}=1, \lambda_{13}=2, \lambda_{14}=1, \lambda_{23}=1, \lambda_{24}=2, \lambda_{34}=1
$$

Assume that repair is implemented immediately when the system reaches state 4 and the repair rate $\mu = 6year^{-1}$. Demand of the system is $w=50s^{-1}$ and the system is in the best state 1 at beginning. Let $P_j(t)=P\{X(t)=j\}, j=1,2,3,4$, and obviously $\{X(t),t\ge 0\}$ is a homogenous CTMC. Probability intensity matrix of $\{X(t),t\ge 0\}$ is:

$$
Q = \begin{bmatrix} -4 & 1 & 2 & 1 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & -1 & 1 \\ 6 & 0 & 0 & -6 \end{bmatrix}
$$

According to Eq.(1), $P_1(t) = 0.231 + 0.651e^{-4.732t} + 0.118e^{-2.268t}$,

$P_2(t) = 0.077 - 0.238e^{-4.732t} + 0.161e^{-2.268t}$,

$P_3(t) = 0.538 - 0.225e^{-4.732t} - 0.314e^{-2.268t}$,

$P_4(t) = 0.154 - 0.188e^{-4.732t} + 0.034e^{-2.268t}$.

For the original Markov process, according to Eq. **(3)**, instantaneous availability $A(t)$ is:

$$A(t) = \sum_{\mathbf{G(t)} \geq w} p_i(t) = p_1(t) + p_2(t) + p_3(t)$$
$$= 0.846 + 0.188e^{-4.732t} - 0.034e^{-2.268t}$$

Also we can get the steady-state availability A when $t \to \infty$, i.e., $A = \lim_{t\to\infty} A(t) = 0.846$. As for the reliability $R(t)$ of the original system, according to Eqs. (7) and (8), a new stochastic process $\{\hat{X}(t), t \geq 0\}$ can be defined and:

$$R(t) = 0.833e^{-t} + 0.5e^{-3t} - 0.333e^{-4t}$$

which is shown as Fig. 6.

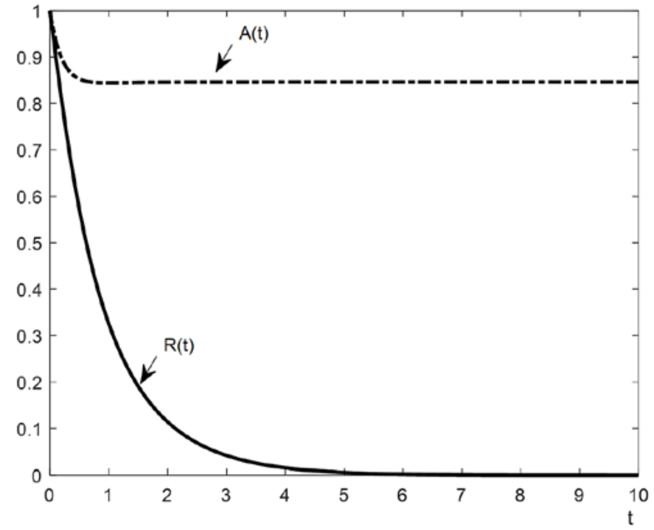

Fig. 6. Availability and Reliability of the multi-state component

From Eq. (10), MTTFF of the multi-state component is:

$$MTTFF = \int_0^{\infty} (0.833e^{-t} + 0.5e^{-3t} - 0.333e^{-4t})dt = 0.916year \approx 330days$$

When the failure time is too short to be detected or the fault will not affect the component's output performance, we can set a threshold value τ for the maintenance. In fact, this phenomenon is rather common in fault-tolerant design. When we use a program or software in a personal computer, we may endure "program nonresponse" for several seconds (similar to τ). As long as the program or software works well during τ , the short fault time can be ignored and does not affect the PC's performance. Here a new stochastic process $\{\tilde{X}(t), t \geq 0\}$ which contains only two states is introduced and first we consider τ a constant 0.6.

From Eqs.(15) and (16), the instantaneous availability of $\{\tilde{X}(t), t \geq 0\}$ is:

$$\tilde{A}(t) = A(t) + P(\tilde{X}(t) = 1, X(t) = 4) = 0.9805 + 0.1716e^{-4.732t} - 0.0082e^{-2.268t}$$

Obviously, when $t \to \infty$, t is always larger than τ , so $\tilde{A} = \lim_{t\to\infty} \tilde{A}(t) = 0.9805$. Fig. 7 shows the change of instantaneous availabilities of two conditions when $\tau = 0.6$.

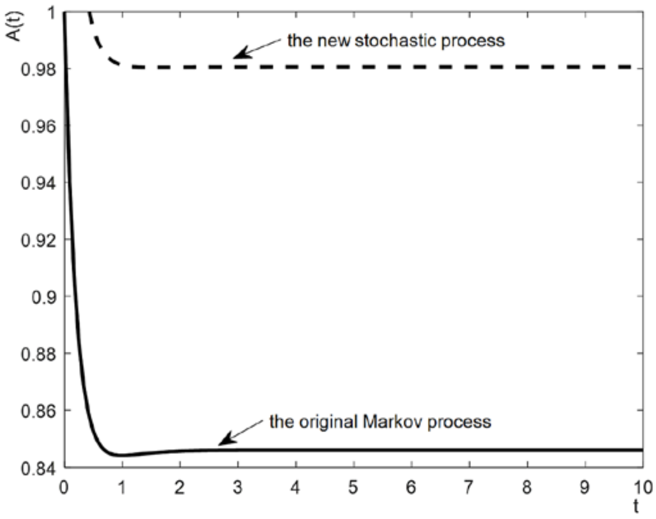

Fig. 7. Change of instantaneous availability when $\tau = 0.6$

In addition, when we have different variable τ , availability of the new system will definitely not be the same. Fig. 8 shows the change of availabilities with different maintenance threshold values with respect to $\tau = 0, 0.2, 0.4, 0.6$.

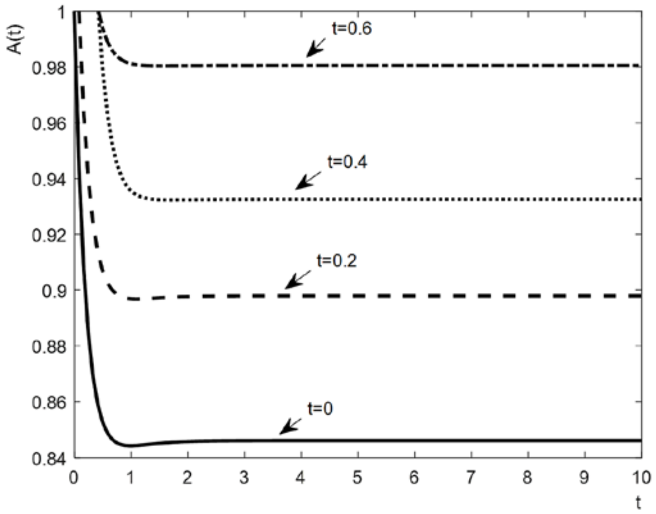

Fig. 8. Change of instantaneous availability with $\tau = 0.6$, 0.4, 0.2, 0

From Fig.8, the higher τ is, the larger the steady-state availability $\tilde{A}$ will be. When $\tau = 0$, the new system is equal to the original system and the new stochastic process is the old Markov process itself. When $\tau \to \infty$, the system will never fail as long as its output performance is larger than demand.

Then we consider the failure threshold τ is a random variable with a Gamma distribution. Suppose the density function of τ is $f(\tau) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\lambda\tau}$ and $\alpha = 1, \lambda = 2$, $dH(\tau) = 2e^{-2\tau} d\tau$. According to Eqs.(17) and (18), $\tilde{A}(t)$ becomes:

$$\tilde{A}(t) = A(t) + P(\tilde{X}(t) = 1, X(t) = 4)$$
$$= 0.9325 - 0.0688e^{-16.732t} + 0.065e^{-12t} - 0.097e^{-10.268t} + 0.0038e^{-8.732t} - 1.0722e^{-8t}$$
$$+ 1.1057e^{-6.732t} + 0.1727e^{-4.732t} + 0.1683e^{-4.628t} - 0.0175e^{-2.268t}$$

Fig.9 shows the change of availability when τ obeys a Gamma distribution $Ga(1,2)$. The steady-state availability is $\tilde{A} = 0.9325$.

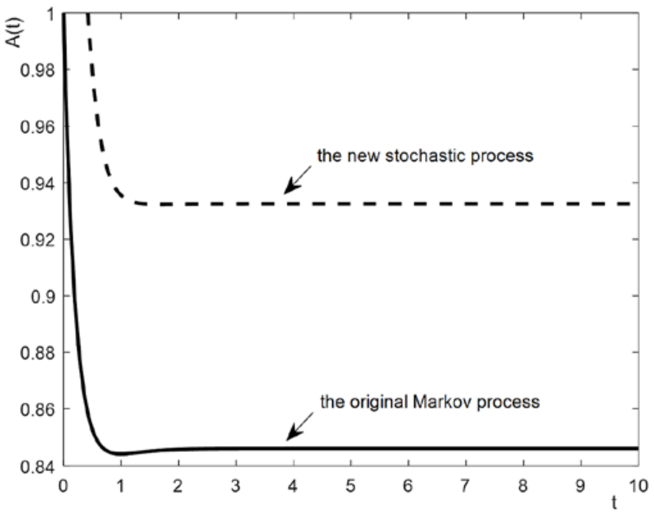

Fig. 9. The curve of availability when $\tau \sim Ga(1,2)$

From Fig. 9, we can see that the steady-state availability becomes 0.9325 compared with the original steady-state availability 0.846. The rational explanation is that we ignore some fault effect when the repair time is less than τ. As a matter of fact, steady-state availability reflects the working proportion of the power generator. Fig. 10 shows the change of steady-state availabilities when fault interval omission is considered or not.

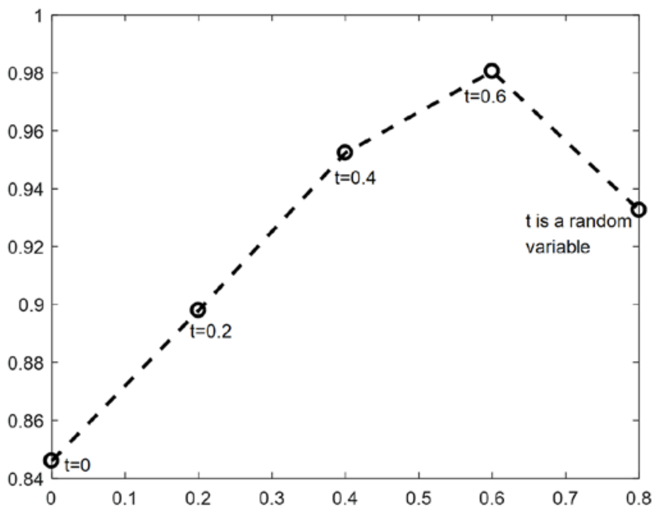

Fig. 10. Steady-state availabilities when fault interval is considered

Next we'll consider the effect on maintenance cost $C(t)$ of fault interval τ, including constants $0, 0.2, 0.4, 0.6$ and random variable with distribution function $H(\tau)$. Suppose the initial replacement cost of the power generation system $c_f = £100$ and the initial preventive cost $c_p = £60$, according to Eq.(18), $C(t) = 100e^{-\alpha t} + 60(e^{\alpha t} - 1)$.

Depreciation rate $\alpha(\alpha > 0)$ is set by reliability engineers and after a serious evaluation of the power generator, $\alpha = 8 \times 10^{-4}$ is suggested. According to Eq.(19), the best maintenance moment $T^* = (1/2\alpha)\ln(c_f / c_p) = 319.27$ days. From Fig. 10, when the fault interval $\tau = 0$, $T_0 = 304.56$ days. As τ grows from $\tau_1 = 0.2$ to $\tau_2 = 0.4$ and $\tau_3 = 0.6$, lifetime of the power generator increases from $T_1 = 323.24$ days to $T_2 = 342.87$ days and $T_3 = 352.98$ days. Similarly, maintenance cost rises from $C(T_1) = £94.92$ to $C(T_2) = £94.947$ and $C(T_3) = £94.976$.

On one aspect, the lifetime of the power generator can prolong as τ grows, while the maintenance cost rises correspondingly. Choosing an appropriate threshold value τ for the fault interval can not only extend the equipment's lifetime but also manage the maintenance cost. In this illustrative example, the best threshold interval τ^* should be $0 < \tau^* < 0.2$ and that parameter is of vital importance to make proper maintenance policies.

5. Conclusions

In this paper, we build a Markov process for the multi-state repairable component which contains N output performances based on a homogenous CTMC. Under the assumption that residence time and repair time are exponentially distributed, Kolmogorov equations are built. Based on the possibility of each state of the multi-state component, availability, reliability and mean time to first failure are deduced.

When the fault time is too short to be detected, a new stochastic process considering neglected fault effect is determined. Though it is associated with the original system tightly, it is not a Markov process any more. When the threshold failure time is a constant or a random variable, we compare the change of instantaneous availability. Numerical examples show that the availability will be larger when repair time omission is considered. At the same time, when maintenance cost is introduced, the best policy of choosing an appropriate threshold is to balance the maintenance cost and the lifetime.

Relevant results can also be used in queuing theory and management science. For example, in a queuing theory problem, whether a customer leaves or not depends on the tolerant interval one can accept. If the endurable time is extremely long, no matter how many people are queuing before him or her, one will always wait for his or her service. At the same time, with the increasing of state numbers, state exploration will definitely come up. Markov method may have a problem in solving differential functions and universal generating function (UGF) technology [13] can be considered. Therefore, the future emphasis is on the mixture of Markov process and UGF with neglected fault effect or delayed failures.

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