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Brief note

INFLUENCE OF HEAT SOURCES AND RELAXATION TIME ON TEMPERATURE DISTRIBUTION IN TISSUES

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In the present study, the temperature fluctuations in tissues based on Penne's bio-heat transfer equation is investigated by applying the Laplace and Hankel transforms. To get the solution in a physical form, a numerical inversion technique has been applied. The temporal and spatial distribution of temperature is investigated with the effect of relaxation time and is presented graphically.

Key words: Penne's bio-heat equation, relaxation time, heat source, Laplace transform, Hankel transform.

1. Introduction

It is essential to know the heat transfer in biological systems as they are of relevant importance in many diagnostic and therapeutic applications that involve changes in the temperature. The tissues are exposed to an electromagnetic or ultrasound energy source for a period of time, and the temperature is measured at various points away from the applicator by thermal probes such as thermistors or thermocouples.

The method for studying temperature distribution so far has been mostly experimental. Different mathematical methods may be applied for the computational study of temperature distributions in the biomass irradiated by a source of electromagnetic radiation. Carslaw and Jaeger (1959), and Lienhard (1987) focused on the heat transport analysis and numerical method. Riu *et al*. (1997), Bowman *et al*. (1975) and Martin *et al*. (1992) discussed the temperature rise for constant perfusion by an analytic method and finite element method based on the bio-heat transfer equation.

Arkin and Holmes (1994) used the bio-heat transfer equation to discuss heat transport in blood perfusion tissue. Erdmann *et al*. (1998) applied the finite element method to optimize the nonlinear bio heat transfer equation for optimizing regional hyperthermia, whereas Yreus and Diederich (2002) studied a two dimensional bio-thermal model of ultrasound applicators based on the bio heat transfer equation. Diller (1998; 1999) Jiang *et al*. (2002), Chan (1992) and Mochnacki and Majchrzak (2003) applied the boundary element and finite difference method to solve bio-heat equations. Recently, Othman *et al*. (2011) applied the normal mode analysis for characterizing the temperature fluctuation in tissues based on Penne's bio-heat transfer equation.

In this paper, the Laplace and Hankel transforms are employed to solve the bio-heat transfer equation analytically. The soft tissue is considered as a viscoelastic medium and the relaxation time has been used in the bio-heat transfer equation. The effect of heat sources and relaxation time on temperature distribution is studied.

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2. Bio-heat transfer equation

The temperature evaluation in biological tissues can be modeled with Penne's bio-heat equation, which is

$$\rho C_E\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t}=k\nabla^2\theta-\omega_b C_b\rho_b\left(\theta-\theta_b\right)+\left(1+\tau\frac{\partial}{\partial t}\right)Q \tag{2.1}$$

where θ is the temperature, ρ is the density of tissue, C_E is the heat capacity of tissue, k is the diffusion due to blood flow, ω_b is the perfusion due to blood flow, C_b is the heat capacity of blood, ρ_b is the density of blood, T_b is the arterial blood temperature, Q is the absorbed power density and τ is the relaxation time, ∇^2 is the Laplacian operator.

The following dimensionless parameters are introduced as

$$\theta=\frac{\left(\theta-\theta_b\right)a_t}{L^2/k},\qquad x'=\frac{x}{L},\qquad y'=\frac{y}{L},\qquad t'=\frac{\alpha_t t}{L^2},\qquad p_f=\frac{\omega_b C_b L^2}{k},$$

$$\tau'=\frac{a\tau}{L^2},\qquad z'=\frac{z}{L},\qquad a_t=\frac{k}{\rho C_E},\qquad Q=a_t Q \tag{2.2}$$

where L is the tissue length.

With the help of dimensionless quantities defined by Eq.(2.2), Eq.(2.1) yields,

$$\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t}=\nabla^2\theta-p_f\theta+\left(1+\tau\frac{\partial}{\partial t}\right)Q. \tag{2.3}$$

Applying the Laplace and Hankel transforms defined by

$$\hat{f}\left(r,z,s\right)=\int_0^\infty f\left(r,z,t\right)e^{-st}dt, \tag{2.4}$$

$$\tilde{f}\left(\xi,z,s\right)=\int_0^\infty \hat{f}\left(r,z,s\right)rJ_n\left(r\xi\right)dr, \tag{2.5}$$

to Eq.(3.3) yields

$$\left[\frac{d^2}{dz^2}-A\right]\tilde{\theta}=-\left(1+\tau s\right)\tilde{Q} \tag{2.6}$$

where

$$A=\xi^2+s\left(1+s\tau\right)+p_f.$$

The roots of Eq.(2.6) are $\pm m$ and making use of radiation conditions that $\tilde{\theta}\to 0$ as $z\to\infty$, the solution of Eq.(2.6) can be written as

$$\tilde{\theta}=A_1 e^{-mz}+\frac{\left(1+s\tau\right)}{m^2}Q. \tag{2.7}$$

3. Boundary conditions

Two types of boundaries are considered:

Case (I): Concentrated heat source

$$Q = Q_0 \frac{\delta(r)\delta(t)}{2\pi r} \qquad \text{at} \qquad z = 0 \tag{3.1}$$

where Q_0 is the constant temperature applied on the boundary, $\delta(\ )$ is the Dirac delta function. Applying the Laplace and Hankel transforms defined by Eqs (2.4) and (2.5) on the boundary condition Eq.(3.1) and with the help of (2.7), we obtain

$$\tilde{\theta} = \frac{1}{2\pi}\left[\left(1 - \frac{(1+s\tau)}{m^2}Q_0\right)e^{-mz} + \frac{(1+s\tau)}{m^2}Q_0\right]. \tag{3.2}$$

Case (II): Continuous heat source

$$Q = Q_0 \frac{\delta(r)H(t)}{2\pi r} \qquad \text{at} \qquad z = 0 \tag{3.3}$$

where $H(\ )$ is the Heaviside unit step function. Applying the Laplace and Hankel transforms defined by Eqs (2.4) and (2.5) on the boundary condition Eq.(3.3) and with the help of (2.7), we obtain

$$\tilde{\theta} = \frac{1}{2\pi s}\left[\left(1 - \frac{(1+s\tau)}{m^2}Q_0\right)e^{-mz} + \frac{(1+s\tau)}{m^2}Q_0\right]. \tag{3.4}$$

4. Inversion of the transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in Eqs (3.2) and (3.4). These expressions are functions of z, the parameters of Hankel transforms s and ξ respectively and hence are of the form $\tilde{\theta}(\xi, z, s)$. To get the function $\theta(r, z, t)$ in the physical domain, first we invert the Hankel transform using

$$\tilde{\theta}(r, z, s) = \int_0^\infty \xi \tilde{\theta}(\xi, z, s) J_n(r\xi) d\xi. \tag{3.5}$$

The last step is to calculate the integral in Eq.(3.5). The method for evaluating this integral as described in Press *et al*. (1992), involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

5. Numerical discussion

In order to illustrate theoretical results obtained in the proceeding section, we now present some numerical results as follows

$$w_b = 0.5\ Kg/m^3 s\ , \quad C_b = 4200\ J/Kg\ ^oC\ , \quad k = 0.5\ W/m\ ^oC \quad \text{and} \quad \tau = 0.02\ .$$

CASE I: Concentrated heat source

It is observed from Fig.1, which is a plot for temperature distribution at different relaxation times that tissue temperature decreases in the initial range, the magnitude of values is greater in the case of $\tau = 0.07$. As the distance x increases, the values of temperature distribution show small variations about zero value.

Figure 2 shows the influence of the thermal conductivity on the tissue temperature. It is observed that initially the tissue temperature decreases with greater magnitude as thermal conductivity decreases. Further, it is noticed that if x increases, the tissue temperature approaches a zero value.

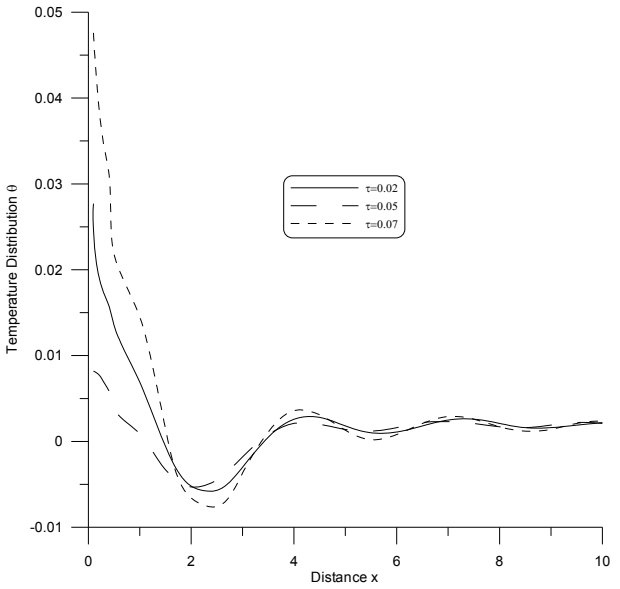

Fig.1. Temperature distribution as a function of different relaxation times ($w_b=0.5$, $C_b=4200$, $k=0.5$).

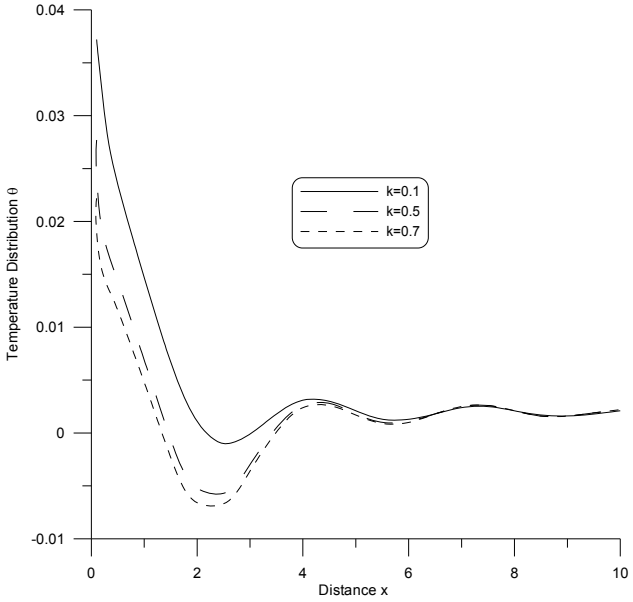

Fig.2. Temperature distribution as a function of different thermal conductivity ($w_b=0.5$, $C_b=4200$, $\tau=0.02$).

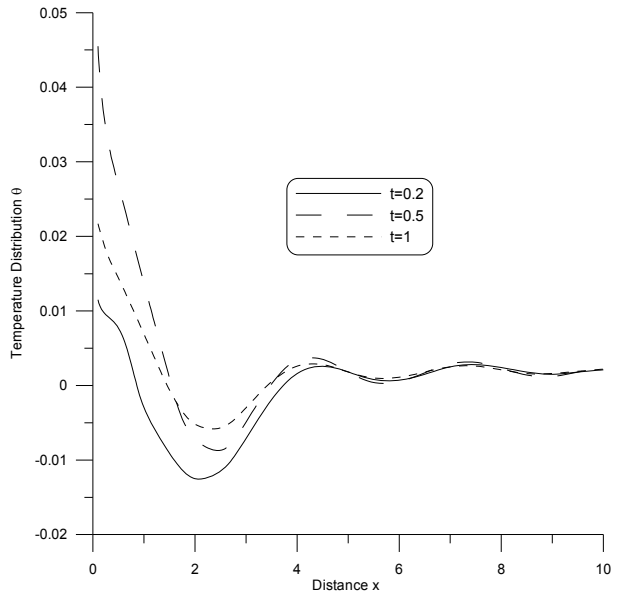

Fig.3. Temperature distribution as a function of blood perfusion ($k=0.5$, $C_b=4200$, $\tau=0.02$).

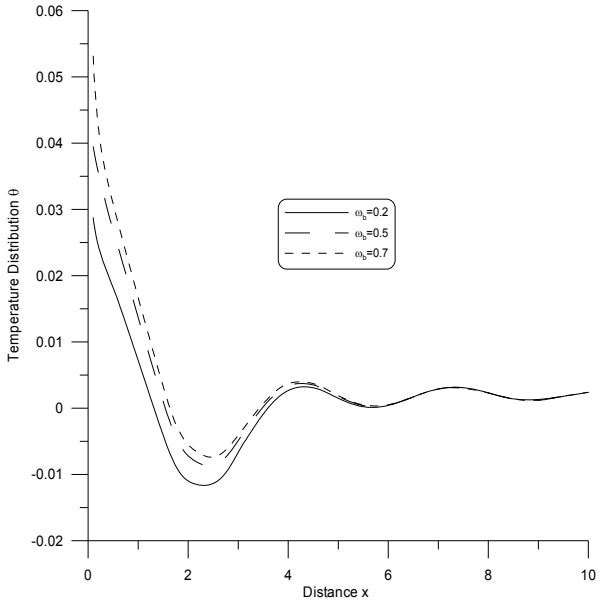

Fig.4. Temperature distribution as a function of time ($w_b=0.5$, $k=0.5$, $C_b=4200$, $\tau=0.02$).

Concentrated heat source

Every person has a varied blood perfusion at various health conditions and hence from Fig.3, which is a plot of the tissue temperature for different blood perfusion values it can be noticed that the greater the blood perfusion, the more the biological body tends to prevent burn injuries.

Figure 4 depicts the tissue temperature at different time, it is noticed that at the early stage of heating the values of temperature distribution decrease with a greater magnitude and as x incresease the tissue temperature approaches a zero value.

CASE II: Continuous heat source

Figure 5 shows the influence of relaxation time on the tissue temperature, it is noticed that as τ increases the tissue temperature decreases with a greater magnitude in the intial range and as x increases the impact of different relaxation time is almost similar.

Figure 6 depicts the impact of different thermal conducitvity, it is noticed that trends are similar as observed for Fig.2 with a significant difference in the magnitude of their values.

It is observed from Fig.7 that the values of tissue temperature decrease with a greater magnitude as blood perfusion increases and with a further increase in x the impact of various blood perfusions is much smaller.

It is noticed from Fig.8 that near the loading surface, the values of tissue temperature decrease with a greater magnitude for small values of time and as x increases the values of tissue temperature approach to a zero value with an oscillatory pattern.

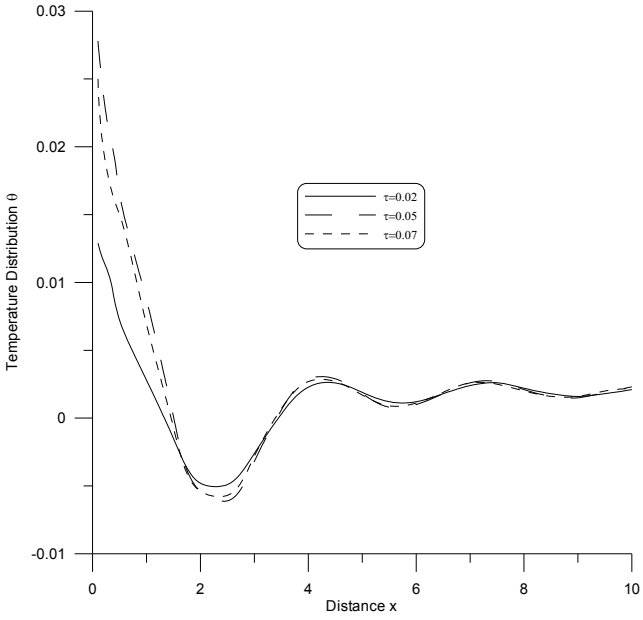

Fig.5. Temperature distribution as a function of relaxation times (w_b=*0.5*, C_b=*4200*, *k=0.5*).

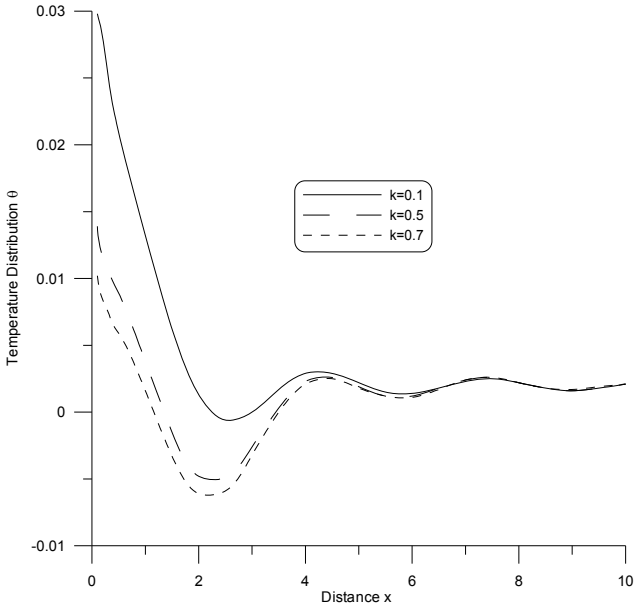

Fig.6. Temperature distribution as a function different of different thermal conductivity (w_b=*0.5*, C_b=*4200*, τ=*0.02*).

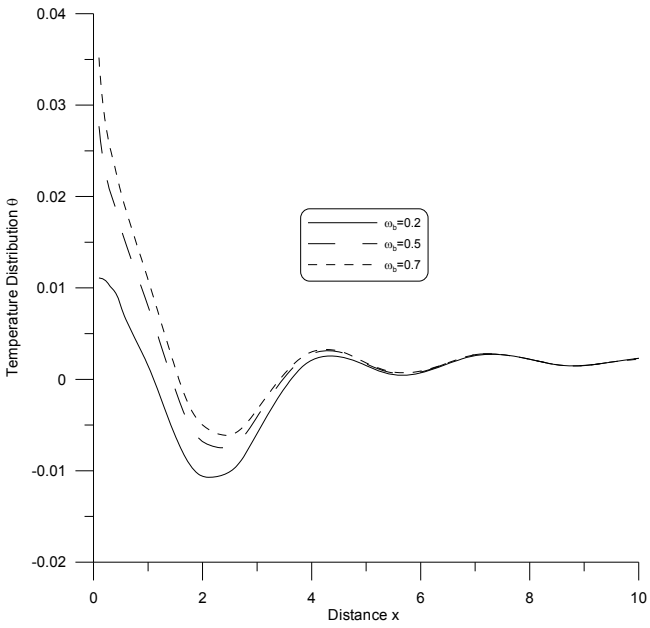

Fig.7. Temperature distribution as a function of blood perfusion ($k=0.5$, $C_b=4200$, $\tau=0.02$).

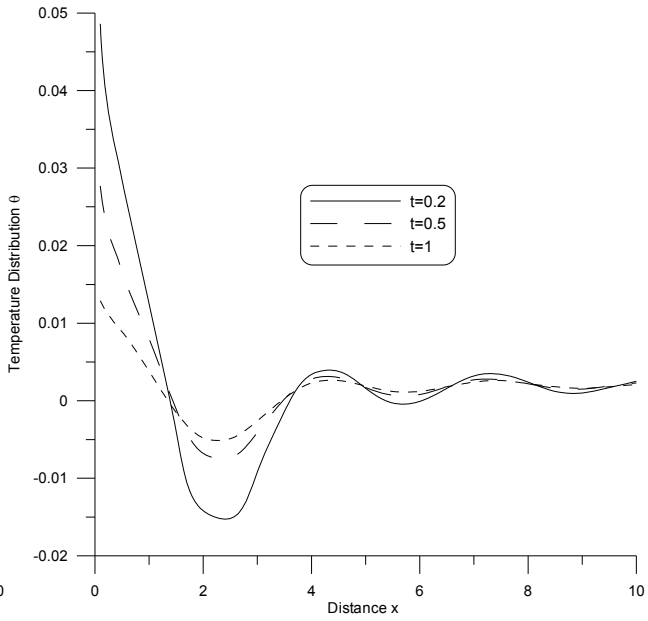

Fig.8. Temperature distribution as a function of time ($w_b=0.5$, $k=0.5$, $C_b=4200$, $\tau=0.02$).

Continuous heat source

Nomenclature

C_b – heat capacity of blood
C_E – heat capacity of tissue
k – diffusion due to blood flow
Q – absorbed power density
T_b – arterial blood temperature
θ – temperature
ρ – density of tissue
ρ_b – density of blood
τ – relaxation time
ω_b – perfusion due to blood flow
∇^2 – Laplacian operator

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ERRATUM

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Unfortunately, one of the co-author's affiliation was incorrectly published in the original publication.

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