Limitations of WSSUS modeling of stationary underwater acoustic communication channel

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Performances of underwater acoustic communication (UAC) systems are strongly related to specific propagation conditions of the underwater channel. Due to their large variability, there is a need for adaptive matching of the UAC systems signaling to the transmission properties of the channel. This requires a knowledge of instantaneous channel characteristics, in terms of the specific parameters of stochastic models. The wide-sense stationary uncorrelated scattering (WSSUS) assumption simplifies the estimation of terrestrial wireless channel transmission properties. Although UAC channels are hardly ever WSSUS, the rationale of such a stochastic modeling is that over a short period of time, and for a limited frequency range, this assumption is reasonably satisfied. The limits of application of the local-sense stationary uncorrelated scattering (LSSUS) model are determined by the stationary time and frequency. This paper presents the results of LSSUS model analysis for UAC channel measurements gathered by the former Research Department for Underwater Acoustics and Marine Geophysics (FWG), now part of the WTD71, during the SIMO experiment in the Bornholm Basin of the Baltic Sea.

Keywords: underwater communications, impulse response, stationary time, stationary bandwidth, adaptive communications, WSSUS modeling

1. Introduction

Underwater acoustic communication (UAC) system performance is strongly related to the specific propagation conditions of the particular underwater channel, which change over time. For the design of the UAC physical layer, it is necessary to determine the time period and frequency range, at which the channel can be considered as stationary [1]. Then it can be modeled with the use of the wide sense stationary uncorrelated scattering (WSSUS) assumption that simplifies stochastic description of the time-varying multipath channel. A set of four parameters, namely delay spread τ_M, Doppler spread ν_M, coherence time $T_C$and coherence bandwidth B_C are used to design the signaling scheme; as much resistant as possible to distortions caused by the time-varying multipath propagation conditions.

As it was shown in [1], UAC channels are hardly ever WSSUS. But over a limited period of time, and for a limited frequency range, this assumption can be satisfied. This approach is called local-sense stationary uncorrelated scattering (LSSUS). It allows the use of two other transmission parameters, namely stationary time $\tau_D$and stationary bandwidth B_D, to describe transmission properties of a non-stationary communication channel.

2. WSSUS indicators

UAC channel can be characterized by a time-variant impulse response $h(t,\tau)$, defined in the window of observation time response t and delay response τ [2]. To test whether a given $h(t,\tau)$ represents a stochastic WSSUS process or not, two indicators are proposed, for the wide-sense stationary (WSS) and uncorrelated scattering (US) features of the channel.

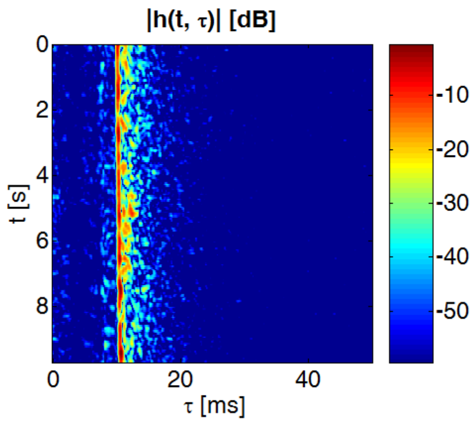

Fig. 1. UAC time-varying impulse response

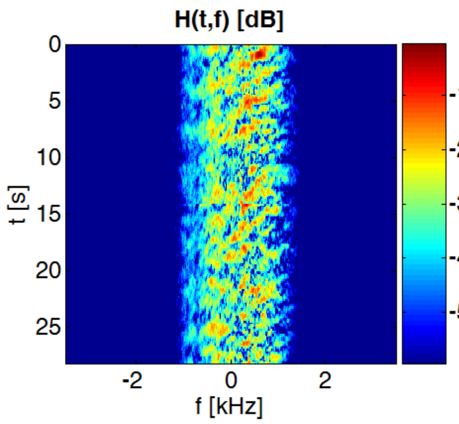

Fig. 2. UAC time-varying transfer function

WSS indicator

UAC channel impulse response, measured using bandpass modulated acoustic pulse, and downsampled into baseband, is represented with a complex-value time process $h(t,\tau)$. It consists of real (in-phase) $h_I(t,\tau)$) and imaginary (quadrature) $h_Q(t,\tau)$ components:

$$h(t,\tau) = h_I(t,\tau) + jh_Q(t,\tau). \quad (1)$$

According to [3], if the $h_I(t,\tau)$ and $h_Q(t,\tau)$ are jointly WSS over observation time t, then its autocorrelation functions $R_I(\Delta t,\tau)$and$R_Q(\Delta t,\tau)$, calculated in t domain:

$$R_I(\Delta t,\tau) = E[h_I(t+\Delta t,\tau)h_I^*(t,\tau)], \quad (2)$$

$$R_Q(\Delta t,\tau) = E\left[h_Q(t+\Delta t,\tau)h_Q^*(t,\tau)\right], \tag{3}$$

and cross-correlation functions$R_{IQ}(\Delta t,\tau)$ and $R_{QI}(\Delta t,\tau)$:

$$R_{IQ}(\Delta t,\tau) = E\left[h_I(t+\Delta t,\tau)h_Q^*(t,\tau)\right], \tag{4}$$

$$R_{QI}(\Delta t,\tau) = E\left[h_Q(t+\Delta t,\tau)h_I^*(t,\tau)\right], \tag{5}$$

satisfy the relations:

$$R_I(\Delta t,\tau) = R_Q(\Delta t,\tau), \tag{6}$$

$$R_{IQ}(\Delta t,\tau) = -R_{QI}(\Delta t,\tau), \tag{7}$$

Additionally, the mean value of impulse response $h(t,\tau)$ should be time-invariant.
Thus, *WSS indicator* is constructed of the mean value of correlation coefficients $c_{IQ}(\tau)$ between $R_I(\Delta t,\tau)$ and $R_Q(\Delta t,\tau)$, expressing their similarity (for a given τ the correlation coefficient is calculated as the maximum value of cross-correlation of $R_I(\Delta t,\tau)$ and $R_Q(\Delta t,\tau)$):

$$c_{IQ}(\tau) = \max_{\Delta t'}\left[E\left[R_I(\Delta t+\Delta t',\tau), R_Q^*(\Delta t,\tau)\right]\right], \tag{8}$$

the mean value of correlation coefficients$c_{IQQI}(\tau)$between $R_{IQ}(\Delta t,\tau)$ and $R_{QI}(\Delta t,\tau)$, which is calculated as:

$$c_{IQQI}(\tau) = \max_{\Delta t'}\left[E\left[R_{IQ}\left(\Delta t+\Delta t',\tau\right), R_{QI}^*(\Delta t,\tau)\right]\right], \tag{9}$$

and the variance of the mean amplitude value of $h(t,\tau)$:

$$\sigma_h^2 = var\left[\underset{t}{mean}(|h(t,\tau)|)\right]. \tag{10}$$

The formula for WSS indicatorI_{WSS} is as follows:

$$I_{WSS} = \sqrt{\frac{\underset{\tau}{mean}(c_{IQ}(\tau)) \cdot \underset{\tau}{mean}(c_{IQQI}(\tau))}{1+\sigma_h^2}} \tag{11}$$

The I_{WSS} indicator can have a value between 0 and 1.

US indicator

The US assumption implies that there is no correlation between the fading coming from different signal scatters. The UAC channel, that fulfills this assumption, has a complex time-varying transfer function $H(t,f)$, calculated as the Fourier transform of $h(t,\tau)$ over delay τ. Similarly, as in the case of impulse response, the time-varying transfer function $H(f,\tau)$can be represented as a sum of its real (in-phase) $H_I(t,f)$ and imaginary (quadrature) $H_Q(t,f)$parts:

$$H(t,f) = H_I(t,f) + jH_Q(t,f), \tag{12}$$

The corresponding correlation functions of $H(t,f)$ have the following form:

$$R_I(t,\Delta f) = E[H_I(t,f+\Delta f)H_I^*(t,f)], \tag{13}$$

$$R_Q(t,\Delta f) = E[H_Q(t,f+\Delta f)H_Q^*(t,f)], \tag{14}$$

$$R_{IQ}(t,\Delta f) = E[H_I(t,f+\Delta f)H_Q^*(t,f)], \tag{15}$$

$$R_{QI}(t,\Delta f) = E[H_Q(t,f+\Delta f)H_I^*(t,f)]. \tag{16}$$

If $h(t,\tau)$ is US process in τ domain, the correlation functions do not depend on the absolute frequency, but only on the Δf [4]. Moreover, they satisfy the following equations:

$$R_I(t,\Delta f) = R_Q(t,\Delta f) \tag{17}$$

$$R_{IQ}(t,\Delta f) = -R_{QI}(t,\Delta f) \tag{18}$$

The proposed US indicator I_{US}takes into account the similarity of the corresponding correlation functions, as well as the variance of the mean absolute value of $H(t,f)$:

$$I_{US} = \sqrt{\frac{\underset{t}{mean}(c_{IQ}(t))\underset{t}{mean}(c_{IQQI}(t))}{1+\sigma_H^2}}, \tag{19}$$

where correlation coefficients $c_{IQ}(t)$and $c_{IQQI}(t)$are determined as:

$$c_{IQ}(t) = \max_{\Delta f'}\left[E\left[R_I(t,\Delta f+\Delta f'), R_Q^*(t,\Delta f)\right]\right], \tag{20}$$

$$c_{IQQI}(t) = \max_{\Delta f'}\left[E\left[R_{IQ}(t,\Delta f+\Delta f'), R_{QI}^*(t,\Delta f)\right]\right], \tag{21}$$

And σ_H^2denotes variance of the mean amplitude value of $H(t,f)$:

$$\sigma_H^2 = var_t\left[\underset{f}{mean}(|H(t,f)|)\right]. \tag{22}$$

As in the case of I_{WSS}, the I_{US} can have a value between 0 and 1.

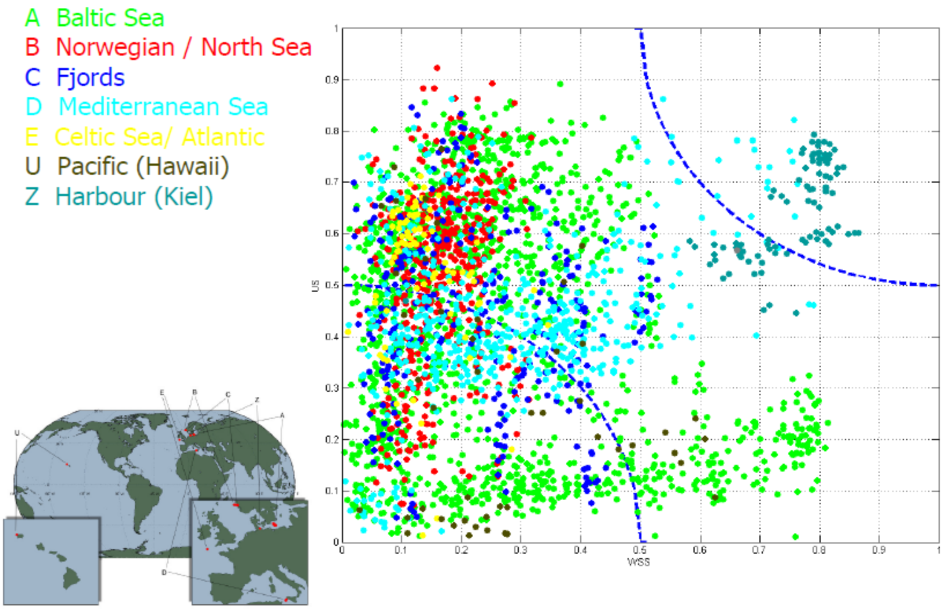

Fig. 3. WSS-US-values for all FWG measurements since 2001

The results WSS and US indicators for UAC channel measured during the WTD 71 A40 sea trial are shown in [1]. Fig. 3. shows the results for other UAC channels, measured by WTD71-FWG during numerous sea trials conducted in different reservoirs since 2001. In most cases the channel does not fulfill the WSS, but satisfies the US assumption. It is assumed that the channel is jointly WSS and US, if I_{WSS} and I_{WSS} satisfy the condition:

$$\sqrt{I_{WSS}^2 + I_{US}^2} \leq 0.5. \tag{23}$$

3. Local sense stationary uncorrelated scattering (LSSUS) assumption

In the case of many underwater communication channels, especially when system terminals are in movement, the WSSUS model is of limited value. The A40 sea trial has shown that in quasi-stationary conditions the UAC channel also doesn't have to meet the WSS and US conditions. For such channels, significant modifications in transmission parameter estimation, are needed. In [5] a local sense stationary uncorrelated scattering (LSSUS) assumption is proposed for characterizing the time-varying radiocommunication channel. This approach may also be used for UAC channel stochastic description.

A process is called local-sense stationary (LSS) if there exist some partitions for which at least one interval, here denoted as J_i, is considered to be WSS, $J_{(V)} = J_i$. Within this 'local' stationary interval J_i the second-order statistics are approximately independent of time, but vary slowly in time, across all other intervals for which $J_{(V)} \neq J_{k \neq i}$. Thus, the autocorrelation is WSS at J_i but non-WSS at all other intervals $J_k \neq J_i$ [5]. Moreover, the process is local-sense uncorrelated scattering (LUS) if its Fourier transform satisfies the LSS assumption.

Stationary time

The stationary interval, wherein the UAC channel impulse response $h(t,\tau)$ can be considered as WSS process, is called stationary time T_D. Based on the WSS process definition [6], it can be determined as the following: let the $R_h(t,\Delta\tau)$ be the autocorrelation function of $h(t,\tau)$ in τ domain. Stationary time defined as maximum observation time interval T_D, for which $R_h(t,\Delta\tau)$ is stationary process and the mean value of $h(t,\tau)$ is constant.
In the case of an autocorrelation function $R_h(t,\Delta\tau)$ being a stationary process in t domain, correlation coefficients $\mathrm{c_h(t_i, t_i + \Delta t)}$ of successive rows of $R_h(t,\Delta\tau)$ have to be computed. Fig. 4 shows the autocorrelation function $R_h(t,\Delta\tau)$ for the impulse response presented in Fig. 1, and values of the correlation coefficients of their rows are shown in Fig. 5. A high value of correlation coefficient $c_h(\mathrm{t_i}, \mathrm{t_i} + \Delta t)$ indicates great similarity of $R_h(t_i,\Delta\tau)$ and $R_h(t_i + \Delta t, \Delta\tau)$.

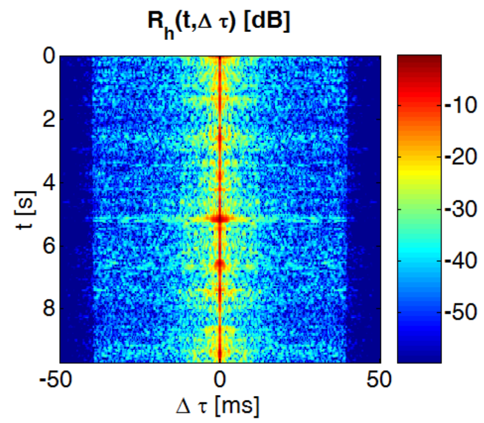

Fig. 4. Autocorrelation function of UAC impulse response $h(t,\tau)$ over τ.

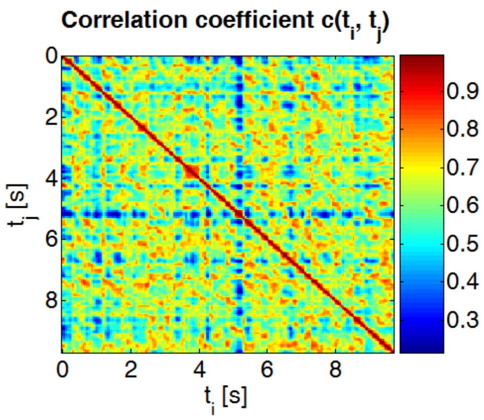

Fig. 5. Correlation coefficients of rows of $\mathrm{R_h(t, \Delta\tau)}$

In [5] the local region of stationarity (LRS) is defined as the region where, starting from its maximum value, the correlation coefficient $\mathrm{c_h(t_i, t_i + \Delta t)}$ does not undergo a threshold

$c_{th} = 0.5$. The width T_{D_i} of such a region for i-th row of $R_h(t, \Delta\tau)$ can be determined as the maximum observation time offset, for which $R_h(t_i, \Delta\tau)$ is highly correlated with $R_h(t_i + T_{D_i}, \Delta\tau)$:

$$T_{D_i} = \max\{\Delta t|_{c_h(t_i, t_i+\Delta t) \geq c_{th}=0.5}\}, \quad (24)$$

Thus, the stationary time T_D is calculated as that minimum of across the T_{D_i}:

$$T_D = \min\{T_{D_i}\} \quad (25)$$

Stationary bandwidth

The stationary bandwidth B_D is a range of frequencies wherein the UAC impulse response can be modeled as a US process. Let the $R_H(\Delta t, f)$ be the autocorrelation function of $H(t, f)$ in the t domain. Stationary bandwidth B_D is defined as the maximum frequency range in which $R_H(\Delta t, \mathrm{f})$ is a stationary process in f domain and the mean value of $H(t, f)$ is constant. Fig. 6 presents the autocorrelation function $R_H(\Delta t, \mathrm{f})$ of a transfer function shown in Fig. 2. In order to test, whether in a given frequency range Δf the transfer function $H(t, f)$ behaves as stationary process, correlation coefficients $c_H(f_i, f_i + \Delta f)$ between the TF columns are calculated. Fig. 7 shows the results. The high value of correlation coefficient $c_H(f_i, f_i + \Delta f)$ indicates, that $R_H(\Delta \mathrm{t}, f_i)$ and $R_H(\Delta \mathrm{t}, f_i + \Delta f)$ are very similar.

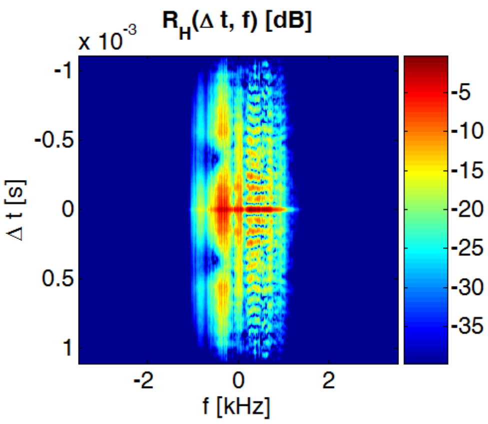

Fig. 6. Autocorrelation function of UAC transfer function $\mathrm{H(t, f)}$ over t.

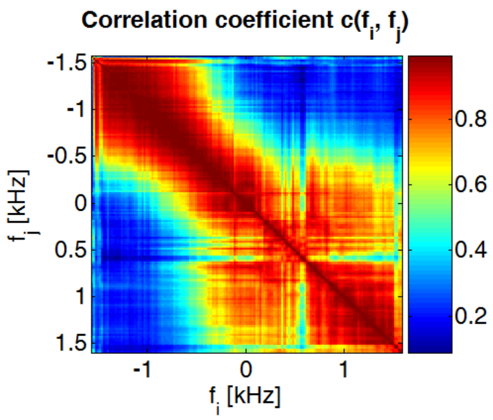

Fig. 7. Correlation coefficients of columns of $\mathrm{R_H(\Delta t, f)}$

The stationary bandwidth is determined as the range on both sides of $f_0 = 0$, being the “middle point” of the transfer function, where, starting from its maximum value, the correlation coefficient $c(0, \Delta f)$ does not undergo a threshold $c_{th} = 0.5$:

$$B_D = \max\{\Delta f|_{c(0,\Delta f) \geq c_{th}=0.5}\}, \quad (26)$$

Table 1 presents stationary bandwidth values of selected UAC channels measured during the run A40 experiment [1]. Both the stationary time and stationary bandwidth parameters, can be used for designing the physical layer of data transmission. They identify local regions of stationarity concerning the channel characteristics in time and frequency domain; and thus, can be the basis for selecting transmitting symbol duration, or subcarrier spacing, in multicarrier modulation schemes.

Tab.1. Stationary bandwidth of selected UAC channels measured with 7 hydrophones (H1 - H7) during the run A40 experiment [1].

Channel	Stationary bandwidth [Hz]						
	H1	H2	H3	H4	H5	H6	H7
A40_20100308193100	3.1111	3.5000	1.6453	3.1111	3.1111	2.4430	2.6923
A40_20100308194600	2.8120	2.9217	2.1439	1.6553	1.6553	1.6154	2.9017
A40_20100308200100	3.3006	3.2707	2.9416	1.2066	1.2066	2.1538	3.1510
A40_20100308201600	1.6254	3.2308	1.6154	1.3362	1.3362	3.1410	3.1111
A40_20100308204600	3.3504	3.5000	3.5000	2.8917	2.8917	2.6823	1.6054
A40_20100308210100	3.2507	1.5456	1.3362	1.1467	1.1467	1.5556	2.1738
A40_20100308213100	3.3604	3.3405	3.4202	3.1410	3.1410	3.3903	1.4858
A40_20100308214600	1.9046	1.5057	2.8219	3.5000	3.5000	2.1439	1.5057
A40_20100308220100	3.4302	1.5356	3.3604	2.9516	2.9516	3.3704	1.7650
A40_20100309044600	3.5000	3.5000	3.4900	0.8775	0.8775	3.4900	3.5000

4. Adaptive use of the indicators

To provide robust underwater acoustic communications, the channel has to be estimated based on adaptive equalization strategies. In this cooperation task the transmitter has to shape the wave transmission in such a way as to minimize the bit-error-rate at the receiver's side. The channel conditions can also change rapidly in stationary situations with motionless transmitters and receivers. Transmit parameters for this adaptive process are, for example, the mapping scheme, the number of sub-carriers, the code rate, guard intervals, the pilot pattern, the source level, etc. The receiver can estimate the channel transmission parameters like time spread and Doppler spread, stationary time and stationary bandwidth, the indicators I_{US} and I_{WSS}, can choose the gain value, and determine the bit error rate over time. Both transmitter and receiver have different knowledge. If they exchange the system identification information, this knowledge is old after traveling through the medium. With a distance of 1 NM the travel time is 1.2 seconds, mostly longer than the stationary time of the channel. On the other hand, the Bit Error Rate depends on the Doppler shift, and the chosen non-stationary channel model [7]. How can we overcome this dilemma with instantaneous channel characteristics?

One approach is the use of a log-book. Important values in network protocols are, for example, the last hop and next hop address [8], transmitted via a separate network header together with the application transfer volume, the user message. If the message receiver add both indicators in the form of two bits, namely WSS bit (1 if $I_{WSS} \geq 0.5$, otherwise 0) and US bit (1 if $I_{US} \geq 0.5$, otherwise 0) into the information header, the transmitter and all other neighbors can store this information in their own log-book. Table 2 and Table 3 present the outline idea of the possibilities of adaptive matching of the OFDM transmission parameters to values of WSS and US bits.

Tab.2. Rules of adaptive matching the OFDM transmission parameters to the value of WSS bit.

WSS Bit	# pilot tones	#sub-carriers	Symbol duration	Statistical model
1: $I_{WSS} \geq 0.5$	↓ could decrease	↑ can extend	↑ can increase	WSS
0: $I_{WSS} < 0.5$	↑ has to increase	↓ should reduce	↓ should reduce	LSS

Tab.3. Rules of adaptive matching of the OFDM symbol guard time to value of US bit.

US Bit	Guard time	Statistical model
1: $\mathrm{I_{US}} \geq 0.5$	↓ can reduce	US
0: $\mathrm{I_{US}} < 0.5$	↑ should extend	CS

The WSS bit can determine the number of OFDM pilot tones, sub-carrier spacing and OFDM symbol duration. The US Bit implies that there is no correlation between the fading coming from different signal scatterers. This has an influence on the multipath spread and thus the US indicator can determine the duration of OFDM symbol guard time.

5. Conclusions

Analysis of UAC channel impulse responses gathered during sea trials conducted by WTD71-FWG has shown that underwater communication channels can hardly ever be modeled as a WSSUS stochastic process. Two new indicators are proposed to classify channels as being WSS/non-WSS, and US/non-US. In case of a non-WSSUS channel, it is possible to define stationary time and stationary bandwidth, determining the period of time and frequency range, in which the WSSUS assumption is locally satisfied. These parameters can be helpful parameters in designing the physical layer of data transmission. However, in an adaptive UAC system working in rapidly varying conditions, the information about instantaneous stationary parameters (that can take values over a wide range) should be exchanged sufficiently frequently, which may not be possible, taking into account the significant latencies in the data transmission system. To overcome this limitation, another approach is proposed, based on defining the instantaneous channel state with the use of WSS and US bits transmitted as part of network protocol, and exploit in system transmitter and receiver, to adapt the signaling scheme to underwater channel propagation conditions.

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Appendix

Matlab script for WSS indicator calculation.

```
function WSSIndicator = wss_indicator(fn, ir_string)
%INPUT: fn - name of a .mat file with impulse response measurement;
%       ir_string - name of an impulse response (e.g. h.h001, h.h002, etc.)
load(fn);
ir = eval(ir_string); ir = ir./max(max(ir));   %impulse response (IR)
var_mean = var(mean(ir));             %variance of mean value of IR
I = real(ir); Q = imag(ir);           %in-phase and quadrature components of IR
ACF_I = [];ACF_Q = [];ACF_C=[]; C_IQ = []; C_IQQI = [];
for i=1:length(tau)
     %autocorrelation of IR in-phase component
     ACF_I = [ACF_I xcorr(I(:,i))/length(t)];
     ACF_I(:,i) = ACF_I(:,i)-mean(ACF_I(:,i));
     %autocorrelation of IR quadrature component
     ACF_Q = [ACF_Q xcorr(Q(:,i))/length(t)];
     ACF_Q(:,i) = ACF_Q(:,i)-mean(ACF_Q(:,i));
     %cross-correlation of IR in-phase and quadrature component
     ACF_C=[ACF_C xcorr(I(:,i), Q(:,i))/length(t)];
     ACF_C(:,i)=ACF_C(:,i)-mean(ACF_C(:,i));
     %maximum value of cross-correlation of ACF_I and ACF_Q
     C_IQ = [C_IQ max(xcorr(ACF_I(:,i), ACF_Q(:,i),'coeff'))];
     %maximum value of cross-correlation of ACF_C and its "mirror reflection"
     C_IQQI = [C_IQQI max(xcorr(ACF_C(:, i), ACF_C(end:-1:1, i),'coeff'))];
end
mn_C_IQ = mean(C_IQ); mn_C_IQQI = mean(C_IQQI);
%WSS indicator:
WSSindicator = sqrt((mn_C_IQ*mn_C_IQQI)/(1+var_mean));
```

Matlab script for US indicator calculation.

```
function USIndicator = us_indicator(fn, ir_string)
%INPUT: fn - name of a .mat file with impulse response measurement;
%       ir_string - name of an impulse response (e.g. h.h001, h.h002, etc.)
load(fn)
ir = eval(ir_string);  ir = ir./max(max(ir));  %impulse response (IR)
Tf = fftshift(fft(ir,[],2));                    %transfer function:
var_mean = var(mean(Tf));                       %variance of mean value of TF
I = real(Tf); Q = imag(Tf);           %in-phase and quadrature components of TF
ACF_I = [];ACF_Q = [];ACF_C=[]; C_IQ = []; C_IQQI = [];
for i=1:length(t)
     %autocorrelation of TF in-phase component
     ACF_I = [ACF_I; xcorr(I(i,:))/length(f)];
     ACF_I(i,:) = ACF_I(i,:)-mean(ACF_I(i,:));
     %autocorrelation of TF quadrature component
     ACF_Q = [ACF_Q xcorr(Q(i,:))/length(f)];
     ACF_Q(i,:) = ACF_Q(i,:)-mean(ACF_Q(i,:));
     %cross-correlation of TF in-phase and quadrature component
     ACF_C=[ACF_C xcorr(I(i,:), Q(i,:))/length(f)];
     ACF_C(i,:)=ACF_C(i,:)-mean(ACF_C(i,:));
     %maximum value of cross-correlation of ACF_I and ACF_Q
     C_IQ = [C_IQ max(xcorr(ACF_I(i,:), ACF_Q(i,:),'coeff'))];
     %maximum value of cross-correlation of ACF_C and its "reflection"
     C_IQQI = [C_IQQI max(xcorr(ACF_C(i,:), ACF_C(i,end:-1:1),'coeff'))];
end
mn_C_IQ = mean(C_IQ); mn_C_IQQI = mean(C_IQQI);
%US indicator:
USindicator = sqrt((mn_C_IQ*mn_C_IQQI)/(1+var_mean));
```

Matlab script for stationary time calculation.

```
function tD = stationary_time(fn, ir_string)
%INPUT: fn - name of a .mat file with impulse response measurement;
%       ir_string - name of an impulse response (e.g. h.h001, h.h002, etc.)
load(fn);
ir = eval(ir_string);  ir = ir./max(max(ir));  %impulse response (IR)
XC = [];
for i=1:length(t)
    XC = [XC xcorr(ir(i,:))]             %autocorrelation of IR over tau
end
XC = XC./max(max(abs(XC)));
R = corrcoef(XC');                       %correlation coefficients of XC rows
TRR = ones(size(R));
for i=1:length(t)
    TR = find(abs(R(i,:))<treshold); TRR(i, TR)=0;   %tresholding R
end
Cth = TRR;
ss = size(Cth); C2th = ones(ss);
for i=1:length(t)
    for k = i+1:length(t)
        C2th(i,k) = Cth(i,k)*C2th(i,k-1);
    end
    for k = i-1:-1:1
        C2th(i,k) = Cth(i,k)*C2th(i,k+1);
    end
end
%Stationary time:
TSj = sum(C2th); TS_ind = min(TSj); TD = TS_ind*t(2);
```

Matlab script for stationary bandwidth calculation.

```
function BD = stationary_bandwidth(fn, ir_string)
%INPUT: fn - name of a .mat file with impulse response measurement;
%       ir_string - name of an impulse response (e.g. h.h001, h.h002, etc.)
load(fn);
ir = eval(ir_string);  ir = ir./max(max(ir));  %impulse response (IR)
Tf = fftshift(fft(ir,[],2));                   %transfer function
XC = [];
for i=1:length(f)
    XC = [XC xcorr(Tf(:,i))]             %autocorrelation of TF over f
end
XC = XC./max(max(abs(XC)));
R = corrcoef(XC);                        %correlation coefficients of XC columns
TRR = ones(size(R));
for i=1:length(f)
    TR = find(abs(R(i,:))<treshold); TRR(i, TR)=0;   %tresholding R
end
Cth = TRR; ss = size(Cth); C2th = ones(ss);
for i=1:length(f)
    for k = i+1:length(f)
        C2th(i,k) = Cth(i,k)*C2th(i,k-1);
    end
    for k = i-1:-1:1
        C2th(i,k) = Cth(i,k)*C2th(i,k+1);
    end
end
%Stationary bandwidth:
BSj = sum(C2th); BS_ind = BSj(ceil(length(f)/2)); BD = BS_ind*fs/length(f);
```