Supporting Decisions on the Forex Market Using Fuzzy Approach

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Abstract:

A new concept of the multicriteria fuzzy trading system using the technical analysis is proposed. The existing trading systems use different indicators of the technical analysis and generate buy or sell signal only when assumed conditions for a given indicator are satisfied. The information presented to the trader – decision maker is binary. The decision maker obtains a signal or no. In comparison to the existing traditional systems called as crisp, the proposed system treats all considered indicators jointly using the multicriteria approach and the binary information is extended with the use of the fuzzy approach. Currency pairs are considered as variants in the multicriteria space in which criteria refer to different technical indicators. The introduced domination relation allows generating the most efficient, non-dominated (Pareto optimal) variants in the space. An algorithm generated these non-dominated variants is proposed. It is implemented in a computer-based system assuring the sovereignty of the decision maker.

We compare the proposed system with the traditional crisp trading system. It is made experimentally on different sets of real-world data for three different types of trading: short-term, medium and long-term trading. The achieved results show the computational efficiency of the proposed system. The proposed approach is more robust and flexible than the traditional crisp approach. The set of variants derived for the decision maker in the case of the proposed approach includes only non-dominated variants, what is not possible in the case of the traditional crisp approach. The reservation point and its impact on the overall results are measured with the use of the sensitivity analysis.

Keywords: *Trading system, Forex, fuzzy membership function, multicriteria analysis*

1. Introduction

In this paper, we propose an extension of the traditional trading systems based on the technical analysis using the concept of fuzziness. This concept can be implemented in a decision support system aiding the trader in making his decisions. As a field for experiments, we selected the Forex market, which is a global, decentralized market with currency pairs as basic instruments. According to the Bank for International Settlements, its average daily turnover reached $5.3 trillion in January 2014 [28]. Unlike other markets, the Forex is completely decentralized and housed electronically. It is considered one of the largest markets in the world, where about 90% of its turnover is generated by currency speculators. Still growing number of instruments available for the trader – decision maker in the last few years makes it very difficult to manually manage even the single transaction.

Three main approaches of financial data analysis are used to forecast prices on the Forex market: the technical analysis, the fundamental analysis, and the sentiment analysis. The fundamental analysis including the text-mining techniques was adapted to the stock market in [20], [34], as well as for the Forex market in [32]. The sentiment analysis called also the opinion mining was presented in [10]. The technical analysis is based on the assumption, that there it is possible to predict future prices on the basis of the historical prices, in other words, that past behavior of the price has an effect on the future prices. The process of construction of such indicator can be considered as a dimension reduction [38]. In such an approach, the initial data is transformed to another domain which may be simpler than the original data. Such action leads to a situation, where price and technical indicator values become an independent example of the problem.

One of the main directions of development of the trading systems based on the technical analysis consists in using various indicators which are mostly some complex formulas used for historical data to extract hidden information from price time series or to reduce some irrelevant noise. Such an approach is used to identify moments to open the positions on the market. The technical analysis is the most popular tool used in the trading. Moreover, its importance is increasing over years [17].

By the trading system, we understand any system (manual or automatic), which with the use of data analyzed from the market calculates values of the market indicators. Such values are further used to generate a signal and open the position related to the selected currency pair. However, there exists a significant drawback, where all selected indicators must give the signal at the same time, thus increasing the number of market indicators leads to more seldom signals. What is obvious, even in the case, where all necessary conditions all fulfilled, there is no guarantee, that derived signal will be profitable.

Newer works like [4] suggest, that especially very volatile markets like Forex may be very difficult to analyze. That leads to various works which limit the application of the fully automatic concept of trading systems for the decision support. There are also manual trading systems which generate signals for opening and closing positions on the market, where the final decision is made by the decision maker. One of the most significant advantages of such manual trading systems is that the decision maker may additionally apply other types of analysis on the market situations. Examples of such systems can be found in [13], [18], [8]. Thus one of the most important advantages of such systems is that they assure sovereignty of the decision maker.

The paper discusses decision support problems in the case of the manual trading systems. A typical system analyses data from the market, calculate technical indicators, generates buy or sell signals when a given set of rules is satisfied for a given indicator. The decision maker observing the signals can make the final decision. Still growing number of instruments available for the decision makers results in a dynamic growth of the decision space. Therefore a new approach capable to handle such difficulties is required. In this article, we investigate problems arising in the case of the traditional crisp trading systems, where the decision is made on the basis of a simple binary function. The first limitation of such systems is encountered, where all initially defined rules should return the ”true” value to open the position at the same time. By increasing the number of rules included in the system results in a reduction of the number of possibilities for the decision maker and often none of the variants are considered as a promising. The traditional system disregard situations when all the indicators are very close to satisfying the assumed rules. Such situations can be in general much more promising than in the case of the signal when the rules are satisfied for only one indicator.

To cover this gap we propose to apply a multicriteria fuzzy approach. In this approach, all considered indicators are considered jointly. Each indicator is represented by a criterion in a multicriteria space of variants. The traditional strict rules are replaced by fuzzy rules. Values of the criteria are calculated by introduced membership functions. The decision maker can control the introduced fuzziness using concepts of aspiration and reservation points adopted from the reference point approach of the multicriteria analysis (see [42]). An algorithm generating non-dominated variants in the criteria space is proposed. Concepts of the domination cons applied in the algorithm ensure the high computational efficiency of the algorithm approved in experiments made on real data from the Forex market.

We introduce the risk profile of the decision maker in the form of a single point in the criteria space. It seems to be consistent with the regulations imposed by the European Union according to the directive MiFID II [1]. This law forces every entity present on the market to estimate the so-called risk profile of the single decision maker. The risk profile is built on the basis of a questionnaire, which is further used to estimate the risk aversion of the decision maker. In our work, we propose to use the so-called reservation point representing the minimal acceptable risk taken by the decision maker during the trading session.

We present the proposed multicriteria approach for n technical indicators. The introduced fuzzy concepts are shown on the example of three indicators: the moving averages (MA), Relative Strength Index (RSI) and Commodity Channel Index (CCI). For the three indicators, the results of computation experiments on real data from the Forex market are presented and analyzed. Examples of the technical indicators are presented in Section 2. Section 3 introduces the concept of the traditional (crisp) trading system. Section 4 includes a detailed description of the proposed fuzzy approach along with definitions of all fuzzy membership functions included in the system. The algorithm which generates the non-dominated variants for the decision maker is proposed. Section 5 contains the results of various experiments with different real-world data sets. The discussion presented along with experiments results points out the weakness of existing crisp systems and emphasizes the advantages of the proposed fuzzy system.

2. Related Works

The existing trading system can be divided into two main groups. The first group includes systems related to the High Frequency Trading (HFT), where strategies are mostly based on the complex algorithms capable to effectively analyze multiple markets. One of their main features is a very short transaction time which leads to small profits. Duration of the single transaction rarely exceeds a few seconds. Despite the efficiency of such systems, their applicability for single decision makers is debatable. Issues related to the mentioned HFT are focused on examining factors like market liquidity and its impact on the system efficiency [23], [21]. There are also articles measuring the impact of the unexpected events on the profits generated by the transaction system. Despite the fact of general high interest in the HFT, there are authors [35] pointed out, that possible outcomes offered by the HFT approach are decreasing.

At the same time, the group of trading systems dealing with the Low Frequency Trading (LFT) gains more attention. Opposite to the HFT approach, in the LFT concept, reaction time pays less important role and duration of the single transaction may last longer than in case of HFT approach. In the following, we discuss the traiding systems applying the LFT approach.

We propose to use in the trading systems the ideas of fuzzy sets theory as were originally introduced by [44]. There are multiple publications related to the use of fuzzy concepts with stock data (rarely with Forex data) such as [39, 19, 40]. In this article, the fuzzy sets view is combined with the multicriteria analysis. One of the crucial concepts in the analysis relates to so-called aspiration levels introduced by Wierzbicki in [41], who provided a mathematical background for satisfying the decision making. In the papers [11, 33] and in the newer one [2] ideas of multicriteria decision making in the case of fuzzy sets are developed.

Systems based on new trading rules generated on the basis of existing technical analysis indicators often rely on approximate mechanisms such as metaheuristics. Examples of such papers include genetic programming [26], grammar evolution [9], and evolutionary algorithms [6]. Such approaches are based on the concept of setting optimal values of the technical indicators. More recent papers in this field rely on the concept of evolutionary multicriteria algorithms [7] and particle swarm optimization [5]. Approximate approaches in this field are presented in [31] and [30].

It is worth noting new combined technical analysis indicators used for decision support, however, there is only a small group of articles in this field. An example of such work may be found in [16], in which a new indicator correlated with the risk factor was presented. An extension of this concept based on the mathematical evidence of trends in financial data was presented in [15]. Another example is a new moving minmax indicator described in [36]. This indicator was used as a chart smoothing tool allowing to ignore small price corrections. It is worth noting that there are very few approaches in which decision support systems are preferred over automated trading. In most of the approaches, the complete trading system is treated like a black box, where the set of input data is transformed into the output data signal; however, in article [25] the authors proposed an approach in which it is possible to set parameters representing the risk aversion.

The rule-based trading systems are not the unique class of systems used on the Forex market. There are also trading systems based on neural networks. Publications related to this subject emerged already in 1995 when a neural network was used to generate a preliminary signal [12]. To the best of the authors' knowledge, using the neural networks on the market was described in [29] as in one of the first articles dedicated to this area. Newer approaches include the use of neural networks for data prediction [43] or applying technical indicators as neural network entry points [37]. A self-organizing map is used as a mechanism of detecting correlations between Japanese candlesticks. A similar concept was proposed in [14], where the k-means algorithm was used to detect some Japanese candlestick patterns. The authors of [27] proposed to use different volatility measures as an input for the support vector machines. The ARIMA model was compared with the artificial neural network for the prediction on the Forex market in [24]. More complex systems involving the use of modern metaheuristics like Cuckoo Search Algorithm [3] or heuristic-based trading systems combining different trading rules [31] were also proposed.

3. Background

We define a trading system as a pair:

$$T_s = \{C, AT\}, \tag{1}$$

where C is the set including the single, or multiple currency pairs analyzed by the trading system, while AT includes all rules generated on the basis of market indicators used in the trading system. In its trivial form, AT may include one up to many different rules, while more complex approach involves using subsets of AT, where each subset relates to C. Thus different sets of rules are applied at the same time to the same set of currency pairs.

Below we introduce a simple division of the trading systems. The division is made on the basis of number of currency pairs and transaction concepts involved in the decision process:

- one currency pair – one set of rules – the simplest concept used on the Forex market. In this approach, there is only one set of rules used with the single currency pair. Often some additional elements correlated directly with the currency pair characteristics are used in such a system.
- one currency pair – n sets of rules – a signal may be generated on a basis of a few different sets of rules. This mechanism is used for example in the social trading, where there is a possibility to copy orders from different transaction systems.
- n currency pairs – one set of rules – this holds the assumption, that the same set of rules is capable to give proper signals to more than one currency pair. In such a situation, the system is active for any considered set of currency pairs.
- n currency pairs and m sets of rules – the most complex mechanism which involves parallel using of different sets of rules. Even in the case of short-term trading it may lead to the concept of portfolio building, where elements of such portfolio are currency pairs.

3.1. Decision Process on the Forex Market

We define three independent phases, where each phase has a crucial impact on the trading process effectiveness. There are different issues referring to the effectiveness. They include not only profit maximization but also minimizing the maximum drawdown, duration of the open position, risk diversification and the time which is devoted to the currency pair analysis. The general schema of opening and managing the position on the Forex market is presented in fig. 1.

In this figure we present three different phases referring to the process of making the decision on the market. Each phase includes different mechanisms capable to effectively manage the position. Below we introduce and shortly describe each of the three phases:

- opening position – a crucial phase which includes concepts related with opening the single or multiple positions. At this phase, there is a need to consider a set of conditions, which must be fulfilled to open the position and move to the next phase. In the most cases, such conditions include the constant set of crisp rules related with different market indicators. Fulfillment of these rules leads to generating the signals. Depending on the type of trading system, this phase ends with opening a set of positions (in the case of the automatic trading system) or deriving

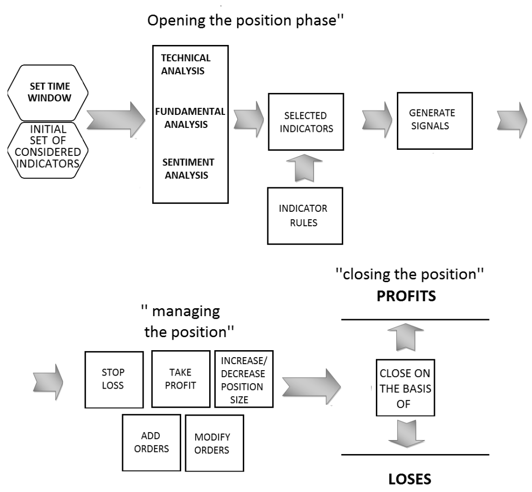

Fig. 1. Managing position phases

the set of signals to the decision maker (in the decision support trading system).

- managing the position phase – the phase focused on managing all opened positions, which, depending on the risk profile of the decision maker may include realizing small profits, opening additional positions, adjusting the taking profit or cutting loses levels and more.
- closing the position phase – in the trivial case, this phase may correspond to closing the position by hitting some price levels defined in the previous phase. Other possibilities may include closing the position on the basis of different market indicator, closing the position as the effect of opening an opposite position, or applying more complex rules related to the concept of portfolio management.

We purport, that crucial phase, which has the greatest impact on the effectiveness of the trading system is the first phase, in which the most preferred currency pair is selected from the set of all possible variants. Thus the first phase will be our main interest in this article.

4. Examples of Technical Indicators

In the following three equally important market indicators frequently used in the transaction systems are presented: the moving average and two oscillators. The proposed approach can, however, include any number of indicators.

The moving average equation is given as follows:

$$MA_p(t) = \frac{\sum_{i=1}^{p} price_i}{p}, \qquad (2)$$

where $MA_p(t)$ is the value of the moving average for period p in time t, $price_i$ is a currency pair value for a given time i, and p is the number of included values. An example concept based on moving averages may be found in [22].

The first technical indicator belonging to the group of oscillators is the Relative Strength Index (RSI):

$$RSI_p(t) = 100 - \frac{100}{1 - \frac{avg_{gain}}{avg_{loss}}}, \qquad (3)$$

where $RSI_p(t)$ is the value of the RSI indicator calculated on the basis of the last p periods in time t, avg_{gain} is the sum of gains over the past p periods and avg_{loss} is the sum of losses over the past p periods. The second oscillator to be used is the Commodity Channel Index (CCI):

$$CCI_p(t) = \frac{1}{c} \cdot \frac{price_{typical} - MA_p(t)}{\sigma(price_{typical})}, \qquad (4)$$

where $CCI_p(t)$ is the value of the CCI indicator calculated on the basis of p periods in time t, $price_{typical}$ is the typical price calculated as the average value of the Close, Low and High price from a given period, σ is the mean absolute deviation and c is the constant value used for scaling the mean absolute deviation value; for $CCI_{20}(t)$ this value is equal to 0.015.

5. Crisp Trading System

Actions of the typical crisp system can be described with the use of a binary activation function. The function takes the value one when a respective condition for a technical indicator is true and takes the value zero otherwise. The signal to open a position on the market is generated only in the first case.

Let us denote these conditions for the considered indicators as $cond_{MA}$, $cond_{RSI}$ and $cond_{CCI}$. A potential BUY signal may be generated for a given currency pair when at least one of the conditions is fulfilled:

$$f_{buy} = true \text{ if } (cond_{MA_{Buy}} = true \lor cond_{RSI_{Buy}} = true \lor cond_{CCI_{Buy}} = true), \qquad (5)$$

where conditions $cond_{MA_{Buy}}$, $cond_{RSI_{Buy}}$, $cond_{CCI_{Buy}}$ refer to the moving averages, RSI and CCI indicators respectively. If neither of the conditions is fulfilled the currency pair is removed from further analysis:

$$f_{buy} = false \text{ if } \neg(cond_{MA_{Buy}} = true \lor cond_{RSI_{Buy}} = true \lor cond_{CCI_{Buy}} = true), \qquad (6)$$

The typical conditions used in the existing trading systems for the considered technical indicators are presented below. The condition for the moving averages takes the form:

$$cond_{MA_{Buy}} = true \text{ if } (MA_{fast}(t) > MA_{slow}(t)) \land (MA_{fast}(t-1) < MA_{slow}(t-1)), \qquad (7)$$

where $MA_{fast}(t-1)$ is the value of the moving average from the lower period in time $t-1$, $MA_{slow}(t-1)$ is the value of the moving average from the higher period in time $t-1$. An example signal is generated if two moving averages cross each other.

In the case of the oscillators RSI and CCI, the binary activation functions are built on the basis of crossing the indicator with some predefined levels. For RSI this level will be 30. In the case of CCI it is -100. The conditions take the form:

$$cond_{RSI_{Buy}} = true \text{ if } (RSI_p(t-1) < 30) \\ \wedge (RSI_p(t) > 30), \tag{8}$$

and

$$cond_{CCI_{Buy}} = true \text{ if } (CCI_p(t-1) < -100) \\ \wedge (CCI_p(t) > -100), \tag{9}$$

where $RSI_p(t-1)$ and $CCI_p(t-1)$ denote respectively the values of RSI and CCI in time $t-1$.

6. Proposed Fuzzy Trading System

In the proposed system the different indicators are considered jointly and the activation conditions are fuzzy. Each currency pair is treated as a variant in a multicriteria decision space. Criteria in this space refer to particular indicators. Values of the criteria are defined by membership functions referring to particular indicators. It is assumed that the membership function for each indicator takes values in the range $\langle 0,1 \rangle$. The membership function takes the value 1 when the value 1 is achieved by the binary activation function in the crisp approach.

In the fuzzy approach, the original signal generated in the case of crisp approach is still included. However, the situation when the conditions for a given indicator are almost satisfied, omitted in the crisp approach, can be included in the fuzzy approach with the use of the membership function.

Let each currency pair c will be treated as a variant y in the decision space $\mathbb{R}^n$; thus every variant y will be denoted as the vector of criteria $y = (y_1, y_2, ..., y_n)$, $y_i \in \langle 0,1 \rangle$, $i = 1,2,...,n$, where n is the number of considered indicators. The criteria are defined by values of the membership function calculated for particular indicators.

Due to limited space, we introduce only membership functions related to the BUY signals. The membership functions for the SELL signals can be defined in a similar way. The membership function for the MMA indicator is proposed in the form:

$$\mu_{MA-BUY}(c) = \begin{cases} \frac{max - f_{low}}{max} \text{ if } (MA_{fast}(t) > MA_{slow}(t)) \\ \wedge (f_{low} < max) \\ \wedge (MA_{fast}(t-1) > MA_{slow}(t-1)) \\ 1 \text{ if } (MA_{fast}(t) > MA_{slow}(t)) \\ \wedge (MA_{fast}(t-1) < MA_{slow}(t-1)) \\ \frac{f_{high}}{max} \text{ if } (MA_{fast}(t) < MA_{slow}(t)) \\ \wedge (f_{high} < max) \\ \wedge (MA_{fast}(t-1) < MA_{slow}(t-1)) \\ 0 \text{ in other case} \end{cases} \tag{10}$$

where max is the maximal number of readings used in the calculations, f_{high} is a function used to count readings above the moving average with a higher period and f_{low} is a function used to count readings below the moving average with a higher period. It is assumed in the above calculations that in the case of reading without the crossover of moving averages the possibility of a trend change would rise while the present trend would continue.

The membership function defined for the RSI indicator is given as follows:

$$\mu_{RSI-BUY}(c) = \begin{cases} \frac{RSI_p(t)}{30} \text{ if } (RSI_p(t) < 30) \\ 1 \text{ if } ((RSI_p(t-1) < 30) \\ \wedge (RSI_p(t) > 30)) \\ \vee (RSI_p(t) = 31) \\ \frac{0.9}{RSI_p(t) - 30} \cdot \alpha \text{ if } (RSI_p(t) > 31) \\ \wedge (RSI_p(t) < 50) \\ \wedge (RSI_p(t-1) \leq 30) \\ 0 \text{ if } (RSI_p(t) > 50) \end{cases} \tag{11}$$

In the case of the CCI indicator the membership function takes the form:

$$\mu_{CCI-BUY}(c) = \begin{cases} 0 \text{ if } (CCI_p(t) < CCI_{min}) \\ \frac{CCI_p(t) - CCI_{min}}{-CCI_{min} - 100} \text{ if} \\ (CCI_p(t) > CCI_{min}) \\ \wedge (CCI_p(t) < -100) \\ 1 \text{ if } (CCI_p(t-1) < -100) \\ \wedge (CCI_p(t) > -100) \\ \frac{CCI_p(t) + 50}{-50} \text{ if } (CCI_p(t) > -100) \\ \wedge (CCI_p(t) < -50) \\ \wedge (CCI_p(t-1) > -100) \\ 0 \text{ if } (CCI_p(t) > -50) \end{cases} \tag{12}$$

where $CCI_p(t)$ is a value of the CCI indicator in the present time, CCI_{max} is the maximal considered CCI value and CCI_{min} is the minimal considered CCI value. A vector of scalar values in the range of $\langle 0;1 \rangle$ is generated in a given time t for all of the given indicators and represents each currency pair as a variant in the multicriteria space. In this space, we made multicriteria analysis and look for the Pareto optimal (non-dominated) variants. Respective domination relations have to be introduced.

The following relations between variants are introduced in $\mathbb{R}^n$ space:

Definition 1 A variant y is at least as preferred as a variant z if each criterion of y is not worse than the respective criterion of z.

$$y \succeq z \Leftrightarrow (y_1 \geq z_1) \wedge (y_2 \geq z_2) \wedge ... \wedge (y_n \geq z_n). \tag{13}$$

Definition 2 A variant y is more preferred (better) than a variant z according to the logical formulae:

$$y \succ z \Leftrightarrow (y \succeq z) \wedge \neg(z \succeq y). \tag{14}$$

An algorithm deriving non-dominated variants is proposed. The following notions are used in the algorithm: the ideal – aspiration point u, the reservation point $x = (x_1, x_2, ..., x_n)\_where x_i \in [0,1]$, the set of all variants Y, the set of points removed from the analysis in the algorithm Y^-, the set of points accepted for further analysis in the algorithm $Y^+ = Y \setminus Y^-$, the set of non-dominated variants ND. The aspiration point u refers to the case when BUY signals are generated for all indicators, i.e. when all the membership functions take the value 1. The reservation point x is defined by the minimum values of membership functions accepted by the decision maker.

The simplified idea of the algorithm is given below:

- **Step 0**. In this initial step, the sets Y and $ND = \emptyset$ are created. The aspiration point $u = (1, \dots, 1)$ and the reservation point x assumed by the decision maker are fixed.
- **Step 1**. The set Y^- is generated as the set of points dominated by the reservation point and removed from further analysis. All other points belong to the set Y^+.
- **Step 2**. If there exists variant $y \in Y^+$, $y = u$, then $ND = \{y\}$. End of the algorithm.
- **Step 3**. Each variant $y \in Y^+$ is checked: if $y \in Y^-$ then it is removed from further analysis, else it is compared with the points in the set ND (it is added to ND if ND is empty). For each point $z \in ND$, if y dominates z, then z is removed from ND, y is added to ND and the set Y^- is extended by the set of point dominated by y; if y is dominated by z then y is removed from analysis, i.e. removed from the set Y^+.

The algorithm ends, when all variants from the set Y^+ are checked.

In the algorithm, the concept of domination cons is used. For each point Y added to the set ND, the set Y^- is extended using the domination cone. The successive extensions of this set of points removed from analysis assure the high computational efficiency of the algorithm.

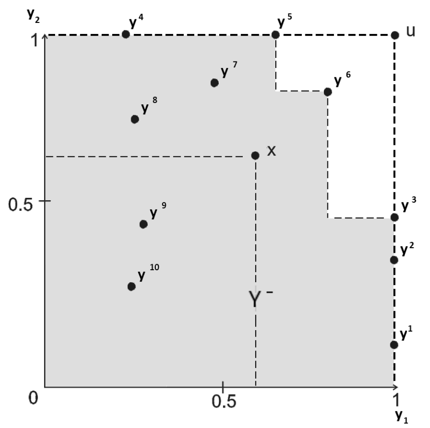

Fig. 2. An illustrative example

It has been proved that the algorithm derives all non-dominated variants in the set of variants non-dominated by the reservation point x. All other variants are eliminated from the analysis.

The algorithm has been implemented in a computer-based trading system to make experiments using real data from the Forex market.

The fuzzy approach introduces additional uncertainty which is not present in the case of the crisp approach. The decision maker selecting one of the variants derived by the fuzzy trading system takes a risk that the variant can be not effective. The uncertainty and risk depend on the distance of the reservation point from the aspiration point u. In the case of the reservation point which is close to the aspiration point, the risk is lower but only a few or even not any variant can be derived by the system. The risk is greater when the distance increases but the system can derive and propose a greater number of non-dominated variants. The decision maker is aware of this decides on the positioning of the reservation point.

Figure 2 presents variants analyzed by the algorithm in a two-dimensional criteria space. The aspiration point u and the reservation point x are shown. The algorithm, from the set of all variants, selects the non-dominated variants y^3, y^5 and y^6. The shadowed area represents the set of points dominated by the variants above.

The classical crisp system generates 5 signals for variants y^1, y^2, y^3, y^4, y^5 not informing, which of them are more or less promising. Let us note, that variants y^1, y^2, y^4 are eliminated and removed from analysis by the proposed fuzzy system.

7. Numerical Experiments

In this section, we present the results of numerical experiments with real data from the Forex market. In the experiments, the proposed fuzzy trading system is compared with the existing crisp trading systems for the three indicators considered above. Our main motivation was to estimate the number of variants potentially interesting for the decision maker, indicated by the different trading systems. We tested 30 successive readings. By a reading, we mean a single situation on the price chart which is observed in a specific time window. The systems derived variants interested for the decision maker in every reading. We selected three different time windows (frames) corresponding to the scalping system with aggressive trading (the length of a single time window was equal to 5 minutes), to the intraday system (the length of a single time window was equal to 1 hour) and finally to the long-term trading with the length of a single time window equal to 1 day. The overall length of the experiments in the case of the scalping system was equal to $30 \cdot 5 = 100$ minutes, for the intraday system the overall length of the experiments was equal to 30 hours, and 30 days for the long-term trading. Information about the selected time windows can be found in Table 1. The number of variants (currency pairs) available and analyzed in every reading was always equal to 68. Systems with two and three indicators were analyzed separately.

Tab. 1. Data sets summary

	Starting Date	Ending Date
5 Minutes	2017 IV 03 8.00	2017 IV 03 10.25
1 Hour	2017 I 02 7.00	2017 I 03 12.00
1 Day	2017 II 03 00.00	2017 III 15 00.00

7.1. Data Example With Two Indicators

We selected an approach with two criteria based on the CCI and RSI indicators and the most popular 1-hour time window and the date of January 3, 2017, which was connected with the opening of the New York trading session. All generated variants are presented in the two-dimensional criteria space shown in Fig. 3a. In the analysis, we focused only on the buy signals. A similar analysis can be performed for short sells.

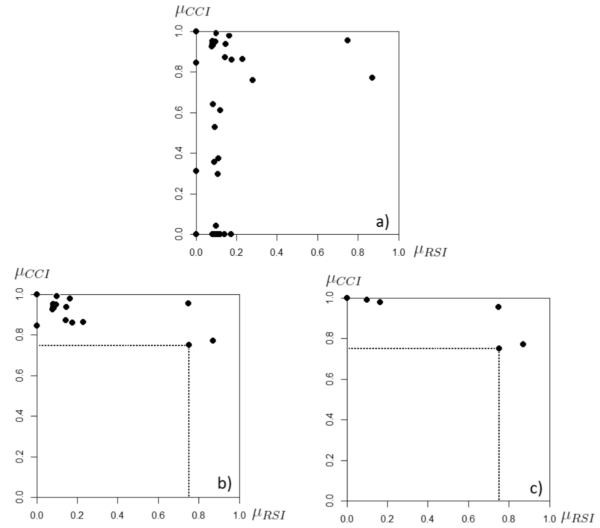

Fig. 3. Two-criteria example with $y_1 = \mu_{CCI}, y_2 = \mu_{RSI}$. a) all 68 variants generated; b) set of considered variants along with the reservation point x; c) set of non-dominated variants derived for the decision maker

A characteristic arrangement of variants in the criteria space can be observed in the case of the system based only on two criteria. In a large number of variants only one criterion has a very high value (close to 1), while the other criterion is often below the acceptable value. The results of the fuzzy approach are presented in Fig. 3b. The dot lines in Fig. 3b and Fig. 3c denote the position of the reservation point $x = (0.75, 0.75)$. First, all variants dominated by the reservation point x are excluded from further analysis. This situation can be observed in Fig. 3b. After obtaining a set of variants that are potentially acceptable to the decision maker, the proposed algorithm is used to generate a set of non-dominated solutions ND. These variants can be seen in Fig. 3c. Finally, 5 non-dominated, different variants were derived for the decision maker.

In the case of the crisp approach, the number of variants derived for the decision maker is relatively small, while the fuzzy approach can be used to extend the set of non-dominated variants derived for the decision maker.

7.2. Number of Variants Derived to the Decision Maker

Selected results of the experiments are presented in Table 2 for the 5-minute time window, in Table 3 for the 1-hour time window and in Table 4 for the largest 1-day time window. The tables present the numbers of the non-dominated variants derived by the proposed fuzzy system for four different values of the reservation point: $x = (x_1, x_2, x_3)$, $\forall_i\ x_i = 0.7$, $x_i = 0.8$, $x_i = 0.9$, $x_i = 0.95$ (the columns are marked by $x = 0.7, x = 0.8, x = 0.9, x = 0.95$ respectively). Theses results are compared to the numbers of signals generated by three versions of the crisp approach: Crisp*, Crisp** and Crisp***, wherein the Crisp* approach a signal is generated and presented to the decision maker when at least one of the conditions defined by the binary activation function is satisfied, in the second considered approach – Crisp** at least 2 conditions must be fulfilled, while in the last considered Crisp*** approach all the 3 conditions have to be satisfied. In the last case, the generated signal corresponds to the variants equal to the aspiration point u.

The Crisp* approach overproduces the number of variants proposed to the decision maker, thus selection of a single variant by the decision maker to make the trading decision may be extremely difficult. A decreasing number of criteria is observed in the case of Crisp** so that it leads to an empty set of variants derived for the decision maker. In the case of Crisp***, which corresponds to the situation, in which a variant equal to the aspiration point u should be found, even a single solution was not observed.

The fuzzy approach generates relatively small sets of non-dominated variants which are far easier to analyze by the decision maker.

We use bold font to indicate in the tables the desirable market situations when the number of variants derived by the fuzzy system for the decision maker is 4, 3 or 2. We use the italic font to indicate situations when the empty set of variants derived for the decision maker by the Crisp** method is observed, e.g. in readings 6, 7, 9, 10, 12, 13 in Table 2; see also readings 1, 3 – 6, 8 in Table 3 and readings 1, 3 – 7, 9, 13 in Table 4.

There were also situations when the proposed fuzzy approach derives a relatively large set of variants, which may be difficult to analyze by the decision maker. In the case of the 1-hour and 1-day time window, such a situation is undesirable, but the decision maker has additional time to perform the analysis. While for the smaller time windows such situations need some additional extension of the proposed approach. Such an extension is planned in further works with the use of respective ranking methods.

It is crucial to understand that all variants derived for the decision maker in the case of the fuzzy approach are non-dominated, while in the case of the Crisp* approach (due to the binary activation function) many variants indicated by the system can be dominated. In the case of the crisp system, the decision maker has no information which of the generated variants is better or worse. This leads to an important observation that in the case of the crisp approach a single variant is treated as acceptable if any criterion is equal to 1. Thus, in the case of two variants, $y^1 = (0, 0.05, 1)$ and $y^2 = (0, 0.95, 1)$, both of them are treated as equally good as $(0, 0, 1)$, while in the fuzzy approach it is possible to distinguish these two variants in favor of y^2 which strictly dominates the first variant.

Similar experiments were conducted for two remaining time windows observed in Table 3 and Table 4. In the case of the reservation point being far from the aspiration point $x = (0.7, 0.7, 0.7)$ the sets of generated variants often exceeded the assumed limits. The fuzzy system generates only a few variants, which can be easily analyzed. In the 1-day tie window, an interesting situation could be observed in readings 4, 18 and 20, where the $Crisp^{**}$ could not deliver even a single variant while $Crisp^{**}$ generated a number of variants that greatly exceeded the analytical capabilities of the decision maker. The fuzzy approach, in turn, once again allowed to obtain a reasonable number of non-dominated variants in successive readings.

The obtained results are also presented in the graphical form for the fuzzy approach with $x = 0.95$, and compared to the $Crisp^{*}$ and $Crisp^{**}$ approaches. The results from Table 3 are presented in Fig. 4, remaining results from Table 4 and 5 are presented respectively in Fig. 5 and Fig. 6. One can easily observe disproportions in the number of variants generated by both crisp methods and a reasonable number of non-dominated variants derive from the proposed fuzzy system.

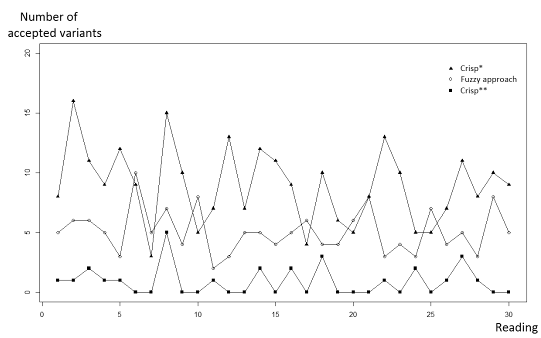

Fig. 4. 5-minute time window linear chart for the fuzzy approach with $x = 0.95$, $Crisp^{*}$ and $Crisp^{**}$

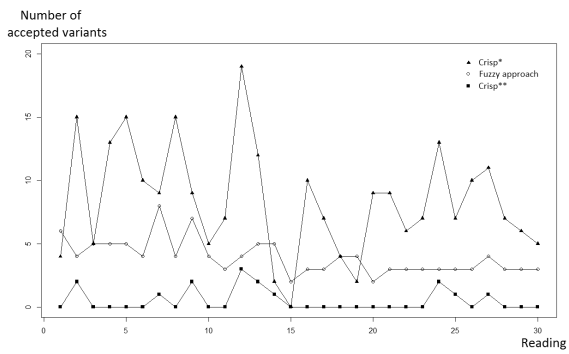

Fig. 5. 1-hour time window linear chart for the fuzzy approach with $x = 0.95$, $Crisp^{*}$ and $Crisp^{**}$

The number of solutions generated in the case of $Crisp^{*}$ fairly exceeds analytical capabilities of the decision maker, while $Crisp^{**}$ often generates no solutions at all, and the most restrictive crisp approach

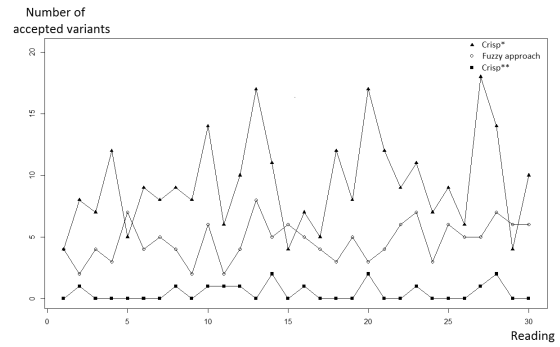

Fig. 6. 1-day time window linear chart for the fuzzy approach with $x = 0.95$, $Crisp^{*}$ and $Crisp^{**}$

$Crisp^{***}$ not delivered any variants at all. The proposed fuzzy approach gives the possibility to control the number of generated variants on the basis of the risk aversion adjusted with the use of the reservation point. It may be easily extended on the trading systems with four and more indicators represented by criteria in a multicriteria space of possible decisions.

7.3. Sensitivity Analysis for the Reservation Point

To investigate the impact of the reservation point x on the results achieved by the fuzzy approach, we additionally performed the sensitivity analysis for this particular parameter. Expected results should lead to the conclusion, that equivalent decreasing the x value for all criteria increases the number of variants derived for the decision maker. At the same time the risk related to these variants will be higher than in the case of the crisp approach. However, decreasing only one component x_i relates to the respective criterion and the risk is increasing only in the case of this criterion. This should also lead to increasing number of variants derived for the decision maker without excessive risk.

The performed sensitivity analysis referring to the reservation point x allows to estimate the impact of x on the overall number of variants generated for the decision maker. It is obvious, that the number of variants will be decreasing, while x is moved towards the aspiration point u. Thus two important questions emerge. First of all, we investigate, whether the selected data set (to be more precise, the length of the time window) has an impact on the overall number of variants in the set ND.

The shape of the chart indicates, that the average number of variants derived to the decision maker gradually rises without any sudden changes, while the reservation point is moved out of the aspiration point (see Fig. 7). It would indicate the positive correlation between the position of the x and the number of variants available for the decision maker without any observable anomalies. Such anomalies would include large leap for some boundary values of the point x.

Results for two out of three analyzed time windows are presented in Fig. 7. While the position of the reservation point x is moved away from the aspiration point u, the average number of variants interesting for the decision maker is increased. Moreover, no visible

Tab. 2. Number of variants available to the decision maker for the 5-minute time window

	x = 0.7	x = 0.8	x = 0.9	x = 0.95	Crisp*	Crisp**	Crisp***
Reading 1	7	6	5	5	8	1	0
Reading 2	6	6	6	6	*16*	1	0
Reading 3	10	10	6	6	*11*	**2**	0
Reading 4	9	8	7	5	9	1	0
Reading 5	6	**4**	**3**	**3**	*12*	1	0
Reading 6	*12*	*12*	*11*	10	9	*0*	0
Reading 7	7	7	6	5	**3**	*0*	0
Reading 8	8	8	8	7	*15*	5	0
Reading 9	7	7	7	**4**	10	*0*	0
Reading 10	9	8	8	8	5	*0*	0
Reading 11	**4**	**4**	**3**	**2**	7	1	0
Reading 12	6	5	**4**	**3**	*13*	*0*	0
Reading 13	8	7	6	5	7	*0*	0
Reading 14	6	5	5	5	*12*	**2**	0
Reading 15	9	9	5	**4**	*11*	*0*	0
Reading 16	10	8	6	5	9	**2**	0
Reading 17	*11*	8	7	6	**4**	*0*	0
Reading 18	6	6	**4**	**4**	10	**3**	0
Reading 19	9	8	5	**4**	6	*0*	0
Reading 20	8	7	6	6	5	*0*	0
Reading 21	*12*	*11*	10	8	8	*0*	0
Reading 22	**4**	**4**	**3**	**3**	*13*	1	0
Reading 23	8	8	6	**4**	10	*0*	0
Reading 24	**3**	**3**	**3**	**3**	5	**2**	0
Reading 25	7	7	7	7	5	*0*	0
Reading 26	5	**4**	**4**	**4**	7	1	0
Reading 27	5	5	5	5	*11*	**3**	0
Reading 28	**4**	**3**	**3**	**3**	8	1	0
Reading 29	9	9	9	8	10	*0*	0
Reading 30	8	7	7	5	9	*0*	0

large leaps are observed.

8. Conclusion

Existing trading systems based on the crisp approach have a number of disadvantages. In this article, we proposed the multicriteria fuzzy trading system including three different technical indicators. Trading rules for both: the classical crisp and the proposed fuzzy trading system were defined. A new concept of the fuzzy trading system including the possibility to generate sets of non-dominated variants derived to the decision maker was introduced as well. All concepts of trading systems were experimentally verified and tested on the limited set of technical indicators.

We experimentally verified, that proposed fuzzy trading system is capable to effectively derive Pareto-optimal variants for the decision maker. The proposed system was compared in the experiments to three versions of the crisp system: Crisp*, Crisp**, Crisp***. In contrary to the fuzzy approach, the crisp system derives very small (or even none) variants in the case of the Crisp** or number of variants is too large to be effectively handled by the decision maker – what was observed in the case of the Crisp*. The third version of the classical trading system Crisp*** was not capable to derive even single variant. One of the most important advantages of the proposed approach is that the fuzzy system is capable to derive sets of non-dominated solutions, which could be further used to develop a system for generating portfolios of variants.

Further works should include the application of methods allowing ranking of the derived variants according to preferences of the decision maker. Besides the further development of the fuzzy concept, a more robust and less computationally expensive algorithm capable to derive a set of non-dominated variants should be developed as well.

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Tab. 3. Number of variants available to the decision maker for the 1-hour time window

	x = 0.7	x = 0.8	x = 0.9	x = 0.95	Crisp*	Crisp**	Crisp***
Reading 1	10	8	7	6	**4**	*0*	0
Reading 2	9	8	5	**4**	*15*	**2**	0
Reading 3	*11*	9	6	5	5	*0*	0
Reading 4	8	6	5	5	*13*	*0*	0
Reading 5	6	6	5	5	*15*	*0*	0
Reading 6	7	6	**4**	**4**	10	*0*	0
Reading 7	10	9	9	8	9	1	0
Reading 8	7	7	5	**4**	*15*	*0*	0
Reading 9	9	8	7	7	9	2	0
Reading 10	7	**4**	**4**	**4**	5	*0*	0
Reading 11	**4**	**3**	**3**	**3**	7	*0*	0
Reading 12	6	6	5	**4**	*19*	**3**	0
Reading 13	10	9	5	5	*12*	2	0
Reading 14	6	5	5	5	**2**	1	0
Reading 15	7	5	**3**	**2**	*0*	*0*	0
Reading 16	**3**	**3**	**3**	**3**	10	*0*	0
Reading 17	**4**	**4**	**3**	**3**	7	*0*	0
Reading 18	6	**4**	**4**	**4**	**4**	*0*	0
Reading 19	6	6	5	**4**	**2**	*0*	0
Reading 20	**4**	**4**	**3**	**2**	9	*0*	0
Reading 21	5	**4**	**3**	**3**	9	*0*	0
Reading 22	7	5	**4**	**3**	6	*0*	0
Reading 23	**4**	**3**	**3**	**3**	7	*0*	0
Reading 24	**4**	**4**	**4**	**3**	*13*	2	0
Reading 25	**4**	**4**	**4**	**3**	7	1	0
Reading 26	6	6	5	**3**	10	*0*	0
Reading 27	6	6	**4**	**4**	*11*	1	0
Reading 28	5	5	**3**	**3**	7	*0*	0
Reading 29	5	**4**	**4**	**3**	6	*0*	0
Reading 30	**3**	**3**	**3**	**3**	5	*0*	0

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Tab. 4. Number of variants available to the decision maker for the 1-day time window

	x = 0.7	x = 0.8	x = 0.9	x = 0.95	Crisp*	Crisp**	Crisp***
Reading 1	9	9	7	**4**	**4**	*0*	0
Reading 2	**3**	**3**	**3**	**2**	8	1	0
Reading 3	6	6	5	**4**	7	*0*	0
Reading 4	**4**	**3**	**3**	**3**	12	*0*	0
Reading 5	7	7	7	7	5	*0*	0
Reading 6	7	6	**4**	**4**	9	*0*	0
Reading 7	8	6	6	5	8	*0*	0
Reading 8	6	5	**4**	**4**	9	1	0
Reading 9	6	6	**2**	**2**	8	*0*	0
Reading 10	9	7	6	6	*14*	1	0
Reading 11	8	5	**3**	**2**	6	1	0
Reading 12	5	**4**	**4**	**4**	10	1	0
Reading 13	9	8	8	8	*17*	*0*	0
Reading 14	8	7	6	5	*11*	2	0
Reading 15	8	6	6	6	**4**	*0*	0
Reading 16	5	5	5	5	7	1	0
Reading 17	**4**	**4**	**4**	**4**	5	*0*	0
Reading 18	6	**4**	**4**	**3**	*12*	*0*	0
Reading 19	6	5	5	5	8	*0*	0
Reading 20	**4**	**4**	**3**	**2**	9	*0*	0
Reading 21	5	**4**	**3**	**3**	9	*0*	0
Reading 22	7	5	**4**	**3**	6	*0*	0
Reading 23	**4**	**3**	**3**	**3**	7	*0*	0
Reading 24	**4**	**4**	**4**	**3**	*13*	2	0
Reading 25	**4**	**4**	**4**	**3**	7	1	0
Reading 26	6	6	5	**3**	10	*0*	0
Reading 27	6	6	**4**	**4**	*11*	1	0
Reading 28	5	5	**3**	**3**	7	*0*	0
Reading 29	5	**4**	**4**	**3**	6	*0*	0
Reading 30	**3**	**3**	**3**	**3**	5	*0*	0

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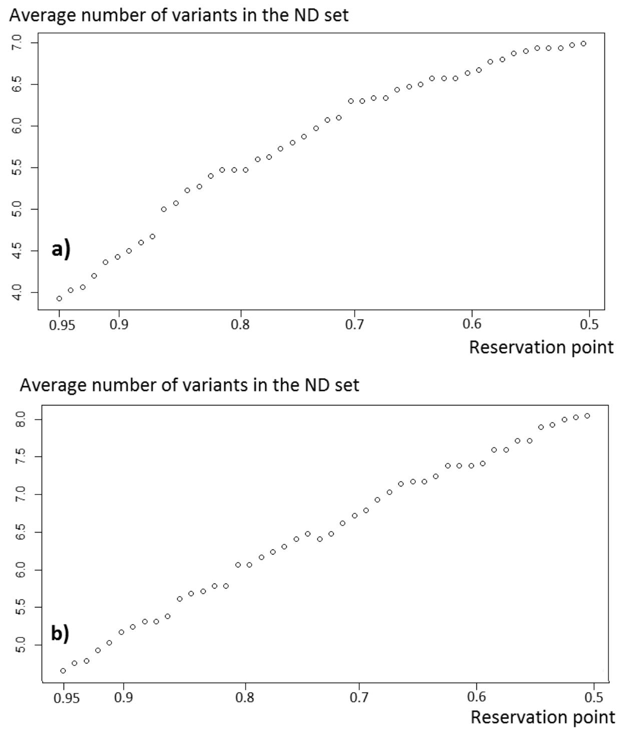

Fig. 7. Sensitivity analysis for two selected data sets. (a) 1 hour time window with starting date 2 I 2017 7.00 and ending date 3 I 2017 2.00; (b) 1 day time window with starting date 3 II 2017 and ending date 5 III 2017

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