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## STATISTICAL ANALYSIS OF SIGNIFICANCE ESTIMATION AND QUANTITY OF THE INFORMATION IN MASS SERVICE

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*Summary*. On the basis of the theory of images, a new approach that allows quantifying the significance and amount of information is proposed. The approaches to the definition and calculation of the basic concepts of information theory, in particular, the amount of information and evaluation of its significance, are based on statistical considerations (classical approach), algorithmic theory (algorithmic approach), and the theory of pattern recognition (figurative approach). Approaches to the processing of fuzzy information are proposed in the conditions of incomplete determination of the vector of input characteristics, are based on the theory of fuzzy sets and fuzzy measures. Their analysis have been carried out, the limits of their use, and areas of effective application, in particular, regarding mass service systems.

Keywords: amount of information, significance of information, fuzzy sets and fuzzy logic, systems of mass service

# 1. INTRODUCTION AND THE PURPOSE OF THE STUDIES

The significance of theoretical results, as a rule, is based on their practical value in applied applications that directly or indirectly confirm the feasibility of their application and do not violate the logic of reasoning. Newly introduced terms or analytical or descriptive formulas that are not operational and do not have practical value over time are excluded from scientific consideration.

The purpose of the work is to show the universality of the approach to quantitative evaluation of the value of information on the basis of the theory of images, to compare it with existing methods and approaches to assessing the value of information, to indicate the limits of practical applications, to determine the diradvantages and disadvantages, the directions of further research.

#### 2. MAIN RESULTS

Getting information about the system is associated with a decrease in its entropy. How much the entropy of the system will decrease, so much the information about it will grow. From this perspective, information can be defined as a negative entropy of the system.

This point of view is formulated as the non-aggressive principle of information in the [1].

There was in the papers [4, 5, 8, 14] proposed approaches to the estimation of the quantity and value of information on the basis of the classical theory of information, of the theory of pattern recognition, fuzzy sets and measures. They considerably extend the area of applications comparison with classical results [3, 10].

### 2.1. APPROACH TO EVALUATION OF QUALITY AND VALUE OF INFORMATION

The classical theory of information, developed by C. Shannon [10], today is a perfect universal apparatus for solving problems related to information encoding, its transformation and optimal transmission over long distance communication channels. The limitation of this theory is that it in no way takes into account the semantics (mode of formation) of information and totally ignores the human factor in the formation of information, that is, the notion of value of information is largely ignored, which is largely a subjective characteristic and depends on the purposes and preferences of the user. Obviously, this is the fee for the ability to calculate the amount of information based on statistical considerations (probability characteristics), regardless of its quality and value.

Note that the amount of useful information is always magnitude positive. In the case of the mutual influence of one system on the other, there appears conditional uncertainty of the states of the system, which may lead to a change in the sign in determining the amount of information. Then we deal with another quality of information – misinformation or false information.

Algorithmic theory, based on the notion of complexity of the algorithm, has made a significant step in the direction of taking into account the method of formation of information. Despite a number of shortcomings of this theory, its main provisions are used in the figurative concept of information theory [4, 8]. According to this concept, the information should be understood as recognizable images that are stored in the memory of a computer or any other cybernetic machine. The image is a signal recorded in the sensor memory of the scanner devices of the cybernetic machine. Thus, in order to obtain information, the procedure for recognizing the input object, the image based on its mapping Q – of a certain standard of this image, created on the basis of arrangements between the sender and the message receiver, should be implemented.

The amount of information  $I_0$  contained in a certain image obtained and successfully recognized by a cybernetic machine can be determined by the formula:

$$I_0 = q^{-1} F[G(O)] \tag{1}$$

where q – the probability of correct recognition of the image; F – the function of obtaining the length of the extended (taking into account the cycles) of the image recognition program; G – the length of the program of recognition of the image, expressed in bits, O – display (image) of the real object.

Obviously, in approach (1), it is possible to calculate the amount of information of complexly structured images, which can be separate words, sentences or texts or figures. In this case, the amount of information will depend significantly on the length of

the extended recognition program, the probability of correct recognition and the functionality of the cybernetic machine that implements the recognition procedure. For the objective calculation of the amount of information according to formula (1), the following should be taken into account:

- a) image recognition programs should be optimal for their own size, performance and functionality;
- b) an increase in the number of operations (commands) that must be performed by the machine for successful recognition, leads to an increase in the information we obtain from the image;
- c) the smaller the probability of the correct recognition of the image, the greater the amount of information it has;
- d) the amount of information in an insignificant message is equal to the length of this message, expressed in bits, multiplied by the length of the program of recognition of one bit of this message and divided by the probability of correct recognition of the message.

The first experiments [8] confirmed the right to exist for such an approach to calculating the amount of information, although its effectiveness can only be realized in the process of producing numerical (computer) experiments with the recognition of complex structured objects. Such an approach has the following disadvantages:

- a) to a large extent, a subjective estimate of the amount of information that carries an image due to both the technical capabilities of the cybernetic machine and the quality of the recognition program;
- b) the property of the additivity of the amount of information that is present in the classical information theory is lost;
- c) formula (1) is too cumbersome, which complicates the carrying out of analytical assessments.

The second disadvantage when calculating the amount of semantic (figurative) information is eliminated if you go to the logarithmic scale:

$$I_0 = -Log_2q + Log_2f(O)] \tag{2}$$

where f is the length of the pattern recognition program, expressed in the number of operations (machine commands) needed to successfully recognize the image.

#### 2.2. THE PRINCIPLE OF COMPUTING VALUES OF INFORMATION ON THE TARGET FUNCTIONS BASIS

With regard to the value of information, then it can be said about it if it is necessary to achieve a certain goal after receiving information by the user, that is, ensuring the implementation of the target function. In work [4] AA Harkevich proposed the value of information to be calculated as:

$$F = Log_2 \frac{p_0}{p_i} \tag{3}$$

where  $p_0$  – the probability of correct solution of the problem to obtain information, and – the probability of correct solution of the problem after receiving information. This approach has the right to exist, although it raises a number of doubts as to its effectiveness and feasibility. First, the unit of measurement of the value of information in this approach is a bit, as in the case of deducting the amount of information. Obviously, when a new value is introduced, a new dimension or calculation should be made in dimensionless

units. Secondly, formula (3) can not claim objectivity, because the estimates  $p_0$ ,  $p_1$  will be made by the user. Third, the methodology for evaluating these probabilities is not obvious. Fourthly, the value of information is a dynamic value [7] and as information arrives, it should change. Obviously, when calculating the value of information in the role of the target function to achieve the goal, you must choose the satisfaction of certain needs of the user (material, spiritual, aesthetic, taste, cognitive, etc.) or to perform certain actions.

The most expedient considering the diversity of the needs of the user is an approach in which the value of information will be calculated as a percentage: 100% – provided the value of information; 0% – provided that the target function is not reached. Thus, the value of the information F for the current value of the function of the goal Z and the achieved efficiency E after receiving the message at the given time can be specified as a rule:

$$F = \frac{100\%, if \quad Z - E = 0}{0\%, if \quad Z - E > 0}$$
(4)

In order to increase the accuracy of the calculation of the value of information it is advisable to have the formed target function and at the next moments of time and to expand the scale of the calculation of the value of information with a certain step. The first condition significantly increases the value of the information after receiving the entire message, and the second can be realized by the formula:

$$F = \frac{E}{Z} \ge 100\%$$
(5)

Formula (5) is a refinement (4), which is confirmed by the consideration of specific examples of consideration of mass service systems [9]. The obtained results testify to the expediency of the applied application of the proposed approach to the quantitative assessment of the value of information, regardless of its content, nor the way of submission. In the example of the mass service system, it is shown that if formula (4) allows estimating the turnover of a commodity and its value per year, then (5) provides scaling of the process with an arbitrary step, which allows to optimize the process of release of goods and to control the financial costs after almost every financial operations.

# 2.3. ESTIMATES OF QUALITY AND VALUE OF INFORMATION IN MASS MEDIA SYSTEMS

#### 2.3.1. Regulatory approach

A fundamentally different approach to determining the quantity and value of information proposed by the Russian scientist AM Kolmogorov [5]. His algorithmic theory of information is based on the notion of the complexity of the algorithm of transforming one object to another. In this approach, it is crucial to establish a mutual relationship between the objects being studied and the length of the program they are processing. The amount of information on the theory of algorithms for the conversion of one object to another is defined as the length of the program, which enables the conversion of object A to object B:

$$I = f[G(A,B)] \tag{6}$$

where G is a program for converting object A into object B; f is a function that defines the length of the conversion program in bits.

Note that the amount of information for such an approach is significantly dependent on the choice of the structural element of the transformation. The number of information is maximal if the pixel is selected as a conversion element, and the minimum is when the element of the whole letter is selected as the element of conversion, and it will be intermediate when selecting the part of the letter by the conversion element. The same amount of information for a particular letter is impossible to calculate, since two objects need to be implemented for the algorithm (8). Comparing the same letter with it itself has no content, because the length of the conversion program in this case will be zero.

Thus, the following drawbacks are evident in the algorithmic approach:

- a) when calculating the amount of information as a parameter, the length of the conversion program is used, which essentially depends on the structure of the elements that make it possible to convert one object to another. The smaller the selected structure of the elements, the longer becomes the conversion program for the same objects;
- b) the same conversion program can be used to process a whole set of objects, the analysis of which requires the execution of a different number of computer commands, while the length of the program remains unchanged;
- c) there is no method for measuring the amount of information when considering a single object, since according to formula (4) they require two;
- d) the property of the additivity of the information is lost when considering interacting systems, which considerably complicates their analysis.

#### 2.3.2. Semantic approach

These shortcomings are also characteristic of the semantic approach in the calculation of information based on the elaboration of logical statements:

$$I = Log_2 L(O) \tag{7}$$

where L is a function that depends on the number of states in logical statements (variables, facts, rules); O - logical statement or predicate.

The calculation of the amount of information in individual letters or words for the approach (7) is impossible in principle, since neither individual letters nor individual words are logical statements.

#### 2.3.3. The approach based on the fuzzy sets and measures theory

The aforementioned disadvantages are eliminated in the approach based on the application of methods of recognition theory and operates with the notion of information, the quantity and value of information that are different from the classical definitions obtained on the basis of probability theory.

The problems of mathematical and computer modeling today lie, in particular, in the impossibility of applying precise logic rules and models of tasks with clearly defined input parameters in cases where, for some reason, there are contradictions, uncertainties or uncertainties of information about the object being studied, system or phenomenon [15].

Uncertainty is known to arise due to a lack of knowledge relating to a specific event [7]. She is present to the experiment. The mathematical model of uncertainty is based on the theory of probabilities, the theory of possibilities, confidence measures, the theory of prediction and prediction, and a number of others.

The phenomenon of fuzziness arises in the process of combining into one whole objects that have a common property  $\varphi$ :

$$X = \left\{ x \, \middle| \, x \text{ owns } \varphi \right\}$$

where x all the elements of a certain universal set run.

Given that there are always elements x in the reality that it is unclear whether they possess the specified property or not, X it is not a plural in the classical sense. Any attempt to interpret the general description leads to fuzzy concepts, since the exact description contains an excess of details. Increasing the accuracy of the description leads to an increase in the amount of information, the content content of which decreases until the time when the accuracy and meaningfulness do not become mutually exclusive. For the first time, LA stressed the need for uncertainty for the transmission of content. Zade [13]. It was the ideas of this American scientist who pushed for the development of "fuzzy mathematics" [6], which, along with the apparatus of fuzzy sets, contains other methods of work with uncertainty.

The application of the theory of fuzzy sets and measures is a step towards the convergence of the precision of classical mathematics with a false inaccuracy of the environment, an attempt to overcome the linguistic barrier between a person whose judgment and evaluation are approximate and fuzzy, and technical means which can only carry out precise instructions [11].

A device that allows you to work with fuzzy logic, "blurred" parameters of models, is a *Fuzzy*-technology device. The *Fuzzy*-Technology division has fuzzy expert systems.

Linguistic (descriptive) variables extend the ability to represent knowledge. They are determined by fuzzy sets whose values are established by membership functions. Membership functions can be obtained through subjective expert assessments [12], or by analyzing fuzzy clusters. According to [12], fuzzy expert systems can be implemented when the cost of acquiring accurate information, that is, information in absolute terms, exceeds the maximum revenue from the restructuring of a model or is virtually impossible.

It is known that the initial stage of constructing artificial intelligence on the basis of the use of natural language is based on ambiguous logic and the mechanism of output with rigid rules.

The modern or second generation of expert systems possesses at least two peculiarities: fuzzy presentation of knowledge and fuzzy deductions. One of the most common problems of logical derivation in the conditions of fuzziness can be formulated as follows:

> *Given (fuzzy)* log *ical rule* "*If A, then B*". *Observed A' (A to some extent).What should be B?*

After receiving the fuzzy set of conclusions, find a specific numerical match (conduct dephasing).

Consider the object with one output and *n* inputs of the form:

$$y = f(x_1, x_2, ..., x_n),$$
 (8)

where the set of values  $x_1, ..., x_n$  – of input variables; y – output variable.

To construct a mathematical model on the basis of establishing the relationship between input and output variables in accordance with experimental data, by conducting a phasing operation, quantitative and qualitative variables are translated into linguistic terms:

$$U_i = \left[\underline{u}_i, \overline{u}_i\right], \quad i = \overline{1, n},\tag{9}$$

$$Y = \left[\underline{y}, \overline{y}\right] \tag{10}$$

where  $\underline{u_i}, \overline{u_i}$  – the smallest and the highest possible value of the variable  $x_i$ ;  $\underline{y}, \overline{y}$  – the smallest and the highest possible value of the output variable y.

To solve the problem (8), it is necessary to apply a method of making a decision by which the fixed vector of input variables  $x^* = (x_1^*, x_2^*, ..., x_n^*)$ ,  $x_i^* \in U_i$  would unambiguously be placed in accordance with the solution  $y^* \in Y$ . For the formal establishment of this dependence we shall consider the input  $x_i$ ,  $i = \overline{1, n}$  and output yparameters as linguistic variables given on universal sets (9), (10). To evaluate linguistic variables  $x_i$ ,  $i = \overline{1, n}$  and y, we will use qualitative terms from the following term sets:

 $A_i = \left\{a_i^1, a_i^2, \dots, a_i^{p_i}\right\} - \text{ term-set of input variable } x_i, \ i = \overline{1, n}; \ D = \left\{d_1, d_2, \dots, d_m\right\} - \left\{d_1, d_2, \dots,$ 

term set of output variable y. To construct term sets it is possible to apply, for example, the method proposed in [15].

For each term of each linguistic variable, based on expert knowledge, the memberships functions  $\mu^{a_i^p}(x)$  and  $\mu^{d_j}(y)$  (trapezoidal, triangular, rectangular, sinusoidal, parabolic, etc.) [13] are constructed based on expert knowledge, where  $\mu^{a_i^p}(x)$  – the degree of belonging of the element  $x \in U_i$  to the term  $a_i^p \in A_i$ ,  $i = \overline{1, n}$ ;  $p = \overline{1, p_i}$ ;  $\mu^{d_j}(y)$  – the degree of belonging of the element  $y \in Y$  to the term  $d_j \in D, j = \overline{1, m}$ .

The definition of linguistic estimates of variables and the membership functions necessary for their formalization is the first stage in the construction of a fuzzy model of the object being studied. In the literature on fuzzy logic, he received the name of the fuzzification of variables [16].

The next step is to create a fuzzy knowledge base.

Let the object (8) know the rules that connect its inputs and output using vectors such as:

$$V_k = (x_1, x_2, \dots, x_n, y), \quad k = 1, N \text{, and } N = k_1 + \dots + k_j + \dots + k_m,$$
(11)

where  $k_j$  – the number of experimental data corresponding to the same value  $d_j$  of the term-set of the output variable y; m – the total number of terms of the output variable, and in the general case  $k_1 \neq ... \neq k_m$ .

We will assume that the number  $N < p_1 p_2 \dots p_n$ , of available experimental data is less than the total overview of various combinations of possible terms of input variables  $p_i$ ,  $i = \overline{1, n}$ . Then the knowledge base is a table formed according to the following rules:

- 1. The table's size is equal  $N \times (n+2)$ , where n+2 number of columns and N the number of rows;
- Each line of the matrix is a combination of input variables assigned by the expert to one of the possible values of the term-set of the output variable y. In this case, the first k<sub>1</sub> lines correspond to the value of the output variable y = d<sub>1</sub>, the following k<sub>2</sub> lines to the value y = d<sub>2</sub>, etc., and the last k<sub>m</sub> lines to the value y = d<sub>m</sub>;
- 3. The first *n* columns of the matrix correspond to the input variables  $x_i$ ,  $i = \overline{1, n}$ ; the

(n + 1)-th – the weight  $W_{jp}$ ,  $j = \overline{1,m}$ ,  $p = \overline{1,k_j}$ , of the rule and (n+2)-th – the value to the output term-set  $d_j$  of the variable y,  $j = \overline{1,m}$ , corresponding to the combination of values in the first (n + 1)-th columns.

4. The element a<sub>i</sub><sup>ip</sup>, located at the intersection of the *i*-th column and the j<sub>p</sub> − line corresponds to the linguistic evaluation of the parameter x<sub>i</sub> in the row of knowledge matrix with the number j<sub>p</sub>. In this case, the linguistic assessment a<sub>i</sub><sup>jp</sup> is chosen with the term-set corresponding to the variable x<sub>i</sub>, i.e. a<sub>i</sub><sup>jp</sup> ∈ A<sub>i</sub>, i=1,m, p=1,k<sub>i</sub>.

When an expert creates linguistic rules such as "IF - THEN" that form the basis of fuzzy knowledge about a particular object, the expert's confidence in each rule may be different. If one rule in the opinion of an expert can serve as an undeniable truth, then according to another rule in the same expert there may be some doubts.

In order to reflect these different degrees of confidence in the base of fuzzy knowledge, the weighting of the rules is introduced - these are numbers from the interval [0, 1] that characterize the expert's confidence in each particular rule chosen by him to make a decision. The general view of the knowledge base is given in Table 1.

| Incoming combination number | Input variables  |  |                       |              | Weight                 | Output variable |
|-----------------------------|------------------|--|-----------------------|--------------|------------------------|-----------------|
| 6                           | $x_1$            |  | <i>x</i> <sub>i</sub> | $X_n$        | w                      | У               |
| 11                          | $a_1^{11}$       |  | $a_{i}^{11}$          | $a_{n}^{11}$ | $W_{11}$               |                 |
| 12                          | $a_1^{12}$       |  | $a_{i}^{12}$          | $a_{n}^{12}$ | <i>W</i> <sub>12</sub> | $d_1$           |
|                             |                  |  |                       |              |                        |                 |
| $1k_1$                      | $a_{1}^{1k_{1}}$ |  | $a_i^{1k_1}$          | $a_n^{1k_1}$ | $W_{1k_1}$             |                 |
|                             |                  |  |                       |              |                        |                 |
| $j_1$                       | $a_1^{j1}$       |  | $a_{i}^{j1}$          | $a_n^{j1}$   | $W_{j1}$               |                 |
| $j_2$                       | $a_1^{j2}$       |  | $a_i^{j2}$            | $a_n^{j2}$   | $W_{j2}$               | $d_{i}$         |
|                             |                  |  |                       |              |                        | $\alpha_j$      |
| jk <sub>j</sub>             | $a_{1}^{jk_{j}}$ |  | $a_i^{jk_j}$          | $a_n^{jk_j}$ | $W_{jk_j}$             |                 |
|                             |                  |  |                       |              |                        |                 |
| $m_1$                       | $a_1^{m_1}$      |  | $a_{i}^{m1}$          | $a_n^{m1}$   | $W_{m1}$               |                 |
| <i>m</i> <sub>2</sub>       | $a_1^{m2}$       |  | $a_i^{m^2}$           | $a_n^{m^2}$  | W <sub>m2</sub>        | $d_m$           |
|                             |                  |  |                       |              |                        | m               |
| $mk_m$                      | $a_1^{mk_m}$     |  | $a_i^{mk_m}$          | $a_n^{mk_m}$ | $W_{mk_m}$             |                 |

Table 1. General view of the fuzzy knowledge base

After building the knowledge base, you need to carefully check in Table 1 the presence of the opposite in the content of the lines, that is, rules that the same input variables have different output values. The introduced matrix of knowledge defines a system of logical utterances such as "IF - THEN, ELSE", which associate the values  $x_1, ..., x_n$  of input variables with one of the possible output values  $d_i, j = \overline{1,m}$ :

IF 
$$(x_1 = a_1^{11})$$
 AND  $(x_2 = a_2^{11})$  AND ... AND  $(x_n = a_n^{11})$  (with weight  $w_{11}$ ),  
OR  $(x_1 = a_1^{12})$  AND  $(x_2 = a_2^{12})$  AND ... AND  $(x_n = a_n^{12})$  (with weight  $w_{12}$ ),  
OR ...  
OR  $(x_1 = a_1^{1k_1})$  AND  $(x_2 = a_2^{1k_1})$  AND ... AND  $(x_n = a_n^{1k_1})$  (with weight  $w_{1k_1}$ ),  
THEN  $y = d_1$ , OTHERWISE  
IF  $(x_1 = a_1^{21})$  AND  $(x_2 = a_2^{21})$  AND ... AND  $(x_n = a_n^{21})$  (with weight  $w_{21}$ ),  
OR  $(x_1 = a_1^{2k_2})$  AND  $(x_2 = a_2^{2k_2})$  AND ... AND  $(x_n = a_n^{2k_2})$  (with weight  $w_{2k_2}$ ),  
THEN  $y = d_2$ , OTHERWISE ...  
IF  $(x_1 = a_1^{n1})$  AND  $(x_2 = a_2^{m1})$  AND ... AND  $(x_n = a_n^{m1})$  (with weight  $w_{m1}$ ),  
OR  $(x_1 = a_1^{nm})$  AND  $(x_2 = a_2^{m1})$  AND ... AND  $(x_n = a_n^{m1})$  (with weight  $w_{m1}$ ),  
OR  $(x_1 = a_1^{mk_m})$  AND  $(x_2 = a_2^{mk_m})$  AND ... AND  $(x_n = a_n^{mk_m})$  (with weight  $w_{mk_m}$ ),  
THEN  $y = d_n$ .

A similar system of logical expressions is called a fuzzy knowledge base. Using the operations (**OR**) and (**AND**) described system of logical statements can be rewritten in a more compact form:

$$\bigcup_{p=1}^{k_j} \left[ \bigcap_{i=1}^n \left( x_i = a_i^{jp} \right) \right] \to y = d_j, \ j = \overline{1, m}.$$
(12)

Thus, the input relation (8), which establishes the connection between the input parameters  $x_i$ ,  $i = \overline{1, n}$  and the output variable, is formalized in the form of a system of fuzzy logical statements (12) based on the created matrix of knowledge. The rules of the described fuzzy system, in which the degree of truth is different from zero, is considered to be active.

In [9], a method is proposed to use fuzzy logic equations that are based on a knowledge matrix or isomorphic system of logical expressions (12) and allow us to calculate the values of the membership functions of the output variable for the fixed values of the inputs of the object.

Linguistic estimates  $a_i^{jp}$  of the variables  $x_1,...x_n$ , contained in the logical statements (12) will be considered as fuzzy sets defined on universal sets (9). We introduce the following notation:

 $\mu^{a_i^{p}}(x_i)$  - the membership function of the parameter  $x_i$  to the fuzzy term  $a_i^{jp}$ ,  $i = \overline{1, m}$ ,  $j = \overline{1, m}$ ,  $p = \overline{1, k_j}$ ;  $\mu^{d_j}(x_1, x_2, \dots, x_n)$  membership function of the vector of the input variables  $x = (x_1, x_2, \dots, x_n)$  of the term of the output variable  $y = d_i$ ,  $j = \overline{1, m}$ .

Thus, we have two types of functions, the relationship between which is determined by the base of fuzzy knowledge (12), on the basis of which you can output a system of logical equations, which can be submitted in a compact form:

$$\mu^{d_{j}}(x_{1}, x_{2}, ..., x_{n}) = \bigvee_{p=1}^{k_{j}} \left( w_{jp} \left[ \bigwedge_{i=1}^{n} \mu^{a_{i}^{jp}}(x_{i}) \right] \right), \quad j = \overline{1.m}.$$
(13)

where  $\vee$  - is the logical **OR**;  $\wedge$  - logical "**AND**".

The decision  $d^* \in D\{d_1, d_2, ..., d_m\}$ , that corresponds to a fixed vector of the values of input variables  $x^* = (x_1^*, x_2^*, ..., x_n^*)$  will be carried out in accordance with the following algorithm constructed using the apparatus of the fuzzy (blurry) logic [9]:

- 1. The possible range of change of controlled parameters is determined, a knowledge base is created with the use of expert data and a system of fuzzy logic equations is derived (13).
- 2. The vector of the values of the input variables  $x^* = (x_1^*, x_2^*, ..., x_n^*)$  is fixed.
- 3. Specifies the function of fuzzy term membership for different controlled parameters.
- 4. Using logical equations (13), the values of many parametric membership functions μ<sup>d<sub>j</sub></sup>(x<sub>1</sub><sup>\*</sup>, x<sub>2</sub><sup>\*</sup>,..., x<sub>n</sub><sup>\*</sup>) of vector X for all values d<sub>j</sub>, j=1,m of the output variable y are calculated. In this case, the logical operations ∨ (**OR**) and ∧ (**AND**) over the membership functions are replaced by the operations max and min:

$$\mu(a) \lor \mu(b) = \max[\mu(a), \mu(b)], \tag{14}$$

$$\mu(a) \wedge \mu(b) = \min[\mu(a), \mu(b)]. \tag{15}$$

That is, first find the minimum values of belonging functions in each rule, and then among them they choose the highest value of the membership function among all rules for each value  $d_j$ ,  $j = \overline{1,m}$ , which corresponds to the original variable y. Thus, the conclusion is made that the origin variable y belongs to a term  $d_j^*$ , whose membership function is maximal.

The proposed algorithm uses the idea of identifying the linguistic term by the maximum of membership function and generalizes this approach to the entire knowledge matrix. The computational part of this algorithm is easily realized by simply applying operations *max* and *min*.

To obtain a clear number from an interval  $[\underline{y}, \overline{y}]$  that corresponds to a fuzzy value of the output variable, it is necessary to apply a dephasing operation. You can define this clear number, for example, by the gravity method:

$$y^{*} = \frac{\int_{Min}^{Max} y \mu(y) dy}{\int_{Min}^{Max} \mu(y) dy}$$
(16)

where *Min* and *Max* is the left and right points of the interval of the fuzzy set of the source variable *y*.

# 2.3.4. Results of statistical analysis on an example of a system of mass service

The task of forecasting the value of unrealized goods of a certain type at the end of the trade season of a trading firm dealing with chemical protection products of plants is considered in the paper. Obviously, minimizing the balance from unrealized goods will increase the efficiency (productivity) of the trading firm.

The experts found that the weighting factors that affect the current balance Y are:  $x_1$  ("balance") – the balance of the previous trading season (in US dollars);  $x_2$  – "New purchases") – the cost of new purchases (in US dollars);  $x_3$  ("Margin") – the average value of trade margins (in percentages);  $x_4$  ("the duration") – the duration of the sale of this drug (in years). The universal sets for the described variables are defined as follows:  $U_1 = [0;600000]$ ;  $U_2 = [20000;1500000]$ ;  $U_3 = [0;50]$ ;  $U_4 = [0;10]$ . The universal set for the predicted value coincides, obviously, with  $U_1$ .

For each input and for output variables, term-sets are constructed:

$$A_1 = \{\text{"small" "medium" "large" "critical"}\} = \{S, M, L, C\}$$

- $A_2 = \{ "small" "medium" "large" \} = \{ S, M, L \};$
- $A_3 = \{$ "small" "medium" "large"  $\} = \{S, M, C\};$
- $A_4 = \{\text{"short""medium", "long lasting"}\} = \{S, M, L\};$
- $D = \{$ "small" "medium" "large" "critical"  $\} = \{S, M, L, C\}$

Based on the information provided by the expert, the following membership functions are constructed for the terms of input and output variables (Figs. 1–4).

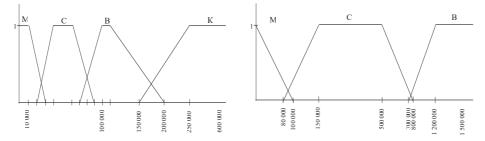


Fig. 1. Membership function of linguistic Fig. 2. Membership function of linguistic variable variable "balance" "new purchases"

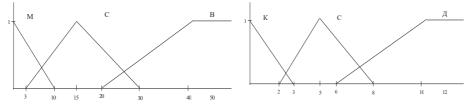


Fig. 3. Membership function of linguistic Fig. 4. Membership function of linguistic variable variable "margin" "the duration"

The next step is to build a fuzzy knowledge base (Table 2).

Table 2. The basis of the fuzzy knowledge of the problem

| Incoming combination     |       | Input v               | ariables | Weight     | Output variable |       |
|--------------------------|-------|-----------------------|----------|------------|-----------------|-------|
| number<br>(logical rule) | $x_1$ | <i>x</i> <sub>2</sub> | Х3       | <i>X</i> 4 | W               | У     |
| 11                       | М     | М                     | М        | К          | 1               |       |
| 12                       | В     | М                     | М        | С          | 0,9             | $d_1$ |
| 13                       | В     | М                     | М        | Д          | 1               |       |
| 14                       | К     | М                     | М        | С          | 1               |       |
| 15                       | М     | М                     | В        | Д          | 0,7             |       |
| 16                       | М     | С                     | В        | C          | 0,7             |       |
| 17                       | С     | М                     | С        | С          | 0,8             |       |
| 18                       | С     | М                     | С        | Д          | 0,8             |       |
| 19                       | С     | С                     | С        | C          | 0,7             |       |
| 20                       | В     | М                     | С        | Д          | 0,9             |       |
| 21                       | В     | М                     | В        | Д          | 0,5             |       |
| 23                       | C     | M                     | В        | Д          | 1               | d2    |
| 24                       | М     | С                     | С        | C          | 1               |       |
| 25                       | М     | В                     | В        | С          | 0,8             |       |
| 26                       | С     | С                     | М        | С          | 0,8             |       |
| 27                       | С     | В                     | М        | В          | 0,7             |       |
| 28                       | С     | В                     | С        | С          | 0,7             |       |
| 29                       | В     | С                     | С        | С          | 0,7             | d3    |
| 30                       | В     | С                     | С        | Д          | 0,5             |       |
| 31                       | В     | С                     | В        | Д          | 0,3             |       |
| 32                       | К     | С                     | В        | Д          | 0,8             |       |
| 33                       | М     | В                     | С        | С          | 0,9             |       |
| 34                       | М     | В                     | С        | К          | 1               |       |
| 35                       | С     | В                     | М        | К          | 1               |       |
| 36                       | С     | В                     | С        | Д          | 0,8             |       |
| 37                       | М     | В                     | В        | С          | 0,8             |       |
| 38                       | M     | В                     | В        | Д          | 0,9             |       |
| 39                       | C     | B                     | В        | C          | 0,9             | $d_4$ |
| 40                       | C     | B                     | B        | Д          | 1               | 4     |
| 41                       | В     | В                     | В        | Д          | 1               |       |

The following calculations were made for the trading company data for the 2011 sales season  $x_1^* = 80000$ ;  $x_2^* = 36000$ ;  $x_3^* = 22$ ;  $x_4^* = 9$ . In this case, the variable  $x_1$  refers to the terms "medium" (with the degree of affiliation  $\mu(x) = 1 - \frac{1}{30000}(x - 60000)$ ) or "high" (with the degree of affiliation  $\mu(x) = \frac{1}{30000}(x - 70000)$ );  $x_2$  – the term "small" (with the degree of belonging  $\mu(x) = 1 - \frac{1}{100000}x$ ); – to the terms "average"

(with degree of affiliation) or "high" (with degree of affiliation);  $x_3$  – to the term "long-term" (with the degree of belonging  $\mu(x) = \frac{1}{4}(x-6)$ ). By simple comparison it is easy to see that the active rules will be rules 18, 20, 21 (lead to the output  $d_1$ ) and 23 (leads to the output  $d_2$ ).

The quantitative value of the output value (the result of defuzzification) was calculated by the method of the center of gravity (16). Finally, the value of the forecast value of the balance of the commodity mass  $y^* = \frac{353865180}{13588,5} \approx 26041,5$  is equal to that which is sufficiently close to the real balance of the goods 23200 by the end of the 2016 season, namely US dollars.

Linking the obtained value  $y y^*$  to the efficiency E, and the predicted value of the output variable with the function of the goal Z, on the basis of formula (4), one can conclude that the value of the firm compared with the previous year is zero in absolute units, since the target function has not been achieved. And according to formula (5), the turnover of the company is about 88%, compared with the previous year.

It should be noted that the predicted values can be more closely approximated to the actual observed (taking into account data for several previous years) by reviewing the established weight coefficients, adjusting membership functions, etc. You can also increase the number of input quantities. However, with their too large numbers, the construction of a fuzzy knowledge base about the unknown dependence becomes a difficult task. This is due to the fact that in the memory of the average statistician at the same time can hold no more than  $7\pm 2$  the notions-signs. In such cases, it is expedient to carry out the classification of input variables and, in accordance with it, construct a derivation tree that defines the system of embedded statements in each other [11, 15].

One can propose a different approach to processing large volumes of fuzzy data under conditions of incomplete certainty of the vector of input variables (primary characteristics). The essence of the approach is based on conducting a simulation of the behavior of the investigated system and an expert assessment of the addition of the existing knowledge base to the new informative data and the establishment of the vector of input characteristics. Obviously, such an approach is iterative and it is necessary to take care of the convergence of the calculation process to achieve the goal with minimal cost and limited error.

When dealing with non-physical data in artificial intelligence problems, the construction of recognition systems, expert systems, medical and parametric diagnostics, the creation of logical-linguistic models, the most successfully adapted declarative programming languages, which in the language of logical statements and functional-logical dependencies provide the opportunity to describe the problem with fuzzy formulated data and obtaining solutions in the form of logical sequences, new functional dependences or probabilistic characteristics with definite mathematical By hope and dispersion of the input sign. Ultimately, the initial vector of primary attributes should be refined, which will ensure reliable processing of fuzzy data.

Therefore, the processing of incomplete or unclear information is, on the one hand, the application of the theory of blurry logic (in particular, the theory of fuzzy sets and measures) and the construction of logic trees or, on the other hand, the formation of logical rules from the functional-logical dependencies with indeterminate variables,

which can take both deterministic and probabilistic values. In the process of processing a priori information and selecting appropriate criteria of likelihood, you can replenish the insufficient data, providing the formation of new knowledge.

Obviously, for implementing the described approach to calculating the value of information, it is advisable to use declarative programming languages (Lisp, Prolog or their modifications depending on a specific objective problem) [15], which are most successfully adapted for the functions of the form (4)–(7), (12), (13). These languages can be both analytical and descriptive (functional, logic rules, fuzzy sets), which allows solving problems associated with qualitative recognition and analysis of objects of complex structure (handwriting recognition, handwriting, psychophysiological state of a person, construction and analysis of storage, processing, information security, automated theorizing, environmental monitoring and decision-making) and implementation of the target functions relevant to a specific applied or scientific task.

## **3. CONCLUSIONS**

The authors propose a new approach to assessing the value of information. This approach is based on the theory of pattern recognition, which extends the scope of its application and can be successfully implemented using declarative programming languages [15] or universal language simulation (UML) [2]. The features of its application and directions of further improvement are noted.

The authors cover three main approaches to assessing the amount of information and assessing its value: classical, algorithmic and figurative. There are given a comparative description of these approaches, considered the advantages and limitations of each of them, and the prospects for their use.

Approaches to the processing of fuzzy information under conditions of incomplete definition of the vector of input characteristics based on the theory of fuzzy sets and fuzzy measures, construction of membership functions and application of declarative programming languages are proposed.

The further development of the proposed approaches can be achieved by conducting specific statistical studies related to the need to assess the quantity and value of information based on a figurative approach.

The proposed approach is advisable to use in applied problems, where the mathematical description is difficult or completely impossible. This approach will contribute to the development, in fact, of the methods of recognition theory and identification, and the theory of information and coding.

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## ANALIZA STATYSTYCZNA SZACUNKU ZNACZENIA I ILOŚCI INFORMACJI W USŁUDZE MASOWEJ

#### Streszczenie

Na podstawie teorii obrazów proponuje się nowe podejście, które pozwala na ilościowe określenie znaczenia i ilości informacji. Podejścia do definiowania i obliczania podstawowych pojęć teorii informacji, w szczególności ilości informacji i oceny jej znaczenia, oparte są na rozważaniach statystycznych (podejście klasyczne), teorii algorytmicznej (podejście algorytmiczne) i teorii wzorca uznanie (podejście oparte na figuratywności). Podejścia do przetwarzania informacji rozmytych proponowane są w warunkach niepełnego wyznaczania wektora charakterystyk wejściowych; oparte są na teorii zbiorów rozmytych i miar rozmytych. Przeprowadzono ich analizę, określono granice ich stosowania oraz obszary skutecznego stosowania, w szczególności w odniesieniu do systemów usług masowych.

Słowa kluczowe: ilość informacji, znaczenie informacji, zbiory rozmyte i logika rozmyta, systemy masowej usługi