Basic equations for analysis of a three-phase three-winding transformer subjected to asymmetric load

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The paper presents a set of equations enabling analysis of operation of three-phase three-winding transformers under asymmetric load. For a selected asymmetric state of such a transformer the computation leading to analysis of characteristic output parameters of the transformer is carried out with the help of the Mathcad software.

1. Introduction

Asymmetric condition of a transformer is caused by asymmetry of the phase loads or supplying voltages. The three-winding transformers used in electric-power systems usually are not loaded with single-phase receivers (with neutral conductor), but often with two-phase receivers. In industrial applications the transformers are used under various receiver connections. The paper presents a set of the equations that describe the effect of various load types on the distribution of currents in the transformer windings and, in consequence, on voltage values in both secondary windings. Making use of the analogies with the power transformers the following definitions were introduced: the high (g), medium (s), and low (d) voltage winding, irrespective of actual values of these voltages. In order to enable researching the effect of three various values on the output parameters of the transformer, the k_1, k_2, and k_3 coefficients are used in the equations. Other coefficients u_1, v_1, and w_1 are introduced with a view to consider various values of the angle φ of the receivers.

2. Basic relationships describing the considered problem

The transformer load is characterized by the impedances connected to particular phases. Therefore, knowledge of the impedance values is necessary for analyzing the transformer operation. It is assumed in the paper that the impedances are known. For the medium voltage winding the impedances are determined as follows (1):

$$Z_{zsu}(k_1) = (k_1 |Z_{odng}| e^{j0.307 u_1 2\frac{\pi}{3}})$$

$$Z_{zsv}(k_2) = (|Z_{odng}| e^{jk_2 0.307 v_1 2\frac{\pi}{3}}) \tag{1}$$

$$Z_{zsw}(k_3) = (k_3 |Z_{odng}| e^{j0.307 w_1 2\frac{\pi}{3}})$$

Application of the transformation matrix (2):

$$\begin{pmatrix} Z_1(k_1,k_2,k_3) \\ Z_2(k_1,k_2,k_3) \\ Z_0(k_1,k_2,k_3) \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} U_{tg1} \\ U_{tg2} \\ U_{tg0} \end{pmatrix} \frac{1}{D(k_1,k_2,k_3)} \tag{2}$$

enables calculating the symmetrical components of the impedance of the medium voltage side (3):

$$Z_{s1}(k_1,k_2,k_3) = (Z_{zsu}(k_1) + aZ_{zsv}(k_2) + a^2 Z_{zsw}(k_3))\frac{1}{3}$$

$$Z_{s2}(k_1,k_2,k_3) = (Z_{zsu}(k_1) + a^2 Z_{zsv}(k_2) + aZ_{zsw}(k_3))\frac{1}{3} \tag{3}$$

$$Z_{s0}(k_1,k_2,k_3) = (Z_{zsu}(k_1) + Z_{zsv}(k_2) + Z_{zsw}(k_3))\frac{1}{3}$$

The currents of particular symmetrical components of the medium and low voltage windings were calculated based on the relationship (4):

$$\begin{pmatrix} I_1(k_1,k_2,k_3 \\ I_2(k_1,k_2,k_3 \\ I_0(k_1,k_2,k_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{10} \\ M_{21} & M_{22} & M_{20} \\ M_{01} & M_{02} & M_{00} \end{pmatrix} \begin{pmatrix} U_{tg1} \\ U_{tg2} \\ U_{tg0} \end{pmatrix} \frac{1}{D(k_1,k_2,k_3)} \tag{4}$$

The values U_{tg1}, U_{tg2}, U_{tg0} in these formulas are the symmetrical components of the high voltage winding (the primary one), while M and D are the elements of the impedance matrix.

Particular terms of the impedance matrix M/D for the medium voltage winding take a form (5):

$$M_{s11}(k_1,k_2,k_3) = (Z_{ugs} + Z_{s0}(k_1,k_2,k_3))(Z_{s0}(k_1,k_2,k_3) + Z_{\mu 0}) - Z_{s1}(k_1,k_2,k_3)Z_{s2}(k_1,k_2,k_3)$$

$$M_{s12}(k_1,k_2,k_3) = Z_{s1}(k_1,k_2,k_3)^2 - Z_{s2}(k_1,k_2,k_3)(Z_{\mu 0} + Z_{s0}(k_1,k_2,k_3))$$

$$M_{s10}(k_1,k_2,k_3) = Z_{s2}(k_1,k_2,k_3)^2 - Z_{s1}(k_1,k_2,k_3)(Z_{s0}(k_1,k_2,k_3) + Z_{ugs})$$

$$M_{s21}(k_1,k_2,k_3) = (Z_{s2}(k_1,k_2,k_3))^2 - (Z_{s1}(k_1,k_2,k_3))(Z_{s0}(k_1,k_2,k_3) + Z_{\mu 0}) \tag{5}$$

$$M_{s22}(k_1,k_2,k_3) = (Z_{ugs} + Z_{s0}(k_1,k_2,k_3))(Z_{s0}(k_1,k_2,k_3) + Z_{\mu 0}) - Z_{s1}(k_1,k_2,k_3)Z_{s2}(k_1,k_2,k_3)$$

$$M_{s20}(k_1,k_2,k_3) = Z_{s1}(k_1,k_2,k_3)^2 - Z_{s2}(k_1,k_2,k_3)(Z_{s0}(k_1,k_2,k_3) + Z_{ugs})$$

$$M_{s01}(k_1,k_2,k_3) = Z_{s1}(k_1,k_2,k_3)^2 - Z_{s2}(k_1,k_2,k_3)(Z_{s0}(k_1,k_2,k_3) + Z_{ugs})$$

$$M_{s02}(k_1,k_2,k_3) = Z_{s2}(k_1,k_2,k_3)^2 - Z_{s1}(k_1,k_2,k_3)(Z_{s0}(k_1,k_2,k_3) + Z_{ugs})$$

$$M_{s00}(k_1,k_2,k_3) = (Z_{s0}(k_1,k_2,k_3) + Z_{ugs}))^2 - Z_{s1}(k_1,k_2,k_3)Z_{s2}(k_1,k_2,k_3)$$

The matrix determinant (6):

$$D_s(k_1,k_2,k_3) = D_{s1}(k_1,k_2,k_3) + D_{s2}(k_1,k_2,k_3) + D_{s3}(k_1,k_2,k_3) \tag{6}$$

where D_{s1}, D_{s2}, D_{s3} are presented as (7):

$$D_{s1}(k_1,k_2,k_3) = (Z_{s0}(k_1,k_2,k_3) + Z_{ugs})(Z_{s0}(k_1,k_2,k_3) + Z_{ugs})(Z_{s0}(k_1,k_2,k_3) + Z_{\mu 0})$$

$$D_{s2}(k_1,k_2,k_3) = -Z_{s1}(k_1,k_2,k_3)Z_{s2}(k_1,k_2,k_3)[3Z_{s0}(k_1,k_2,k_3) + (Z_{ugs} + Z_{ugs} + Z_{\mu 0})$$

$$D_{s3}(k_1,k_2,k_3) = Z_{s1}(k_1,k_2,k_3)^3 + Z_{s2}(k_1,k_2,k_3)^3 \tag{7}$$

Particular terms of the impedance matrix M/D for the low winding voltage take a form (8):

$$M_{d11}(k_1,k_2,k_3) = (Z_{ugd} + Z_{d0}(k_1,k_2,k_3))(Z_{d0}(k_1,k_2,k_3) + Z_{\mu 0}) - Z_{d1}(k_1,k_2,k_3)Z_{d2}(k_1,k_2,k_3)$$

$$M_{d12}(k_1,k_2,k_3) = Z_{d1}(k_1,k_2,k_3)^2 - Z_{d2}(k_1,k_2,k_3)(Z_{\mu 0} + Z_{d0}(k_1,k_2,k_3))$$

$$M_{d10}(k_1,k_2,k_3) = Z_{d2}(k_1,k_2,k_3)^2 - Z_{d1}(k_1,k_2,k_3)(Z_{d0}(k_1,k_2,k_3) + Z_{ugd})$$

$$M_{d21}(k_1,k_2,k_3) = (Z_{d2}(k_1,k_2,k_3))^2 - (Z_{d1}(k_1,k_2,k_3))(Z_{d0}(k_1,k_2,k_3) + Z_{\mu 0}) \tag{8}$$

$$M_{d22}(k_1,k_2,k_3) = (Z_{ugd} + Z_{d0}(k_1,k_2,k_3))(Z_{d0}(k_1,k_2,k_3) + Z_{\mu 0}) - Z_{d1}(k_1,k_2,k_3)Z_{d2}(k_1,k_2,k_3)$$

$$M_{d20}(k_1,k_2,k_3) = Z_{d1}(k_1,k_2,k_3)^2 - Z_{d2}(k_1,k_2,k_3)(Z_{d0}(k_1,k_2,k_3) + Z_{ugd})$$

$$M_{d01}(k_1,k_2,k_3) = Z_{d1}(k_1,k_2,k_3)^2 - Z_{d2}(k_1,k_2,k_3)(Z_{d0}(k_1,k_2,k_3) + Z_{ugd})$$

$$M_{d02}(k_1,k_2,k_3) = Z_{d2}(k_1,k_2,k_3)^2 - Z_{d1}(k_1,k_2,k_3)(Z_{d0}(k_1,k_2,k_3) + Z_{ugd})$$

$$M_{d00}(k_1,k_2,k_3) = (Z_{d0}(k_1,k_2,k_3) + Z_{ugd}))^2 - Z_{d1}(k_1,k_2,k_3)Z_{d2}(k_1,k_2,k_3)$$

The matrix determinant is given by the equation (9):

$$D_d(k_1,k_2,k_3) = D_{d1}(k_1,k_2,k_3) + D_{d2}(k_1,k_2,k_3) + D_{d3}(k_1,k_2,k_3) \tag{9}$$

where D_{s1}, D_{s2}, D_{s3} are presented as (10):

$$D_{d1}(k_1,k_2,k_3) = (Z_{d0}(k_1,k_2,k_3) + Z_{ugd})(Z_{d0}(k_1,k_2,k_3) + Z_{ugd})(Z_{d0}(k_1,k_2,k_3) + Z_{\mu 0})$$

$$D_{d2}(k_1,k_2,k_3) = -Z_{d1}(k_1,k_2,k_3)Z_{d2}(k_1,k_2,k_3)[3Z_{d0}(k_1,k_2,k_3) + (Z_{ugd} + Z_{ugd} + Z_{\mu 0})$$

$$D_{d3}(k_1,k_2,k_3) = Z_{d1}(k_1,k_2,k_3)^3 + Z_{d2}(k_1,k_2,k_3)^3 \tag{10}$$

Assuming that only asymmetric impedances of the receivers are considered and only positive-sequence voltage of the value equal to the rated voltage U_{tng} is taken into account, the currents of particular symmetrical components of the medium voltage winding are determined by the relationships (11):

$$I_{s1}(k_1,k_2,k_3) = (M_{s11}(k_1,k_2,k_3)U_{ntg})\frac{1}{D_s(k_1,k_2,k_3)}$$

$$I_{s2}(k_1,k_2,k_3) = (M_{s21}(k_1,k_2,k_3)U_{ntg})\frac{1}{D_s(k_1,k_2,k_3)} \tag{11}$$

$$I_{s0}(k_1,k_2,k_3) = (M_{s01}(k_1,k_2,k_3)U_{ntg})\frac{1}{D_s(k_1,k_2,k_3)}$$

The load impedances of particular phases of the low voltage winding are written down in the form (12):

$$Z_{zdu}(k_1) = (k_1|Z_{odng} \cdot 1|e^{-j0.45\cdot 2\frac{\pi}{3}})$$

$$Z_{zdv}(k_2) = (|Z_{odng} \cdot 1.5|e^{-jk_2 0.2\cdot 2\frac{\pi}{3}}) \tag{12}$$

$$Z_{zdw}(k_3) = (|Z_{odng} \cdot 0.8|e^{-j0.3\cdot 2\frac{\pi}{3}})$$

Application of the transformation matrix enabled calculating symmetrical components of the load impedance of the low voltage side defined by the equations (13):

$$Z_{d1}(k_1,k_2,k_3) = (Z_{zdu}(k_1) + aZ_{zdv}(k_2) + a^2 Z_{zdw}(k_3))\frac{1}{3}$$

$$Z_{d2}(k_1,k_2,k_3) = (Z_{zdu}(k_1) + a^2 Z_{zdv}(k_2) + aZ_{zdw}(k_3))\frac{1}{3} \tag{13}$$

$$Z_{d0}(k_1,k_2,k_3) = (Z_{zdu}(k_1) + Z_{zdv}(k_2) + Z_{zdw}(k_3))\frac{1}{3}$$

Under the assumptions identical to the ones formerly adopted for the medium voltage side, the currents of particular symmetrical components of the low voltage winding are determined by the relationship (14):

$$I_{d1}(k_1,k_2,k_3) = (M_{d11}(k_1,k_2,k_3)U_{ntg})\frac{1}{D_d(k_1,k_2,k_3)}$$

$$I_{d2}(k_1,k_2,k_3) = (M_{d21}(k_1,k_2,k_3)U_{ntg})\frac{1}{D_d(k_1,k_2,k_3)} \tag{14}$$

$$I_{d0}(k_1,k_2,k_3) = (M_{d01}(k_1,k_2,k_3)U_{ntg})\frac{1}{D_d(k_1,k_2,k_3)}$$

Once the currents of medium and low voltage sides are calculated, the current of high voltage winding (I_g) should be calculated. In order to find these currents the currents of medium and low voltage windings should be summed up, that is presented by the equations (15):

$$I_{g1}(k_1,k_2,k_3) = I_{s1}(k_1,k_2,k_3) + I_{d1}(k_1,k_2,k_3)$$

$$I_{g2}(k_1,k_2,k_3) = I_{s2}(k_1,k_2,k_3) + I_{d2}(k_1,k_2,k_3) \tag{15}$$

$$I_{g0}(k_1,k_2,k_3)=I_{s0}(k_1,k_2,k_3)+I_{d0}(k_1,k_2,k_3)$$

The phase currents (a,b,c) of particular windings are obtained in result of transformation of the symmetrical components to the phase form. This is made with the help of the transformation matrix (16):

$$\begin{pmatrix} I_a(k_1,k_2,k_3) \\ I_b(k_1,k_2,k_3) \\ I_c(k_1,k_2,k_3) \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} I_1(k_1) \\ I_2(k_2) \\ I_0(k_3) \end{pmatrix} \tag{16}$$

The phase currents of the medium voltage winding are determined by the relationships (17):

$$\begin{aligned} I_{sa}(k_1,k_2,k_3) &= I_{s1}(k_1,k_2,k_3)+I_{s2}(k_1,k_2,k_3)+I_{s0}(k_1,k_2,k_3) \\ I_{sb}(k_1,k_2,k_3) &= a^2 I_{s1}(k_1,k_2,k_3)+aI_{s1}(k_1,k_2,k_3)+I_{s0}(k_1,k_2,k_3) \\ I_{sc}(k_1,k_2,k_3) &= aI_{s1}(k_1,k_2,k_3)+a^2 I_{s2}(k_1,k_2,k_3)+I_{s0}(k_1,k_2,k_3) \end{aligned} \tag{17}$$

The phase currents of the low voltage winding are described by the equations (18):

$$\begin{aligned} I_{da}(k_1,k_2,k_3) &= I_{d1}(k_1,k_2,k_3)+I_{d2}(k_1,k_2,k_3)+I_{d0}(k_1,k_2,k_3) \\ I_{db}(k_1,k_2,k_3) &= a^2 I_{d1}(k_1,k_2,k_3)+aI_{d1}(k_1,k_2,k_3)+I_{d0}(k_1,k_2,k_3) \\ I_{dc}(k_1,k_2,k_3) &= aI_{d1}(k_1,k_2,k_3)+a^2 I_{d2}(k_1,k_2,k_3)+I_{d0}(k_1,k_2,k_3) \end{aligned} \tag{18}$$

The phase currents of the high voltage winding are determined by the relationships (19):

$$\begin{aligned} I_{ga}(k_1,k_2,k_3) &= I_{g1}(k_1,k_2,k_3)+I_{g2}(k_1,k_2,k_3) \\ I_{gb}(k_1,k_2,k_3) &= a^2 I_{g1}(k_1,k_2,k_3)+aI_{g1}(k_1,k_2,k_3) \\ I_{gc}(k_1,k_2,k_3) &= aI_{g1}(k_1,k_2,k_3)+a^2 I_{g2}(k_1,k_2,k_3) \end{aligned} \tag{19}$$

The equations (20) present the relationships necessary for calculating the symmetrical components of the output voltages of the medium and low sides.

$$\begin{aligned} E_1(k_1,k_2,k_3) &= U_{ntg}-(I_{s1}(k_1,k_2,k_3)+I_{d1}(k_1,k_2,k_3))Z_{ug1} \\ U_{2tg}(k_1,k_2,k_3) &= I_{g2}(k_1,k_2,k_3)Z_{ug2} \\ U_{0tg}(k_1,k_2,k_3) &= (I_{s0}(k_1,k_2,k_3)+I_{d0}(k_1,k_2,k_3))Z_{ug0} \\ U_{s1}(k_1,k_2,k_3) &= E_1(k_1,k_2,k_3)-I_{s1}(k_1,k_2,k_3)Z_{ug1} \\ U_{d1}(k_1,k_2,k_3) &= E_1(k_1,k_2,k_3)-I_{d1}(k_1,k_2,k_3)Z_{ud1} \\ U_{s2}(k_1,k_2,k_3) &= I_{s2}(k_1,k_2,k_3)Z_{us1} \\ U_{d2}(k_1,k_2,k_3) &= I_{d1}(k_1,k_2,k_3)Z_{ud1} \\ U_{s0}(k_1,k_2,k_3) &= I_{s0}(k_1,k_2,k_3)Z_{us1} \\ U_{d0}(k_1,k_2,k_3) &= I_{d0}(k_1,k_2,k_3)Z_{ud1} \end{aligned} \tag{20}$$

The phase voltages of the medium voltage side are calculated according to the formulas (21)

$$\begin{aligned} U_{sA}(k_1,k_2,k_3) &= U_{s1}(k_1,k_2,k_3)+U_{s2}(k_1,k_2,k_3)+U_{s0}(k_1,k_2,k_3) \\ U_{sB}(k_1,k_2,k_3) &= a^2U_{s1}(k_1,k_2,k_3)+aU_{s2}(k_1,k_2,k_3)+U_{s0}(k_1,k_2,k_3) \\ U_{sC}(k_1,k_2,k_3) &= aU_{s1}(k_1,k_2,k_3)+a^2U_{s2}(k_1,k_2,k_3)+U_{s0}(k_1,k_2,k_3) \end{aligned} \tag{21}$$

The phase voltages of the low voltage side are calculated according to the relationship (22)

$$\begin{aligned} U_{dA}(k_1,k_2,k_3) &= U_{d1}(k_1,k_2,k_3)+U_{d2}(k_1,k_2,k_3)+U_{d0}(k_1,k_2,k_3) \\ U_{dB}(k_1,k_2,k_3) &= a^2U_{d1}(k_1,k_2,k_3)+aU_{d2}(k_1,k_2,k_3)+U_{d0}(k_1,k_2,k_3) \\ U_{dC}(k_1,k_2,k_3) &= aU_{d1}(k_1,k_2,k_3)+a^2U_{d2}(k_1,k_2,k_3)+U_{d0}(k_1,k_2,k_3) \end{aligned} \tag{22}$$

3. Example calculation results

The formulas derived above have been used for calculating selected output parameters of a three-winding transformer of the power of 20/10/10 MVA and the voltages 110, 33, 6.6kV. In order to carry out the example calculation it was assumed that the transformer is subject to asymmetrical load at the low voltage side, and to symmetrical load at the medium voltage side. The values of the output parameters are the best illustrated by the characteristics plotted against the load parameters.

The Figures 1-3 present the relationship between the asymmetry factor U_{d2}/U_{d1} and the parameters k_1, k_2, and k_3, respectively. Figures 4-6 show the plots of the currents in the high, low, and medium voltage sides, respectively, against the k_1 parameter. The Figures 7 and 8 show the plots of the currents in the medium and low voltage sides, respectively, against the k_1 parameter.

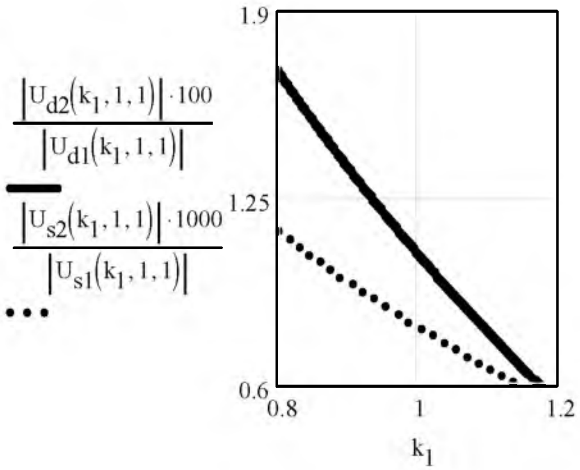

Fig. 1. The relationship between the asymmetry factor and the k_1 parameter

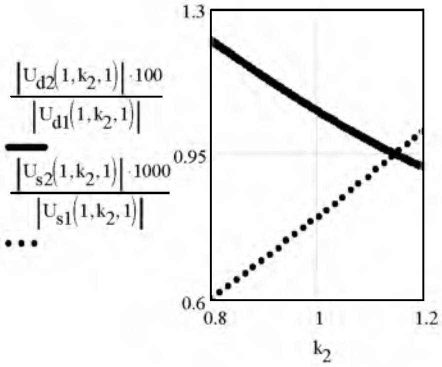

Fig. 2. The relationship between the asymmetry factor and the k_2 parameter

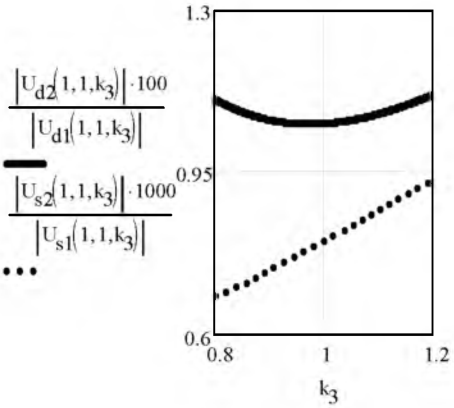

Fig. 3. The relationship between the asymmetry factor and the k_3 parameter

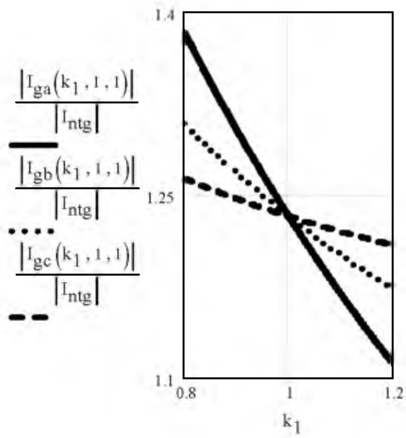

Fig. 4. The relationship between the current in the high voltage side and the k_1 parameter

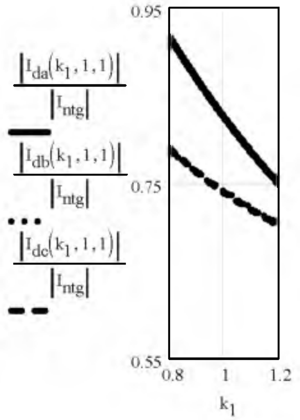

Fig. 5. The relationship between the current in the low voltage side and the k_1 parameter

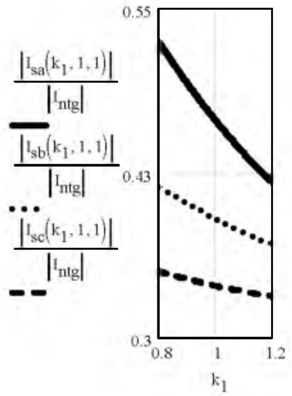

Fig. 6. The relationship between the current in the medium voltage side and the k_1 parameter

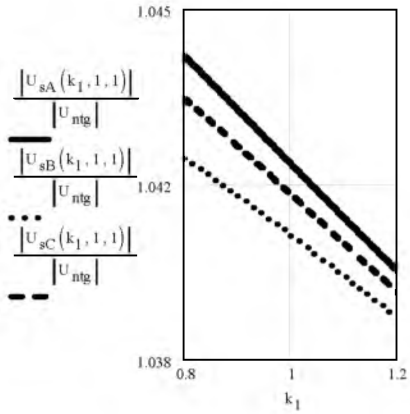

Fig. 7. The relationship between the voltage at the medium voltage side and the k_1 parameter

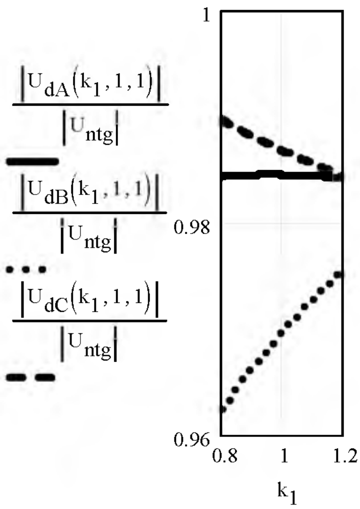

Fig. 8. The relationship between the voltage at the low voltage side and the k_1 parameter

4. Conclusions

Impedance asymmetry of the receivers at the medium and low voltage sides induce asymmetric current distribution in the transformer windings. In result thereof the output voltages and, in consequence, the voltage asymmetry factors are asymmetric too. Therefore, the load impedances should be so chosen as to avoid not only the excess of the rated current level at the high voltage side caused by the asymmetric load but, first of all, as to keep allowable values of the asymmetry factors U_2/U_1.

References

[1] Kasprzyk L., Stein Z., Zielińska M., Analiza wpływu niesymetrycznych obciążeń transformatora na jego charakterystyczne wielkości wyjściowe. Materiały Konferencji „Zastosowania Komputerów w Elektrotechnice”, ZKwE 2010, Poznań, 2010.