

THE APPLICATION OF COGNITIVE COMPUTER GRAPHICS TO ECONOMIC DATA EXPLORATION

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Abstract:

This paper describes the concept of cognitive graphics and its importance for the analysis and visualization of economic data. A brief description of cognitive graphics systems is presented. Particular attention is given to the NovoSpark Visualizer system that not only enables data exploration but also allows color transformation of numerical data. An experiment is conducted to demonstrate how the application of color spectrum results in activation of cognitive thinking, and thus allows the researcher to quickly find a solution to the problem.

Keywords: *cognitive computer graphics, color spectrum, NovoSpark Visualizer*

1. Introduction

People's desire to communicate their ideas and concepts, vision of the world and knowledge in the form of a graphic image is observed from the beginning of civilization. People exchange information by visual means, both abstract and information with direct basis in reality. Data visualization is based on the unique ability of a person to process huge amounts of visual information in a short period of time. Nowadays, visualization helps in conducting scientific research, is routinely used in technical disciplines, medicine, economics and management, in art, that is, whenever we are struggling with problem solving and decision making.

Ensuring adequacy of the decisions is possible only on condition of proper interpretation of data analysis results, which usually is impossible without data visualization and analysis, especially when we try to process a large number of parameters simultaneously. In this case, visual analysis becomes more complicated because of the increased dimensionality of the input data. Additional graphics objects and verbal and numerical information describing these objects are emerged in the image and, therefore, increase the number of tries to interpret the results of the analysis. So, the need for multidimensional data visualization as a single image arises. This image allows for easy interpretation of the results. Combination of computer graphics and cognitive approach provides a way to solve the problems quickly and increase efficiency of the tasks.

The main purpose of the article is to present the cognitive approach for visualization of multidimensional data and to show the effects of its application. An example of problem solving with the NovoSpark Visualiser tool is provided.

2. The Concept and Characteristics of Cognitive Graphics

The concept of cognitive graphics was first presented by the Russian scientist A. Zenkin in the paper describing the research on the properties of various concepts in the number theory. Cognitive graphics is a set of means and methods of graphic presentation of the problem, which allows you to see the solution right away or at least get a hint [Zenkin, 1991]. Graphics is the direction of cognitive machine graphics that combines visualization mapped on a computer screen with the cognitive processes taking place in the human brain. Cognitive graphics allows the user to see a new direction or approach to solving a problem that was not visible when using traditional tools of data visualization. Verbal or symbolic knowledge about an object, phenomenon or situation is rarely able to provide such a clear perception and understanding as visual perception, for which a right hemisphere of the human brain is responsible.

There are three main cognitive functions of cognitive graphics [Zenkin, 1991]:

1. Creation of the models of knowledge representation, which would be able to provide uniform methods of objects presentation, typical for logical thinking and images created with imaginary thinking;
2. Visualization of knowledge, we have not yet found text descriptions;
3. Finding ways to move from the observed images to formulation of hypotheses about the mechanisms and processes that are hidden in the dynamics of the observed patterns.

So, in other words, the key functions of cognitive systems are illustrative and cognitive function. The first one allows you to build various types of diagrams, charts, histograms, charts, while the other helps the user to exploit the natural ability to think in terms of spatial images.

Visualization of multidimensional data in a single image not only allows performing qualitative analysis, but also using the results of visualization to confirm or refute hypotheses. Significant effects can give the application of cognitive approach for visualization of multidimensional data. At present, there are no standard cognitive principles of information visualization, but there is an understanding that graphic images contain a compressed available for the user information to help in solving the problem. Each image accumulates the information in the life cycle of an object, and then an expert interprets it using the experience and knowledge.

In the recent years a number of systems with cognitive approach have been created to explore and visualize data. These systems have found their applications in medicine, economics and management, statistics, construction, architecture, mathematics, physics, microelectronics, etc. [Novoselov, 2008; Rakczeeva, 1993]. These systems support the problems salvation connected with graphs, with three-dimensional map of the terrain, with structures of complex technology, with constructions of various surfaces and even parts of the human body.

Interfaces with cognitive graphics can also be used for educational purposes, such as learning and knowledge management. The effectiveness of using such a technology is based on involvement of the organs of human perception in learning and control processes. For example, in the process of astronauts training the graphical tools for creation of 3D models and three-dimensional worlds are used.

When designing systems using cognitive computer graphics, intelligent tools are needed that perform the key functions: cognitive and illustration. Such systems should fit into different areas of application and at the same time should be sufficiently mobile in different operating systems. Moreover, such a tool should create and display three-dimensional graphical objects with orientation for modern software environment, to add any information to graphic objects, to provide the ability to use graphic materials from other graphics systems.

3. The Visible Color Spectrum and its Application in Computer Graphics

As previously stated, the most important element of cognitive graphics systems is adequate representation of the data. The color spectrum is very often used in cognitive graphic systems for visualization of multidimensional data. A human being can see and understand all colors in a spectrum in a natural intuitive way. The spectrum of colors is part of the electromagnetic spectrum that can be detected by the human eye. The spectrum of colors consists of the following simple colors: violet, indigo, blue, cyan, green, yellow, orange and red. A simple color is a visual impression caused by an electromagnetic wave with a specified length between 380 nm and 740 nanometers.

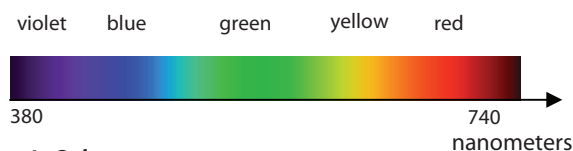


Fig. 1. Color spectrum

Each color has specific meaning. Cool colors are usually not reassuring. Violet is a combination of blue and red. Red brings dynamism and activity to distanced blue. Light blue often has an effect of calmness and relaxation. Green has the same calming effect as well as blue, but it also contains some of the energy of yellow. Generally, the green has the balancing and harmonizing impact. Warm colors convey emotions from simple optimism or hope to anger. These are typically energetic and positive colors. Yellow is the first warm color in the spectrum. It carries the warmth, joy, and

at the same time it is gentle. The orange color usually creates excitement. Red has long been recognized as the color of strong emotions, and even aggression.

Because of the intuitive color perception by the human, in cognitive graphics systems suitable colors reflect the values of various parameters. So, purple indicates minimal value of the selected parameter, red indicates the maximum value of the parameter, and green – the average. The similar scheme has been used in a variety of computer graphics systems, and among others the NovoSpark Visualizer system presented in the next part of this article.

4. NovoSpark Visualizer System – Conception and Methodology

The necessity of displaying three or more related parameters/variables using traditional methods of data visualization (charts, diagrams, icons, and others) cannot cope with the requirements for mapping and, most importantly, subsequent interpretation of resulting images [Chernoff, 1973; Bajaj, 1999; Cleveland, 1993; Friendly, 2008; Friedman, 2008; D'Ocagne, 1885; Inselberg, 1985; Inselberg, 2009]. The main disadvantages and difficulties of traditional methods are:

- Determination of dimensionality;
- Different scales of parameters/variables;
- Detection of data trends;
- Ambiguity in mapping of dynamic changes in parameter/variable values.

The maximum dimensionality of images is limited by dimensionality of the data presentation tools, such as paper or computer screen. Therefore dimensionality of an image representing static state cannot exceed three, and, in the case of dynamic data (when one of the measurements specifies a time interval) maximum number of dimensions that can be shown on an image is limited to two.

Visualization of the integral system state must meet the following requirements:

- Usage of all state parameter values to create an image without loss of information;
- Dimensionality of a state image must not exceed two dimensions;
- Produce a single static image for state dynamics.

In mathematics transformation of a set of discrete values in a continuous two-dimensional space [Fourier, 1808] has been known for years. The first attempt to apply such transformation for visualization of multidimensional data has been described in [Andrews, 1972] and was named “Andrews Curves”. Later, independently of this research Volovodenko *et al.* [Volovodenko, 1991] proposed a new approach that allowed visualization of both static and dynamic data on one integral image. The proposed methodology has been adapted to use modern computer technologies that resulted in implementation of the NovoSpark Visualizer tool [Novospark 2007, Novospark 2009].

The method of visualizing multidimensional objects and processes is based on two isometric spaces, whereby objects from one space (the data space) are called *the originals*, while objects from the other space play the role of *images*.

4.1. The Image of a Multidimensional Observation

Let us select a point-observation A in N -dimensional affine point-vector space R_N of the originals $A = (a_0, a_1 \dots a_{N-1})$ and form linear combination $f_A(t)$ of functions $\{P_i(t)\}_0^\infty$ by using the following rule:

$$f_A(t) = \sum_{i=1}^{N-1} a_i P_i(t) \quad (1)$$

Where $P_i(t)$ are Legendre polynomials, i.e. orthogonal polynomials with weight 1 defined on the segment $t = [0, 1]$.

Relationship $A \leftrightarrow f_A(t)$ is a one-to-one correspondence and, hence, changes in parameters/coordinates a_i imply the corresponding changes in the function $f_A(t)$.

Figure 2 shows images of the multidimensional observations $A = (0,0,0,0,0,0,1)$ and $B = (0,0,0,0,0,0,-1)$ in the two-dimensional coordinate system $\{f, t\}$.

Since image of a point in a multidimensional space is a function-curve, it can be "painted" in accordance with the function values. Applying a color palette emphasizes similarities and/or differences in images and allows viewing these images in the coordinate system $\{z, t\}$, where one can render an arbitrary image "width" along the z-axis. Such representation of an image $f_A(t)$ as a surface is called a "spectrum" of the multidimensional point-observation. Figure 3a shows colored images of the two multidimensional observations A and B in the coordinate system $\{f, t\}$

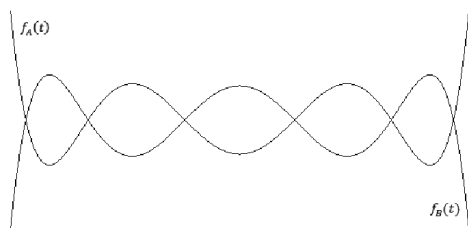


Fig. 2. Images of multidimensional observations A and B

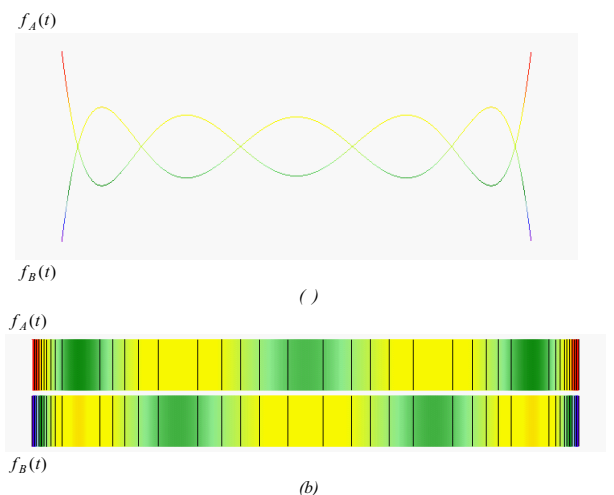


Fig. 3. Images of multidimensional observations A and B: (a) in the coordinate system $\{f, t\}$ and (b) in the coordinate system $\{z, t\}$

and the corresponding spectrums, 3bd in the coordinate system $\{z, t\}$, where vertical lines on the spectrums correspond to arbitrarily selected spectrum surface isolines.

4.2. The Image of a Multidimensional Process

A multidimensional process can be considered as a set of sequential transitions from one multidimensional observation (state) to another; or as a set of multidimensional segments sequentially connecting system states – points in the multidimensional space of the originals.

If we connect origin 0 in R_N to the point A by the radius vector ρ_A , then projections ρ_A will coincide with the coordinates of the point A , i.e. $\rho_A = vector(a_0, a_1 \dots a_{N-1})$. The vector ρ_A is called the finite representing the vector of polynomial $(f_A(t))$. Here one can see the connection with the method of representing vectors [Volovodenko, Koryakina, 1980].

Selecting another point B in multidimensional space R_N produces the radius vector: $\rho_B = vector(b_0, b_1 \dots b_{N-1})$. Now we can describe a multidimensional segment AB with fixed vertices, where one can define the radius vector for any point X belonging to the segment AB :

$\rho_X = vector(x_0, x_1 \dots x_{N-1})$, which satisfies the following equation:

$$\rho_X = \rho_A + z(\rho_B - \rho_A) = \rho_X =, \quad (2)$$

$$= vector(x_0(z) \ x_1(z) \dots x_{N-1}(z))$$

where $z \in [0,1]$. Similarly to (*) we can define an image of the point X at a position z :

$$X = X(z) \leftrightarrow f_X(t) = \sum_{i=0}^{N-1} x_i(z) P_i(t) = f_X(t, z), \quad (3)$$

where $x_i(z) = a_i + z(b_i - a_i)$.

The set of the equations above justifies the transition from the two-dimensional images $f_X(t)$ in the coordinate system $\{f, t\}$ to the three-dimensional images $f_X(t, z)$ in the coordinate system $\{f, t, z\}$.

Due to the linearity of the transformation (1), the multidimensional segment AB corresponds to a smooth continuously changing surface F , as shown in the Fig. 4.

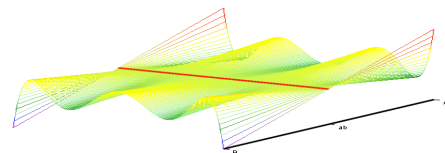


Fig. 4. The image of a multidimensional segment AB with the point $ab=(0,0,0,0,0,0)$ in the coordinate system $\{f, t, z\}$

Let us look at the above surface F in more details. The cross-section $z=0$ produces the function $f(t,0) = f_A(t)$. Likewise, the cross-section $z=1$ generates the function $f(t,1) = f_B(t)$. The intermediate surface cross-sections correspond to the inter-

mediate segment points and produce the functions $f_{X^*}(t) = f_X(t, z^*)$ for $z = z^*$.

Thus, there is a one-to-one correspondence between points of the segment AB in the N -dimensional space of originals R_N and cross-sections of the surface F in the 3-dimensional space of images R_3 .

Since the domain of the arguments f , t and z is strictly defined, it is possible to precisely scale the axes of the coordinate system $\{f, t, z\}$, which is required for displaying the images on a graphical device (e.g. computer monitor).

Although the image of a multidimensional process is defined in the coordinate system $\{f, t, z\}$, identification of process patterns can be easily done in the coordinate system $\{t, z\}$ using a color palette on the process state images as shown below in Fig 5.

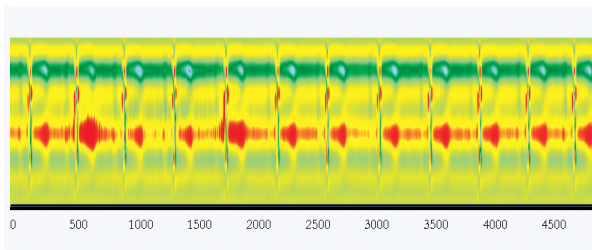


Fig. 5. A fragment of the image of a 12-channel ECG in the coordinate system $\{t, z\}$

Using the coordinate system $\{f, t, z\}$ as a 3-dimensional image space allows defining and displaying the value of a “distance” between multidimensional objects from the original space. That is, two-dimensional images (in the coordinate system $\{f, t\}$) of multidimensional objects are rendered along the z -axis (in the coordinate system $\{f, t, z\}$) according to their distance from the selected origin.

Thus, the z -axis in the coordinate space of images $\{f, t, z\}$ is called the “distance-axis”.

Any known distance metric can be used for the z -axis values, like Euclidean distance, Mahalanobis distance, time interval (to display a process) and/or others.

Practical application of abovementioned methodology will be examined by the example shown in the following part of article.

5. An Example of Using NovoSpark Visualizer System in Decision-making

In order to prove that cognitive graphics system can help in decision making, authors provide an example of health and fitness center problem solving. Promotion and marketing department is given the task to analyze the demand for various center services and create a new price offer taking into account the results of analysis. The number of customer visits at the specified time was calculated. During the research large volumes of the source data were collected. To facilitate the analysis the paper presents only the average number of visits grouped by days of week. The obtained data is imported into the NovoSpark® Visualizer system. Table 1 presents an average number of customers using the pool facilities in 2011 grouped by all days of week.

Table 1. Pool customers' average attendance in 2

Time intervals	M	T	W	T	F	S	S
8.00 – 9.00	8.3	12.2	10.0	13.3	14.5	22.5	10.1
9.00 – 10.00	8.6	14.2	15.4	17.1	16.3	28.9	33.5
10.00 – 11.00	22.1	26.3	20.5	25.5	21.1	35.4	36.3
11.00 – 12.00	25.0	26.9	22.2	29.5	35.5	60.5	51.2
12.00 – 13.00	21.3	23.5	25.9	31.1	34.7	72.3	55.5
13.00 – 14.00	25.8	19.0	30.2	27.4	37.4	68.4	60.4
14.00 – 15.00	25.6	18.4	31.1	29.5	39.4	73.5	57.7
15.00 – 16.00	33.4	34.2	32.7	35.3	42.6	66.6	55.1
16.00 – 17.00	34.4	45.1	38.5	46.5	65.2	63.7	45.5
17.00 – 18.00	42.2	51.8	43.3	56.6	64.7	41.9	42.4
18.00 – 19.00	50.1	55.7	48.6	58.2	60.5	34.9	25.3
19.00 – 20.00	38.9	40.1	46.2	42	37.3	45.1	18.1
20.00 – 21.00	27.5	26.0	32.2	40.2	24.2	22.9	17.2
21.00 – 22.00	12.6	18.9	16.1	25.7	10.3	12.8	15.8
22.00 – 23.00	5.3	8.9	7.7	10.3	2.1	12.2	8

Firstly, the traditional visualization has been made. Then, on the basis of visualization the attendance analysis was made. After that, authors provide the interpretation using color spectrum and compare the results.

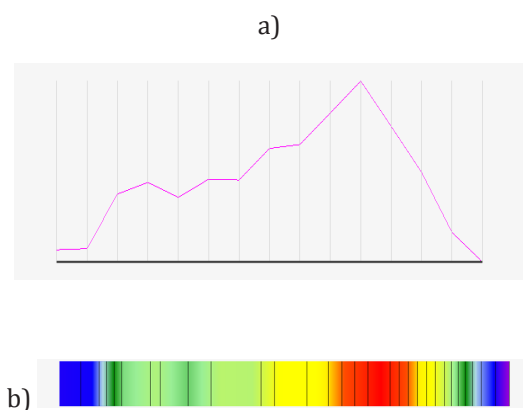


Fig. 6. Visualization of the observation “Swimming Pool – Monday” a) traditional interpretation b) spectral interpretation

Figure 6a shows the data on average number of Monday visits. The x-axis shows the hourly intervals, the y-axis – number of clients. Although the traditional graph shows quite clearly the dynamics, the spectral interpretation (Fig. 6b) provides more information about the function value in the specified period of time. The curve variable shape interferes in making comparisons of values.

The problem deepens when one image maps the functions for the entire week (Fig. 7). The x-axis shows the hourly intervals, the y-axis – number of clients. The curves comparison is a very complicated and does not indicate directly the answer to the question on which days and hours there is the greatest demand and in which – the least. The only thing you can easily see that in the morning and evening the demand is smaller than in the afternoon.

In the next step the data from Table 1 was standardized and three-dimensional visualization was created. The result of these operations is shown in Figure 8. At this stage the system capabilities allow to see the importance of the selected parameter at each point on the image. The color interpretation makes the image much more transparent. At the beginning, the image from Figure 7 is presented without dividing the days to draw a picture of the situation in its entirety (Fig. 9).

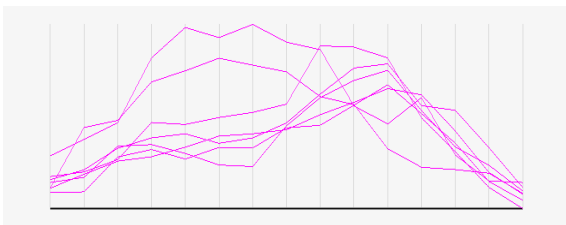


Fig. 7. Visualization of the observation “swimming pool” – traditional interpretation

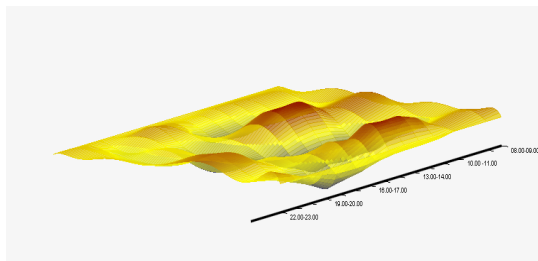


Fig. 8. The image of multidimensional observation for “swimming pool” (standardized data)

In this figure all days of week are labeled sequentially, i.e 0 represents Monday, 1 – Tuesday, etc. To create this picture parallel coordinates were used. At first look, we can easily see that demand for services is much higher on weekends. The small number of people uses the pool in the morning. Evening hours are apparently not popular too. Furthermore, it is easy to see low demand by 15.00 on weekdays.

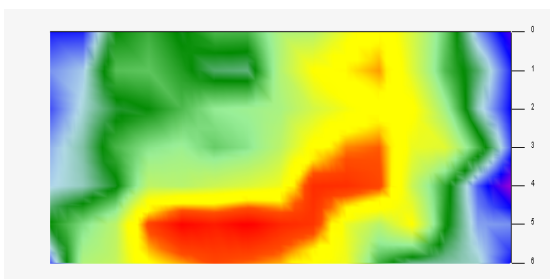


Fig. 9. Visualization of the observation “swimming pool” – spectral interpretation

To obtain more accurate information, authors divide the image into days (Fig. 10). Days of the week placed on the y-axis from Monday at the top, to Sunday at the bottom. Time intervals placed on x-axis. Now we see that the greatest demand occurs on Saturdays from 11.00 to 17.00 and Friday from 16.00 to 19.00. The lowest demand is on Mondays and Wednesdays. The number of people using the pool in the morning is not large, except Saturday, when demand is slightly higher. Furthermore, it is easy to see low demand by 15.00 on weekdays and after 15.00 on weekends. The lowest demand in the morning hours we notice on Monday and in the evening hours – on Friday.

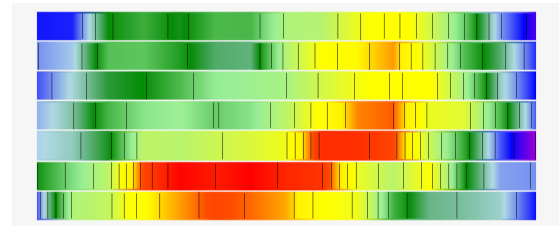


Fig. 10. Visualization of the observation “swimming pool” – spectral interpretation divided into days

A different situation we would see if we analyze the SPA customer data (Tab. 2).

Table 2. SPA customers’ average attendance in 2011

Time intervals	M	T	W	T	F	S	S
8.00 – 9.00	7.2	11.2	11.7	13.6	14.5	2.2	2.8
9.00 – 10.00	8.5	17.5	14.5	14.4	16.7	8.7	3.8
10.00 – 11.00	21.1	24.2	22.8	14.9	21.3	15.1	16.2
11.00 – 12.00	26.5	26.5	23.4	18.5	25.1	20.5	21.7
12.00 – 13.00	20.3	22.7	27.6	20.2	34.4	22.1	25.1
13.00 – 14.00	27.5	20.6	31.9	28.5	37.5	28.5	25.4
14.00 – 15.00	24.7	16.6	32.7	35.3	39.8	33.9	27.3
15.00 – 16.00	33.5	35.4	35.5	28.4	42.2	26.3	25.9
16.00 – 17.00	35.6	31.5	37.5	27.7	35.5	23.3	23.7
17.00 – 18.00	24.2	26.9	34	22.2	34.7	21.1	22.1
18.00 – 19.00	21.3	26.7	21.7	19.7	30.2	24.5	15.5
19.00 – 20.00	17.5	22.3	14.5	10.2	27.1	15.3	8.5
20.00 – 21.00	6.3	7.7	11.1	11.2	14.1	12.4	7.6
21.00 – 22.00	5.4	5.8	7.4	6.3	5.5	6.6	3.1
22.00 – 23.00	0	0.5	0.7	0.7	1.3	2.7	0.5

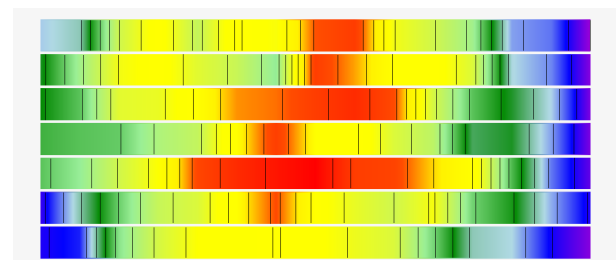


Fig. 11. Visualization of the “SPA” observation – spectral interpretation

The evening hours are the least loaded, while the highest demand is in the afternoon (Fig. 11). In the morning, the demand subsides only on weekends.

Analyzing the fitness data from Table 3, we see that the demand is lowest on Monday (Fig. 12).

Table 3. Fitness customers' average frequency in 2011

Time intervals	M	T	W	T	F	S	S
8.00 – 9.00	6.2	8.4	10.6	7.7	14.3	22.7	10.2
9.00 – 10.00	8.7	14.4	15.5	9.5	16.1	28.5	23.8
10.00 – 11.00	12.2	16.2	20.1	15.3	21.2	35.7	26.5
11.00 – 12.00	15.3	16.5	22.2	19.5	25.5	30.2	21.6
12.00 – 13.00	11.5	13.7	25.5	21.1	24.4	32.4	25.6
13.00 – 14.00	15.5	19.3	20.3	27.5	27.6	38.1	27.1
14.00 – 15.00	20.2	18.6	21.1	29.8	29.8	33.5	30.2
15.00 – 16.00	23.7	24.2	22.7	35.6	32.2	33.6	25.4
16.00 – 17.00	24.1	25.5	28.8	32.2	35.5	33.3	25.8
17.00 – 18.00	22.3	31.1	33.5	36.5	34.4	31.2	25.3
18.00 – 19.00	20.1	25.6	28.9	28.6	30.1	24.9	22.1
19.00 – 20.00	18	20.1	26.3	22.1	17.2	21.3	8.4
20.00 – 21.00	17.5	16.9	22.6	20.2	14.5	12.4	3.9
21.00 – 22.00	8.9	12.8	11.1	15.4	10.2	2.5	0.7
22.00 – 23.00	0.1	1.2	0.2	1.0	0.4	2.2	0.3

Generally, beginning of the week is not very popular. Definitely more customers use gym services on Fridays and weekends. On Sunday the morning and south are popular. Demand for services decreases significantly every day at 19.00.

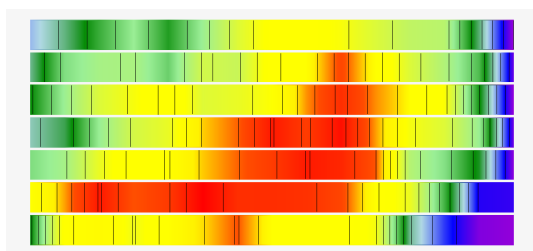


Fig. 12. Visualization of the "fitness" observation – spectral interpretation

After analyzing all services separately, it seems reasonable to compare them within a single image to create a promotional package. If we use traditional visualization we get a picture shown in Figure 13a, which allows you to inquire about the general trends of demand changes. The x-axis shows the hourly intervals, the y-axis – number of clients.

After making three-dimensional visualization we get a totally different picture. Figure 13b clearly shows the time intervals with the highest and lowest frequency. For each point on the surface, we can do analysis of the parameter value's impact on the color of that point.

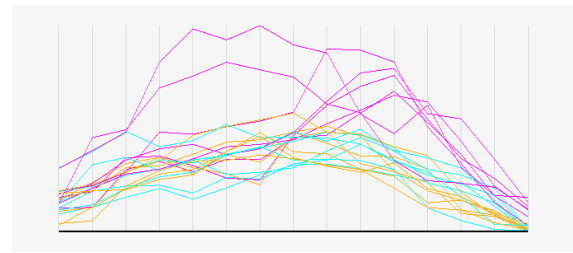


Fig. 13a. Visualization of all observations of the health and fitness centre – traditional interpretation

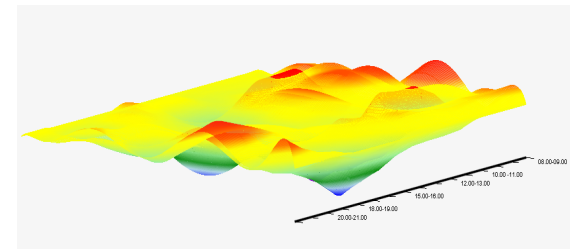


Fig. 13b. Multidimensional image of all services of the health and fitness centre

Observation of a multi-dimensional image works well in the detailed analysis of the data. However, if we want to find the source of a problem quickly, it is worthwhile to make a spectral interpretation. In Fig. 14 on the x-axis the first 7 columns reproduce demand for SPA services, next 7 – for the swimming pool service, the last – for the fitness service. The y-axis shows the hourly intervals. The data is neither normalized nor standardized, and therefore the demand for swimming pool in the figure is much higher than demand for other services.

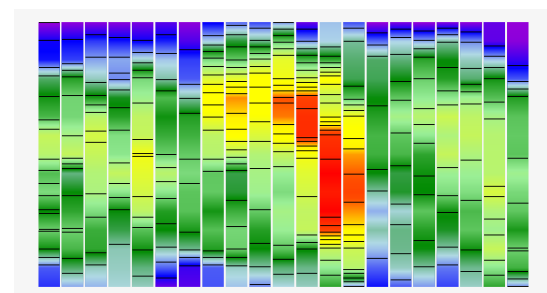


Fig. 14. The observations of all services of the health and fitness centre

The figure allows you to see the similarity of SPA and fitness frequency. On the x-axis the first 7 columns show the demand for spa services, the next 7 – for pool service, the last 7 – for SPA service. The worst situation is in the evening (Fig. 15), especially on weekends. There is, therefore, a possibility to create a promotional offer by connecting these hours. In the morning, the demand for spa and fitness is different, so there is little point in joining the promotion in these hours. Because the number of customers is minimal from 22.00 to 23.00 in the entire center, it is useful to consider its closing an hour earlier.

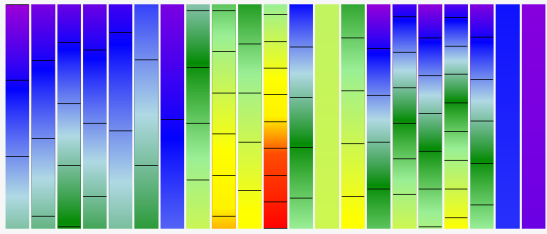


Fig. 15. The observations of all services of the health and fitness centre from 19.00 to 23.00

In the research experiment it was found that the visualization made using the color spectrum has a significant advantage over traditional visualization. The more observations were taken into account during the experiment the less clear the traditional charts were. The NovoSpark Corporation's application enabled not only to look at the situation in its entirety and immediately see the source of the problem, but also make detailed analysis of the selected aspects, for example to compare attendance at specific hours and days in order to make optimal decisions.

6. Conclusion

By introducing an element of cognitive intelligence to traditional technologies modern cognitive graphics systems often contribute to the creation of absolutely new knowledge about the researched object. The research performed in this article represents only a small fraction of the capabilities of cognitive graphics. These systems can become a unique tool to study patterns and processes of the imagination in the future. So, they can make the competition with many systems using the neuromarketing and neuroeconomy concept, because cognitive graphics systems do not create the possibility of manipulation with mind. They just help people to find the source of the problem in a short period of time.

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