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# Design of aspheric spectacle lenses using non-dominated sorting genetic algorithm

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**Significance**: An effective algorithm for optimization of lens parameter can greatly eliminate the aberration and reduce the thickness, making the wearer more comfortable. **Aim**: We proposed a non-dominated sorting genetic algorithm (NSGA-II) for generating sets of base curves and aspheric coefficients to minimize the residual astigmatism and aberration of the lenses, while satisfying the constraints on the lens thickness and power. **Approach**: By simulating natural selection using the NSGA-II algorithm, the design parameters considered the inventory of the semi-finished blank. A comparison of aspheric and spherical spectacle lenses with –8 diopters was designed, simulated, processed, and measured. **Results**: The measured spherical and cylindrical power distributions were consistent with the simulated results with corrected oblique astigmatism and distortion. **Conclusions**: The aspheric spectacle lenses had the required aesthetic shape and weight reduction compared to a spherical lenses of the same power. It is verified that this paper puts forward an effective NSGA-II algorithm for the optimization of lens parameters.

Keywords: aspheric ophthalmic lens, genetic algorithm, multi-objective optimization, aberrations.

## 1. Introduction

The prevalence of myopia has been increasing in East Asia and other parts of the world, with rates of myopia and high myopia projected to reach approximately 50% and 10% of the global population, respectively, by 2050 [1]. Visual disorders, such as myopia or hypermetropia, are conventionally corrected with spectacles, contact lenses, or refractive surgery. Compared with other correction methods such as contact lenses and refractive surgery, spectacle lenses are still the current mainstream and safe correction methods. Thus, in ophthalmic lens design, images are afflicted with various aberrations, and those that are of significance to the spectacle wearer are transverse chromatic aberration, oblique astigmatism, curvature of field, and distortion. In spherical spectacle lens design, there is only one major design variable: the shape factor. The lack of design variables means that the designer has little hope of simultaneously reducing aberrations. In many cases, aspherizing one or both surfaces of spectacle lenses allows the correction of off-axis power errors and reduces thickness [2].

Large-scale digital computers have been used for optical ray tracing and the mechanization of existing optical design techniques. New techniques have been developed for the design and optimization of aspheric spectacle lenses. MIKS *et al.* performed a complex analysis of the third-order design of spectacle lenses considering astigmatism as well as all types of primary aberrations and their combinations [3]. QIN applied the particle swarm algorithm (PSO) to aberration correction for a single aspheric lens and showed that PSO is a simple and effective tool for spherical aberration correction, and it is easy to find a series of good design results [4]. FAN *et al.* presented an efficient and robust approach to retrieve the optimal first-order design of a double-sided telecentric zoom lens based on the particle swarm optimization (PSO) algorithm [5]. MENKE proposed a novel method for the automatic design of optical systems, based on the PSO algorithm, and proved that the proposed method is an excellent tool for the optimization of free-form systems with a large number of variables [6]. GUO *et al.* applied the PSO idea of a stochastic solution to a glass map in an optical system design and discussed how to improve the effectiveness of glass optimization by combining the least squares method with PSO which can greatly reduce the dependence on the experience of the designer and immensely lessen the human effort in the design work [7]. In the optical design process, the most commonly used optimization algorithm is damping least squares (DLS), and SUN *et al.* used the traditional DLS method to optimize the aspherical lenses [8]. Although the astigmatism of the positive and negative lenses in a certain power range was eliminated and the thickness reduced, the distortion was even larger than that of the spherical lenses. However, the results tend to fall into a local minimum, and evolutionary algorithms can solve such problems. The evolutionary algorithm, is an algorithm that simulates the biological evolution process, has a strong global search capability, mainly including the genetic algorithm (GA), particle swarm optimization (PSO), and ant colony algorithm (ACO). YEN and JIN integrated the GA and commercial optical design software to optimize an artificial intraocular lens to reduce aberrations in the human eye [9]. YEN and YE combined GA with a neural network (NN) to improve the optical performance, which made contact lenses comfortable to wear [10].

Relying on computationally intensive ray tracing, optical design can greatly benefit the optimization approach if suitable parallelizable optimization algorithms can be implemented. In our work, a –8.00 diopter (D) spectacle lens with a back aspheric surface was taken as an example. The ray-tracing method was used to calculate the oblique astigmatism, refractive power error, and distortion, and the non-dominated sorting genetic algorithm (NSGA-II) was used to optimize the base curve and conic and high -order coefficients of the back surface as structural variables, which are also design parameters determining the values of aberrations. Compared with the spherical spectacle lenses, four lenses with the focal design, Percival design, minimum distortion design, and balance design were simulated using the optical simulation software FFV, processed according to the optimized parameters with a computer numerical control (CNC) optical generating machine, and measured.

## 2. Multi-objective optimization method of aspheric lenses

### 2.1. Significant variables recognition

An aspheric surface is usually described in terms of its sagitta z, which can be considered as the deviation from a plane at its vertex [11]:

$$Z = \frac{cr^2}{1 + \sqrt{1 - (1 + k)c^2 r^2}} + Br^4 + Cr^6 + Dr^8 + \ldots \tag{1}$$

where $Z$ is the displacement of the surface from the vertex at radial distance $r$ from the optical axis. Parameter $c$ is the curvature of the rear surface (which is the inverse of the radius of curvature at the vertex) and is determined by the curvature of the base curve $C_{\mathrm{BC}}$ for the front surface, and $k$ is known as the conic constant. The terms $B$, $C$, and $D$ are known as the 4th, 6th, and 8th order aspheric coefficients [12].

### 2.2. The NSGA-II algorithm

As the optimization problem of an aspheric lens involves multiple objectives, NSGA-II seems to be the most suitable algorithm for this type of problem, as shown in Fig. 1. NSGA-II has been implemented in commercial software packages and has been widely adopted. NSGA-II was chosen with a population size of 200 and generation number of 500, the crossover and mutation probabilities were set as 0.9 and 0.1, respectively, and the optimal design was obtained from the Pareto front. Population $\mathrm{P}_0$ is initialized first [10–12]. Non-dominated sorting operators are then used to sort the population into each front. To calculate the crowding degree of the objective function, tournament selection is executed to select the parents of the next population. The selected population, which is a structural parameter with smaller distortion, generates offspring $\mathrm{Q}_0$ using gene crossover and mutation operators. $\mathrm{P}_0$ and $\mathrm{Q}_0$ are merged in the next generation and fast non-dominant ranking operators are carried out. The crowding degree is then calculated, and only the best $N$ of the parents and offspring are selected by the elite selection strategy, where $N$ is the population size. The process is repeated to generate subsequent generations until the iteration reaches the maximum [13–15].

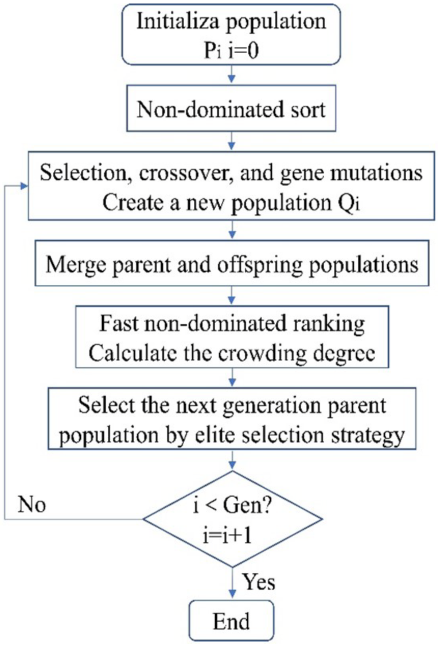


Fig. 1. NSGA-II program flow chart.

The performance indices improved through optimization are called objective functions, which correspond to the crowding degree of the algorithm. This study has three objective functions: oblique astigmatism error (OAE), mean oblique error (MOE), and distortion. The expression for OAE can be defined as

$$F_{\mathrm{OAE}} = \frac{1}{f'_{\mathrm{T}}} - \frac{1}{f'_{\mathrm{S}}} = F'_{\mathrm{T}} - F'_{\mathrm{S}} \tag{2}$$

where $f'_{\mathrm{T}}$ and $f'_{\mathrm{S}}$ are the tangential and sagittal focal lengths, respectively.

The MOE is formulated as [8]:

$$F_{\mathrm{MOE}} = \frac{1}{2}(F'_{\mathrm{T}} + F'_{\mathrm{S}}) - F'_{\mathrm{v}} \tag{3}$$

where $F'_{\mathrm{v}}$ is the back vertex power.

The distortion is expressed as [16]:

$$\begin{cases} \mathrm{MQ} = -\dfrac{n \tan u_1}{(n - tF_1)F'_{\mathrm{v}}} \\ \mathrm{MQ}' = -\dfrac{(1 - l'_2 F'_{\mathrm{v}}) \tan u_2}{F'_{\mathrm{v}}} \\ \eta' = \dfrac{\mathrm{MQ}' - \mathrm{MQ}}{\mathrm{MQ}} \times 100\% \end{cases} \tag{4}$$

After the three objective functions, the multi-objective optimization problem can be written in the following mathematical form: $\min|F_{\mathrm{MOE}}(x)|$, $\min|F_{\mathrm{OAE}}(x)|$, and $\min|\eta'(x)|$, where $x$ represents the decision variable vector obtained from the results in Section 3.1. Thus, the aim of this multi-objective optimization work is to obtain a set of design parameters to minimize the OAE, MOE, and distortion of the spectacle lenses [8, 12].

The design process was divided into two steps. First, the curvature of the base curve $C_{\mathrm{BC}}$ and $k$ are taken as independent variables, whereas OAE, MOE, and distortion were chosen as objectives. A reasonable range of $C_{\mathrm{BC}}$ and its corresponding $k$ were selected after obtaining multiple sets of parameters using NSGA-II. The value of $C_{\mathrm{BC}}$ was determined based on the production conditions. Second, $k$ and the high-order coefficients $B$, $C$, $D$, and $E$ were considered as independent variables to obtain the optimal structure. Finally, the parameters of minimum aberration at different $C_{\mathrm{BC}}$ values were determined considering the thickness of the lens.

## 3. Analysis of optimization results

### 3.1. Parameter optimization

The basic parameters of the aspheric lens were a back vertex power of –8.00 D, distance between the rotation center of the eye and back vertex point of the lens of 27 mm, central thickness of 1.2 mm, refractive index of 1.663, rotation angle of 30°, and diameter of 70 mm. Ray tracing was used to calculate the aberrations and edge thickness vari-

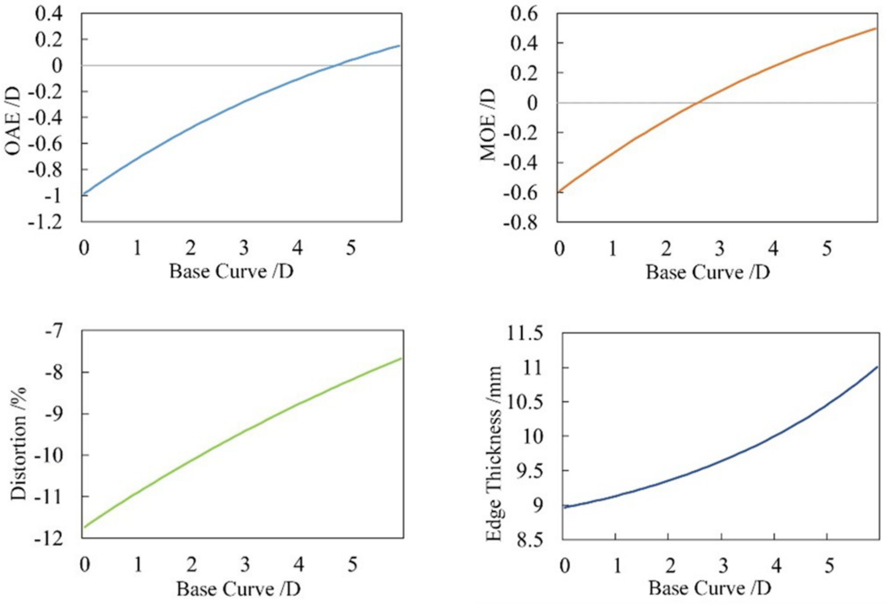


Fig. 2. Variation of each performance with the change of –8.00 D spherical lens base curve.

ation of –8.00 D spherical lenses when the $C_{BC}$ varied in the range of approximately 0–6.00 D, as shown in Fig. 2.

It is found that when the curvature in the range of approximately 1.70–2.60 D, the absolute value of the OAE for the aspherical lens is less than 0.20 D and the distortion is between 8.80% and 9.50%, whereas the absolute value of the OAE and the distortion for the spherical lens are between approximately 0.30−0.55 D and 9.70−10.40%, respectively. The above analysis shows that the optical performance of the aspheric lens is significantly better than that of a spherical lens with the same curvature. In this study, the absolute values of OAE and MOE are both less than 0.25 D, which is regarded as the index of excellent optical performance because the range of 0.25 D can be accepted by the human eye, which is also the commonly used step gradient in the process of lens fitting.

Based on the above analyses and previous values reported in the literature [8], we set the back surface as aspheric, and the approximate ranges of the $C_{BC}$, $k$, and high -order term coefficients are listed in Table 1.

Table 1. Range of the parameters of the –8 D aspheric lenses.

| | CBC [D] | $K$ | $B$ | $C$ | $D$ | $E$ |
|---|---|---|---|---|---|---|
| Min | 0.5 | –5 | $-1\times10^{-7}$ | $-1\times10^{-10}$ | $-1\times10^{-13}$ | $-1\times10^{-18}$ |
| Max | 2.6 | 5 | $1\times10^{-5}$ | $1\times10^{-7}$ | $1\times10^{-9}$ | $1\times10^{-15}$ |

The NSGA-II algorithm was used to calculate a $C_{BC}$ that could present an acceptable range of aberration and estimate the corresponding $k$. The optimized $C_{BC}$ values were all in the range of approximately 1.7–2.60 D, as shown in Table 2. Because the initial population was randomly generated, the calculation results were not the same each time. Analyzing dozens of results after multiple calculations and taking the inventory of the semi-finished blank in actual production into consideration, the base curve was 2.19 D.

Subsequently, the $C_{BC}$ was determined. $k$ and the high-order coefficients $B$, $C$, $D$, and $E$ were considered as independent variables to obtain the optimal structure. Two hundred parameters were calculated, and the results are shown in Fig. 3. The three axes represent the OAE, MOE, and distortion. Each of the 200 circles represents a set of

Table 2. OAE and distortion correspond to different $C_{BC}$ of –8.00 D lenses.

| $C_{BC}$ [D] | Aspheric lenses | | | Spherical lenses | |
|---|---|---|---|---|---|
| | $K$ | OAE [D] | $\eta'$ [%] | OAE [D] | $\eta'$ [D] |
| 1.797 | −0.205 | −0.003 | −9.480 | −0.549 | −10.347 |
| 2.090 | −0.111 | 0.062 | −9.261 | −0.484 | −10.129 |
| 2.201 | 0.116 | −0.006 | −9.332 | −0.460 | −10.048 |
| 2.270 | 0.142 | 0.004 | −9.290 | −0.445 | −9.997 |
| 2.296 | −0.217 | 0.186 | −8.978 | −0.439 | −9.979 |
| 2.572 | 0.004 | 0.173 | −8.897 | −0.382 | −9.782 |

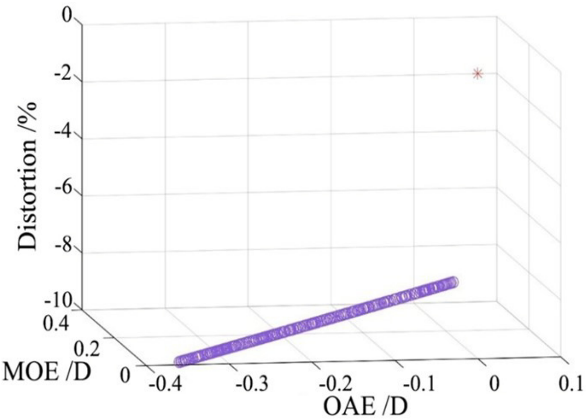


Fig. 3. Performance of –8.00 D lens obtained by NSGA-II algorithm optimization.

corresponding structural parameters, and the asterisk represents an ideal lens free of aberrations. The closer it is to the asterisk, the better the design. The 200 results were divided into four groups according to the design method: the focal design for eliminating astigmatism ($|\mathrm{OAE}| < 0.1$), Percival design for eliminating MOE ($|\mathrm{MOE}| < 0.1$), minimum distortion design, and balance design for each aberration being small but relatively thin. The representative parameters of the four designs are listed in Table 3.

The edge thicknesses of the lenses were obtained, as listed in Table 3. It can be observed that the thickness of the spherical lens was approximately 4 mm thinner than that calculated by SUN *et al.* for proper selection of the base curve. The OAE and distortion of the four aspheric designs were smaller than those of the spherical lens, and their thicknesses were approximately 8.60 mm. Compared with that of a spherical lens, the thickness can be reduced by up to 8.49%. Considering the results of non-dominated sorting and edge thickness, the parameters of the balance design were selected. The ab-

Table 3. Four designs of –8.00 D aspherical lenses parameters obtained by NSGA-II algorithm.

| Design types | Focal design | Percival design | Balance design | Minimum distortion design | Spherical lens |
|---|---|---|---|---|---|
| $k$ | –0.99 | –0.94 | –0.97 | –0.97 | 0.00 |
| $B$ | $7.11\times10^{-9}$ | $1.49\times10^{-7}$ | $1.51\times10^{-8}$ | $2.37\times10^{-9}$ | 0.00 |
| $C$ | $5.51\times10^{-11}$ | $2.38\times10^{-11}$ | $8.99\times10^{-10}$ | $6.20\times10^{-17}$ | 0.00 |
| $D$ | $2.16\times10^{-13}$ | $2.48\times10^{-13}$ | $6.58\times10^{-13}$ | $7.77\times10^{-19}$ | 0.00 |
| $E$ | $7.69\times10^{-21}$ | $2.13\times10^{-21}$ | $9.10\times10^{-23}$ | $6.96\times10^{-23}$ | 0.00 |
| OAE [D] | 0.00 | –0.36 | –0.18 | 0.03 | –0.46 |
| MOE [D] | 0.34 | 0.00 | 0.18 | 0.38 | –0.10 |
| $\eta'$ [%] | –9.33 | –9.90 | –9.61 | –9.27 | –10.05 |
| CT [mm] | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 |
| ET [mm] | 8.59 | 8.63 | 8.61 | 8.60 | 9.39 |
| Reduction [%] | 8.49 | 8.32 | 8.08 | 8.37 | – |

solute values of OAE and MOE were less than 0.25 D, which was within the acceptable range for human eyes; the distortion was also smaller than that of the spherical lens. In summary, all four types of lenses satisfied the design requirements.

### 3.2. Simulation

The aberrations in Table 3 are optimized and calculated for a gaze angle of 30°, and the performance within this range is of greater concern. Taking the focal design as an example, the relationship between the optical performance of the aspherical lens and gaze angle is shown in Fig. 4.

The OAE first increases and then decreases, but the maximum is less than 0.1 D, whereas the OAE of the spherical lens increases with the increase in gaze angle and reaches 0.45 D at the edge of 30°. The distortion was smaller than that of a spherical lens. It can be observed that the astigmatism in the peripheral area of the aspheric lens is smaller than that of the spherical lens, and the image quality in the entire field of view can meet the design requirements.

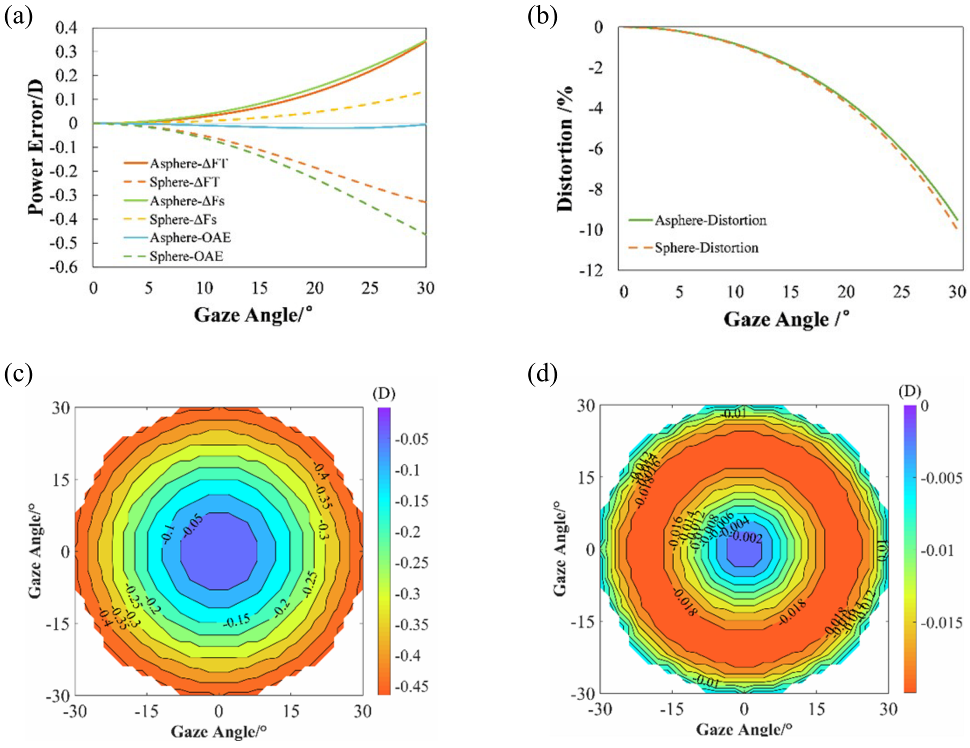
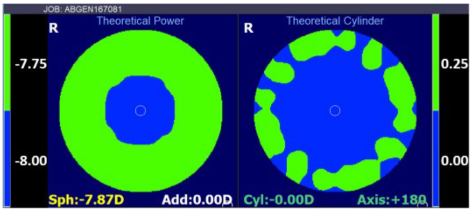
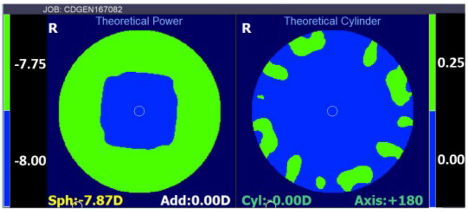


Fig. 4. Optical performance distribution of –8.00 D lens in 30° obtained by genetic algorithm. (a) Aberrations of spherical lens and aspheric lens *vs*. gaze angle. (b) Distortion of spherical lens and aspheric lens *vs*. gaze angle. (c) OAE distribution of spherical lens out to 30°. (d) OAE distribution of aspheric lens out to the 30°.

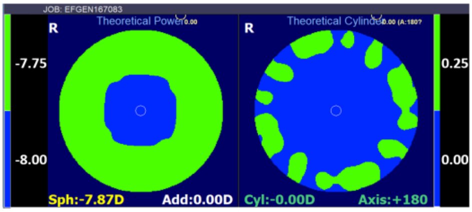
Focal design



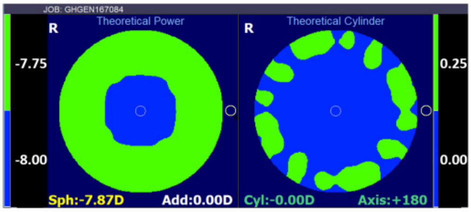
Percival design



Balance design



Minimum distortion design



Spherical lens

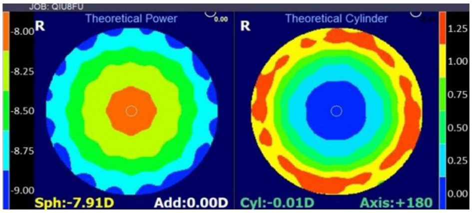


Fig. 5. Simulation of four sets of –8.00 D aspheric and spherical lenses obtained by the NSGA-II algorithm.

Figure 5 shows the simulations for each case based on the use of the Freeform Verifier software (FFV) (ROTLEX, Israel), which is commercial software used as a moiré deflectometer with a point source. Moiré fringes were produced and analyzed, and a simulation of the power distribution was obtained. The parameters of the four lenses were similar, and as a result, their optical performance was similar. The central power was –7.87 D, with a difference of 0.13 D from the target power. The spherical lens was –7.91 D, with a deviation of 0.09 D from the target. The power of the peripheral zone was –9.00 D, and the astigmatism varied by 1.25 D from the center to the periphery, which may cause dizziness for the wearer. The astigmatism of the aspheric lens changed by only 0.25 D. This trend of change is gentle and suitable for use as a spectacle lens.

### 3.3. Processing

The lenses were milled using a CNC optical-generating machine (Satisloh VFT-orbit, Germany). The final products are shown in Fig. 6. The back vertex power was measured with a lensmeter (Nidek-LM1800P, Japan), which has a measurement accuracy

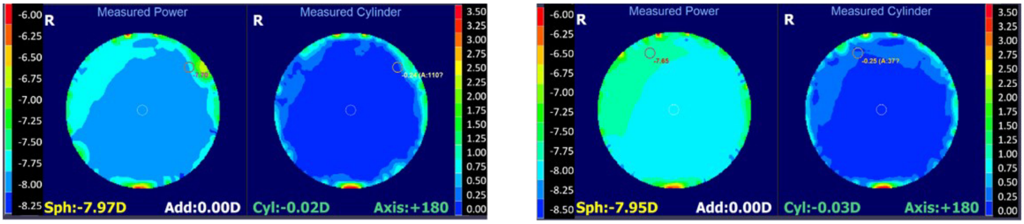

Front of the lens

Edge of the lens

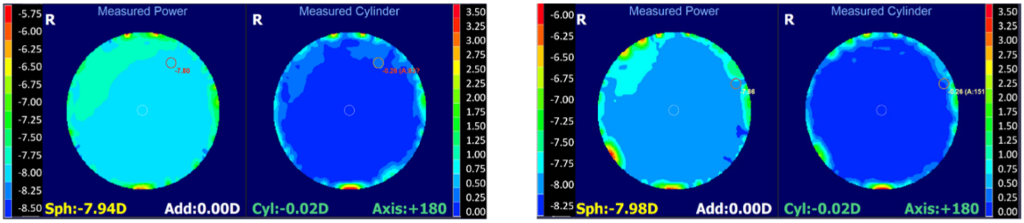
Fig. 6. Four groups of –8.00 D aspheric lenses physical picture obtained by the NSGA-II algorithm.

of 0.01 D. Each lens was measured three times and the average value was taken as the final measurement result. The powers for the four lenses were –8.10, –8.08, –8.07, and –8.04 D. The power errors were within the tolerance range of ±0.12 D specified in the production standard. The edge thicknesses were 8.13, 8.17, 8.00, and 8.03 mm, which were thinner than the theoretical values, and the difference with the theoretical values was approximately 0.53 mm, which may have been caused by processing errors and edge beveling.

A free-form verifier measurement module was used to measure the actual power distribution. From the measurement results in Fig. 7, it can be observed that the four designs have similar power distributions. The power is mostly –8.00 D, and the peripheral part gradually decreases. There were some red parts at the edge, which may be associated with light scattering during the measurement. According to the processing experience, for every 1 D change in spherical power, the corresponding change in astigmatism on the right side within 0.75 D is regarded as an acceptable standard.


Focal design

Percival design


Balance design

Minimum distortion design

Fig. 7. Four groups of –8.00 D aspheric lenses by the NSGA-II algorithm.

The area of the small astigmatic part ($\leq 0.50$ D) was approximately 95%, which met the design requirements. Compared with the simulation results in Fig. 5, the overall distribution change trend is the same, but the area of the optical power of –8.00 D part in the measured results is larger than the simulation results, and there is a small part of astigmatism at approximately 0.75 D from the edge, which is not reflected in the simulation results. In terms of optical performance, the production and processing measurements of these four lenses were as expected and met the design requirements.

## 4. Conclusion

In this study, we present an approach for optimizing the design of aspheric lenses to reduce astigmatism and aberrations by using the multi-objective optimization NSGA-II algorithm with ray tracing.

Furthermore, we investigate the differences between the design of aspheric and spherical lens. The analysis of optical performance from both FFV simulations and machine processing demonstrates that aspheric lenses perform with better effectiveness, since the proposed algorithm can find high-quality parameters that meet the requirements of the design process by combining suitable variables. Additionally, our method does not require building the initial structure and the target evaluation function can be easily designed by substituting it into the program, making it simple to operate and yielding better results which is designer-friendly.

In particular, higher-order coefficients can be optimized to further investigate the effects on the design of aspheric lens design. Although we only used the back surface as an aspheric surface in this study, the proposed method can also be applied to the design of a double-sided aspheric lens to obtain lenses with better optical performance and reduced thickness.

Therefore, the proposed method is able to optimize the design of aspheric lens to improve the visual experience for users which has promising applications in lens designs.

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**Disclosures**

The authors declare no conflicts of interest.

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