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STUDY OF STARTING MODES OF SINGLE-PHASE INDUCTION MOTORS WHEN CHANGING THE PARAMETERS OF THE STATOR WINDINGS, PHASE-SHIFTING CAPACITOR AND SUPPLY VOLTAGE

Suad Omar Aldaikh[1], Mohannad O. Rawashdeh[2], Lina Hani Hussienat[1], Mohamed Qawaqzeh[3], Oleksiy Iegorov[4], Olga Iegorova[5], Mykola Kundenko[5], Dmytro Danylchenko[6], Oleksandr Miroshnyk[7], Taras Shchur[8]

[1]Al-Balqa Applied University, Department of Technical Science, Al Salt, Jordan, [2]Al-Balqa Applied University, Department of Mechanical Engineering, Al Salt, Jordan, [3]Al-Balqa Applied University, Department of Electrical and Electronics Engineering, Al Salt, Jordan, [4]O. M. Beketov National University of Urban Economy in Kharkiv, Department of Alternative Energy, Kharkiv, Ukraine, [5]National Technical University "Kharkiv Polytechnic Institute", Department of Heat Engineerine and Energy-efficient Technologies, Kharkiv, Ukraine, [6]National Technical University "Kharkiv Polytechnic Institute", Department of Electrical Energy Transmission, Kharkiv, Ukraine, [7]State Biotechnological University, Department of Electricity Supply and Energy Management, Kharkiv, Ukraine, [8]Cyclone Manufacturing Inc, Mississauga, Ontario, Canada

Abstract. *Single-phase induction motors (SPIM) are widely used in household appliances, agriculture, trade, medicine and other areas where a cheap unregulated electric drive powered by a single-phase AC network is required. They are produced in millions of pieces per year. Therefore, significant attention has always been paid to research aimed at reducing resource consumption in the production and operation of these engines, improving the initial characteristics and increasing their competitiveness. The article conducted a study of the starting torque when starting single-phase induction motors depending on the initial phase of the voltage of the single-phase network supplying the stator winding, on the phase angles of the network voltage at fixed phase angles of the starting winding. It is also analysed how the active resistance and inductive leakage resistance of the stator winding and the equivalent rotor winding, and the resistance of the capacitor capacitance affect the starting characteristics of the engine.*

Keywords: single-phase induction motors, starting torque, starting current, wind resistance, capacitor

BADANIE TRYBÓW ROZRUCHU JEDNOFAZOWYCH SILNIKÓW ASYNCHRONICZNYCH PRZY ZMIANIE PARAMETRÓW UZWOJEŃ STOJANA, KONDENSATORA PRZESUWAJĄCEGO FAZĘ I NAPIĘCIA ZASILANIA

Streszczenie. *Jednofazowe silniki asynchroniczne znajdują szerokie zastosowanie w sprzęcie AGD, rolnictwie, handlu, medycynie i innych dziedzinach, gdzie wymagany jest tani nieregulowany napęd elektryczny zasilany z jednofazowej sieci prądu przemiennego. Produkowane są w milionach sztuk rocznie. Dlatego zawsze dużą wagę przywiązywano do badań mających na celu zmniejszenie zużycia zasobów w produkcji i eksploatacji tych silników, poprawę parametrów wyjściowych i zwiększenie ich konkurencyjności. W artykule przeprowadzono badania momentu rozruchowego przy rozruchu jednofazowych silników asynchronicznych w zależności od fazy początkowej napięcia sieci jednofazowej zasilającej uzwojenie stojana, od kątów fazowych napięcia sieciowego przy ustalonych kątach fazowych rozruchu. meandrowy. Analizowano także wpływ rezystancji czynnej i indukcyjnej rezystancji uzwojenia stojana i zastępczego uzwojenia wirnika oraz rezystancji pojemności kondensatora na charakterystykę rozruchową silnika.*

Słowa kluczowe: silniki indukcyjne jednofazowe, moment rozruchowy, prąd rozruchowy, opór powietrza, kondensator

Introduction

Large volumes of SPIM production lead to high costs of energy resources and materials. Energy consumption by household appliances is comparable to industrial production, and the consumption of active materials is comparable to the consumption of materials in the production of generators for thermal and hydraulic power plants. Therefore, significant attention is paid to research aimed at reducing resource costs in the production and operation of these engines, improving output characteristics and increasing their competitiveness.

SPIM have become widespread due to such positive properties as:

- the possibility of contactless transmission of torque through the air gap, regardless of the direction and speed of the rotor [6];
- natural stability of the motor moment;
- possibility of direct start and reverse under load;
- the possibility of constructive use of the finished stator of small motors in the corresponding standard sizes of other types of electric motors: synchronous, controlled reluctance and valve motors;
- simplicity of technological implementation of the laminated magnetic circuit of these electrical machines with a small air gap.

1. Analysis of calculation models

Since SPIM are asymmetrical machines, generally possessing all types of asymmetry: electrical, spatial and magnetic, a number of scientists have developed methods for studying these machines that make it possible to take this asymmetry into account when calculating characteristics.

The practice of designing electrical machines can be based on the use of analytical and numerical (field) calculation methods. Analytical methods can be based on a combination of field theory and electrical circuit theory [8,16]. Numerical methods are mainly used to calculate the magnetic field of an electric machine; the integral parameters determined after numerical calculation can then be used as parameters of electrical and magnetic circuits for equivalent circuits.

The theory of a generalized electromechanical energy converter is one of the most universal; with its help, any types of electrical machines, including induction ones, can be considered [18–22].

The main analytical methods of circuit theory used to model asymmetrical alternating current machines include the following:

- method of two reactions (longitudinal and transverse fields);
- method of two rotating fields;
- method of symmetrical components.

The two-reaction method [24,25] is that each of the variables of a multiphase winding system is decomposed into two components along two mutually perpendicular axes. When calculating, MMF and magnetic fluxes are considered separately for each axis. As a result, instead of a multiphase system, a two-phase and, accordingly, two systems of equations are obtained. This method is convenient for mathematical modelling and study of transient processes of SPIM on a computer. The two-reaction method was used in [12]; however, it does not have sufficient clarity to represent the physical processes occurring in the machine.

The method of two fields rotating in opposite directions [13, 14, 28] is as follows. Regardless of the number of windings, type and operating mode of an induction machine with a sinusoidal change in magnetic induction, the magnetic field in the air gap for each phase can be considered as consisting of forward and reverse traveling fields with half amplitudes. The number of these fields is equal to twice the number of phases. The interaction of the resulting forward and reverse fields with the rotor currents creates two torques directed in opposite directions. The resulting moment is determined by their algebraic difference. The method was further developed in [4]. There, the decomposition of pulsating spatial harmonics of the MMF phases into rotating waves is used, in relation to an equivalent unsaturated machine with an air gap that provides the average value of the flux present in the real machine. In [31, 32], the general case of asymmetry of the stator winding of a multiphase induction machine is considered, when each of the stator phase windings contains a different number of turns, has an arbitrary structure and is shifted relative to other phases by any angle. In [1], the rotating field method was used together with the tooth contour method.

The method of symmetrical components [15] is based on the decomposition of asymmetrical variables of a multiphase motor into symmetrical systems, each of which creates its own circular rotating field acting on the rotor of the machine, and a zero sequence that creates a pulsating field for harmonics, the orders of which are in machines with three-phase winding is a multiple of three. The resulting torque on the motor shaft is determined by the algebraic addition of the moments from the individual components. The method received significant development in [3], where the method of symmetrical components was generalized for machines with magnetic and spatial asymmetry. In [7], general principles of the theory of asymmetrical machines were developed, which make it possible to study any operating modes of alternating current machines with an arbitrary spectrum of spatial harmonics.

In [9, 17], the theory of induction motors was developed for any degree of winding asymmetry and a corresponding calculation method was proposed.

The choice of one method or another depends on the tasks being solved and does not have significant priorities. At the moment, for solving SPIM design problems, the most common is method of symmetrical components, which has the following advantages:

- relatively easy determination of equivalent circuit parameters;
- obtaining simple and convenient mathematical expressions for the operating and starting characteristics of the machine in any mode of operation;
- sufficient clarity when analysing the influence of a particular parameter on the indicators and characteristics of SPIM;
- the ability to take into account the influence of higher harmonics on the characteristics of the machine. Method of symmetrical components is mainly used to study steady-state processes, but it can also be used to study transient processes in electrical machines [10].

To calculate the characteristics of an electric machine using an equivalent circuit, it is necessary to have predetermined active and inductive parameters, and the accuracy of determining the parameters significantly affects the adequacy of the calculated characteristics of the machine. The most accurate calculation of parameters – coefficients in differential equations – can be carried out using field theory methods based on Maxwell's equations. The use of this method requires a very complex description of the boundary conditions of the calculated areas and large expenditures of computing resources. The calculation is usually performed using the finite difference method or the finite element method [2, 18]. Today, despite the rapid development of computer technology and universal software calculation tools (Ansys, Flux Faraday), such calculations still remain very complex and therefore must be used in combination with analytical methods for calculating the magnetic field.

Analytical methods for calculating the magnetic field are associated with a number of assumptions [5], but significantly simplify it. When calculating the magnetic field in the air gap, the cogging of the stator and rotor cores is taken into account using the air gap coefficient (Carter coefficient), obtained by the conformal transformation method.

With complex shapes of the areas under study, analytical calculation methods do not provide acceptable accuracy; the introduction of correction factors is often required. In addition, due to the assumptions underlying the analytical methods, taking into account saturation, changes in the field shape and other factors cannot be taken into account at all.

Advanced capabilities for calculating the magnetic field are provided by the tooth contour method [29, 30]. The essence of the method is to divide the two-dimensional region of the air gap field into zones of so-called tooth contours, covering one tooth of the core. Under the accepted assumptions about the two-dimensionality of the field and the infinite magnetic permeability of the core material, the field of the tooth circuit is independent of the fields of other circuits. The total magnetic field in the air gap is represented as the sum of the fields from the tooth circuit currents. Under special boundary conditions, the field of the tooth contour can be calculated by some numerical method. Based on this calculation, the flux through the gear division is determined and the specific magnetic conductivity between the stator and rotor circuits is calculated. The tooth contour method is universal and quite accurate; it can be used to calculate most types of electrical machines. It is not entirely applicable only when the magnetic circuits are significantly saturated and the field distribution in the axial direction is uneven.

Summarizing what has been said, it can be argued that the best in terms of accuracy and at the same time acceptable in terms of resource intensity calculation methods are obtained by combining analytical and numerical methods. They are usually the basis for computer-aided design systems.

2. Research results

As already noted, capacitor induction motors belong to asymmetrical electrical machines that have significant nonlinearity of functional connections. This is the starting point for the mathematical description of an electric machine and significantly complicates the analysis of their operating modes.

In this work, IM with a squirrel-cage rotor operating from a single-phase network are investigated. Starting of engines is carried out using methods known from the theory of electrical machines. The phenomena that occur during the starting, reversing and braking of the motor are very complex in nature and represent electromechanical transient processes, since in this case simultaneous changes in the rotor rotation speed, currents in the windings and magnetic fluxes occur.

When compiling a mathematical model of induction motors operating from a single-phase network, a capacitor induction motor was taken as the initial one, as it has a more complex circuit. By making certain assumptions, the SPIM equations can be simply converted into the SPIM equations with a starting winding on the stator, or even into the equations of three-phase IM operating from a symmetrical three-phase network.

Considering that an induction motor is a complex electrical system, when it is started and reversed, an electromechanical transient process occurs, the study of which leads to cumbersome nonlinear differential equations [11]. Solving such equations is associated with certain difficulties and is not always effective from the point of view of practical application. Experience in analysing transient processes in IM shows that it is not necessary to take into account all factors influencing transient processes [17–20]. When drawing up a mathematical model, it is necessary to identify the main factors in complex phenomena and exclude secondary ones from consideration. This is due to the fact that it is often not possible to take into account all influencing factors in the mathematical description of electromechanical transient processes or is associated with significant difficulties.

Therefore, in practice it is necessary to consider some idealized electrical machine, for which the following assumptions are usually made [27, 28]:

1. The air gap is uniform.
2. The magnetic permeability of steel is assumed to be infinitely large. In research, this allows the principle of superposition to be applied. Steel saturation is taken into account by choosing the parameters of the electric motor.
3. Losses due to hysteresis and eddy currents are not taken into account when drawing up equations.
4. Only the fundamental harmonics of the spatial distribution of the MMF and magnetic fields created by the windings are taken into account. The MMF and induction curves in the air gap are distributed according to a sinusoidal law.
5. The real multi-loop squirrel-cage winding of the rotor is replaced by two equivalent squirrel-cage loops. When replacing the real rotor winding with equivalent circuits, we proceed from the equality of the fundamental harmonic MMFs created by the real and equivalent windings.
6. An induction motor operates from a source of infinite power, adjustable frequency and amplitude, alternating voltage supplied to the stator windings sinusoidally.
7. The parameters of all phases of the stator and rotor windings are reduced to the effective number of turns of the stator winding phase.

The exclusion of the above assumptions or the introduction of new ones will be specifically discussed in the future.

Let's consider the electromechanical transient processes of an electric drive with a capacitor IM. The SPIM diagram is shown in Fig. 1. The stator has two windings, which, as a rule, have a different number of turns (*Wa*, *Wb*) and are displaced in space by 90 electrical degrees. Both stator windings are connected to a single-phase network. Capacitor *C* is connected in series with winding *B*.

The system of differential equations describing the behaviour of an electric drive with SPIM includes the voltage equations of the windings and the equations of motion of the electric drive.

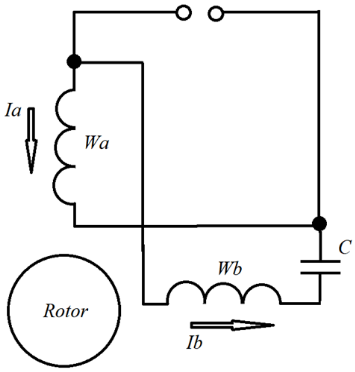


Fig. 1. Scheme of capacitor SPIM

When drawing up voltage equilibrium equations, it is first advisable to use the natural (phase) system of coordinate axes *a* and *b* – the phase axes of the main and starting stator windings *d* and *q* – the axes of the equivalent rotor windings. Counter clockwise rotation is taken as the positive direction of rotation of the electric drive. It is assumed that the starting windings of the stator b and the rotor are connected to the main stator winding of phase *a*, which makes it possible to use the same system of relative units for rotor and stator quantities.

Let us compose the equilibrium equations for the voltage of the windings of the induction motor under consideration in the phase (natural) coordinate system. Taking into account the above, we present the axes of the windings in the form shown in Fig. 2.

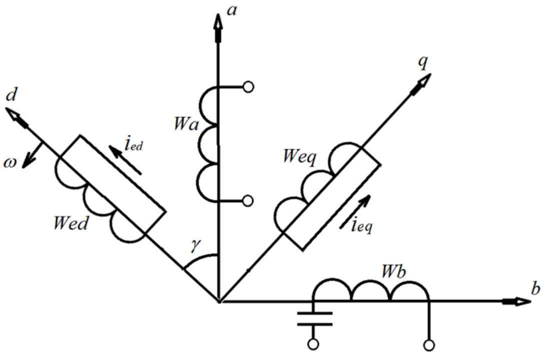


Fig. 2. System of natural coordinates

Then the voltage equations of the windings in the natural coordinate system can be written as:

$$\left.\begin{aligned} u_a &= \frac{d\psi_a}{dt} + r_a i_a; 0 = \frac{d\psi_{ed}}{dt} + r_{ed} i_{ed} \\ u_b &= \frac{d\psi_b}{dt} + r_b i_b; 0 = \frac{d\psi_{eq}}{dt} + r_{eq} i_{eq} \end{aligned}\right\} \qquad (1)$$

Considering that in the system of differential equations describing operation of the SPIM, all parameters of the stator and rotor windings must be reduced to the effective number of turns of the main winding *a* through the reduction coefficient [19]. In what follows, for convenience of notation, we omit the prime, considering that the given parameter values are included in the equations below.

The flux linkage of the windings is determined by the expressions:

$$\left.\begin{aligned} \psi_a &= L_a i_a + M_{aed} i_{\text{э}d} + M_{aeq} i_{eq}; \\ \psi_b &= L_b i_{\text{в}} + M_{bed} i_{ed} + M_{beq} i_{eq}; \\ \psi_{ed} &= L_{aed} i_a + M_{bed} i_b + L_{ed} i_{ed}; \\ \psi_{eq} &= L_{aeq} i_a + M_{beq} i_b + L_{eq} i_{eq}; \end{aligned}\right\} \qquad (2)$$

The following notations are used in equations (1) and (2): u_a, u_b – instantaneous values of voltages supplied to the stator phase windings; ψ_a, ψ_b, ψ_{ed}, ψ_{eq} – instantaneous values of the flux linkages of the stator windings and equivalent rotor windings along axes *a* and *b*, *d* and *q*; r_a, r_b, r_{ed}, r_{eq} – active resistance of the stator windings and equivalent rotor windings; i_a, i_b, i_{ed}, i_{eq} – respectively, currents in the stator and rotor windings Fig. 1, 2; L_a, L_b, L_{aed}, L_{aeq} – inductance of stator and rotor windings; M_{aed}, M_{bed}, M_{aeq}, M_{beq} – respectively, mutual inductances between the stator and rotor windings.

In (2) it is taken into account that there is no mutual inductance between windings *a* and *b*, as well as *ed* and *eq* due to the uniformity of the air gap and the symmetry of the structure [13]. There is mutual inductance only between windings moving relative to each other.

Mutual inductances are equal:

$$\left.\begin{aligned} M_{aed} &= M\cos\gamma; \\ M_{beq} &= M\cos\gamma; \\ M_{aeq} &= M\cos\left(\frac{\pi}{2} - \gamma\right) = M\sin\gamma; \\ M_{aed} &= M\cos\left(\frac{\pi}{2} + \gamma\right) = -M\sin\gamma; \end{aligned}\right\} \qquad (3)$$

where *M* is the maximum value of mutual inductance between the windings of the stator and rotor phases.

Substituting (3) into equations (2), we obtain:

$$\left.\begin{aligned} \psi_a &= L_a i_a + M\cos\gamma i_{ed} + M\sin\gamma i_{eq};\\ \psi_b &= L_b i_b + M\cos\gamma i_{ed} + M\sin\gamma i_{eq};\\ \psi_{ed} &= M\cos\gamma i_a - M\sin\gamma i_b + L_{ed} i_{ed};\\ \psi_{eq} &= M\sin\gamma i_a + M\cos\gamma i_b + L_{eq} i_{eq}; \end{aligned}\right\} \qquad (4)$$

Let's move on to writing equations in relative units. Equation (4) in the system of relative units will have the form:

$$\left.\begin{aligned} \psi_a &= x_a i_a + x_m\cos\gamma i_{ed} + x_m\sin\gamma i_{eq};\\ \psi_b &= x_b i_b - x_m\sin\gamma i_{ed} + x_m cos\gamma i_{eq};\\ \psi_{ed} &= x_m\cos\gamma i_a - x_m\sin\gamma i_b + x_{ed} i_{ed};\\ \psi_{eq} &= x_m\sin\gamma i_a + x_m\cos\gamma i_b + x_{eq} i_{eq}; \end{aligned}\right\} \qquad (5)$$

where x_m is the mutual induction resistance between the phases of the stator and rotor windings; x_a, x_b, x_{ed}, x_{eq} – total inductive resistance of the stator windings and equivalent rotor windings, which can be presented as follows:

$$\begin{aligned} x_a &= x_{\sigma a} + x_m;\\ x_b &= x_{\sigma b} + x_m;\\ x_{ed} &= x_{\sigma ed} + x_m;\\ x_{eq} &= x_{\sigma eq} + x_m \end{aligned} \qquad (6)$$

where $x_{\sigma a}$, $x_{\sigma b}$, $x_{\sigma ed}$, $x_{\sigma eq}$ are the inductive leakage resistances of the stator windings and equivalent rotor windings.

The power consumed by the SPIM from the network, in relative units, can be defined as:

$$\begin{aligned} P_1 &= u_a i_a + u_b i_b = \left(\frac{d\psi_a}{d\tau} + r_a i_a\right) i_a + \left(\frac{d\psi_b}{d\tau} + r_b i_b\right) i_b =\\ &= r_a i_a^2 + r_b i_b^2 + \frac{d\psi_a}{d\tau} i_a + \frac{d\psi_b}{d\tau} i_b \end{aligned} \qquad (7)$$

equation (5) is used and it is obtained:

$$\begin{aligned} \frac{d\psi_a}{d\tau} &=\\ &= x_a \frac{di_a}{d\tau} + x_m\left(\cos\gamma \frac{di_{ed}}{d\tau} + i_{ed}\frac{d\cos\gamma}{d\tau}\right) +\\ &+ x_m\left(\sin\gamma \frac{di_{eq}}{d\tau} + i_{eq}\frac{d\sin\gamma}{d\tau}\right);\\ \frac{d\psi_b}{d\tau} &=\\ &= x_b \frac{di_b}{d\tau} - x_m\left(\sin\gamma \frac{di_{ed}}{d\tau} + i_{ed}\frac{d\sin\gamma}{d\tau}\right) +\\ &+ x_m\left(\cos\gamma \frac{di_{eq}}{d\tau} + i_{eq}\frac{d\cos\gamma}{d\tau}\right); \end{aligned} \qquad (8)$$

where

$$\begin{aligned} \frac{d\cos\gamma}{d\tau} &= \frac{d\cos\gamma}{d\gamma}\frac{d\gamma}{d\tau} = -\sin\gamma\omega;\\ \frac{d\sin\gamma}{d\tau} &= \frac{d\sin\gamma}{d\gamma}\frac{d\gamma}{d\tau} = \cos\gamma\omega \end{aligned} \qquad (9)$$

Thus, the system of differential equations describing the transient electromechanical processes of SPIM in relative units, in the natural (phase) coordinate system [12], reduced to normal form, is written in the form:

$$\left.\begin{aligned} \frac{d\psi_a}{d\tau} &= \beta\cos\left(\alpha\tau + \varphi_0\right) - r_a i_a\\ \frac{d\psi_b}{d\tau} &= \beta k\cos\left(\alpha\tau + \varphi_0\right) - u_c - r_b i_b\\ \frac{d\psi_{ed}}{d\tau} &= -r_{ed} i_{ed}\\ \frac{d\psi_{eq}}{d\tau} &= -r_{eq} i_{eq}\\ \frac{du_c}{d\tau} &= x_c i_{в}\\ \frac{d\gamma}{d\tau} &= \omega\\ \frac{d\omega}{d\tau} &= \frac{1}{H}\left(M - M_C\right) \end{aligned}\right\} \qquad (10)$$

where M_C is the moment of resistance on the motor shaft; H – mechanical constant of rotating masses; u_c – voltage on the capacitor with relative units:

$$u_c = x_c \int i_b d\tau$$

System of equations (10) is a nonlinear system of differential equations with variable (periodic) coefficients, consisting of 7 equations. The system of equations includes 7 variables: ψ_a, ψ_b, ψ_{ed}, ψ_{eq}, ω, u_c, γ. The solution of the system of equations (10) due to its nonlinearity and the presence of periodic coefficients is possible only by numerical methods [26].

When designing an SPIM with given characteristics, it is necessary to calculate transient electromagnetic moments, namely the starting electromagnetic moment [23]. The current in the motor stator in the first periods has a maximum value, which determines the maximum value of the electromagnetic torque.

The article provides a computational assessment of the influence of the initial phase of the mains voltage on the maximum value of the electromagnetic torque when starting a motor. The first and second equations of the system of differential equations (10) include the initial phase of the network voltage, the electromagnetic torque depends on the voltage switching phase. The engine can be started with different phase values of the mains voltage. In addition, in automated electric drives with electronic switches, it is possible to simultaneously turn on the starting and main stator windings.

In the Mathcad program, a calculation algorithm has been created for determining the starting characteristics of a single-phase induction motor using the method described in this article.

Also, a calculation model of a power motor with a power of 1.5 kW, with the number of poles 2p = 4, was created in the Ansys Maxwell program (Fig. 3). During the calculations, only one parameter was changed, and the rest were taken constant and equal to the base values.

A comparison of the results of the starting characteristics, which were obtained analytically in Mathcad and the finite element method in Ansys Maxwell, allowed us to conclude that when designing the SPIM, it is possible to use analytical calculations using the method proposed by the authors. The discrepancy between the results is within 4–6%.

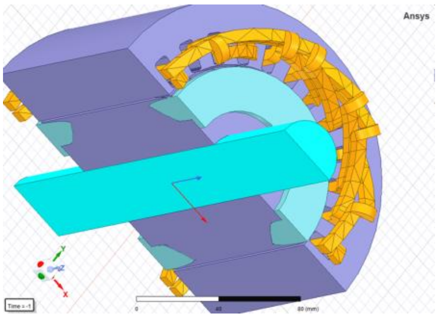


Fig. 3. Calculation model of SPIM

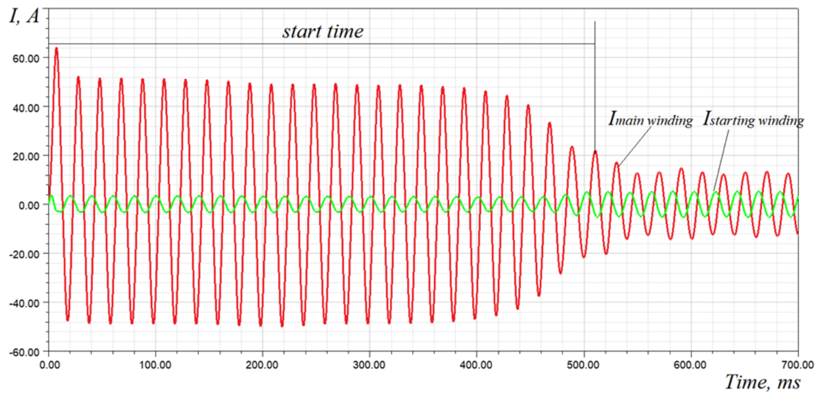


Fig. 4. Calculation of currents in the starting mode of sthe SPIM

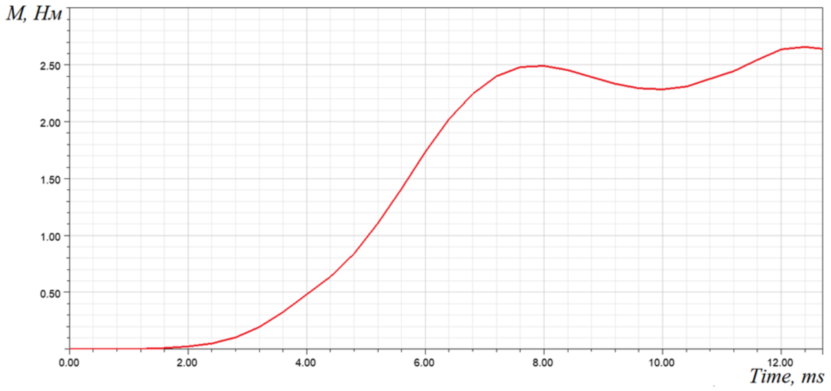
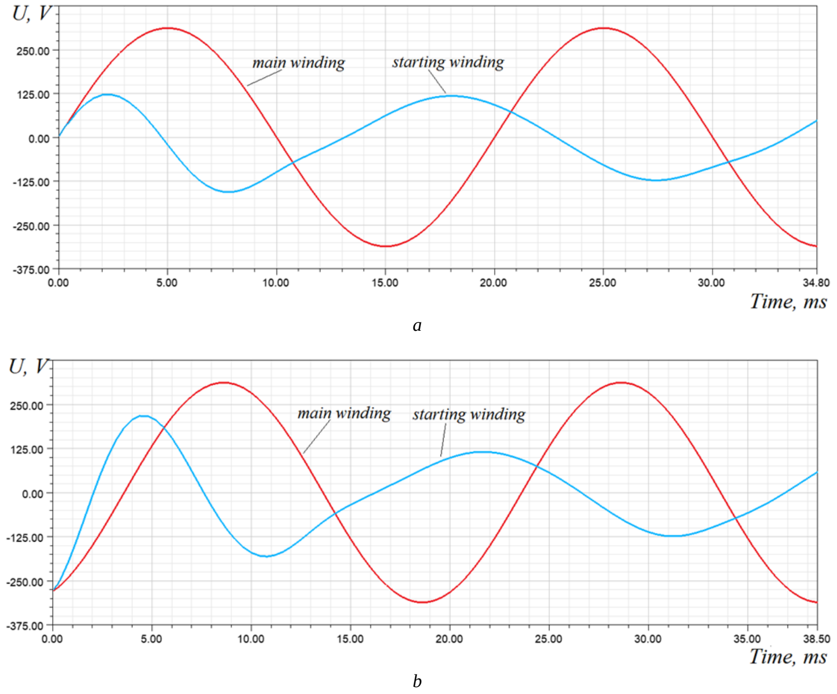


Fig. 5. Calculation of the starting torque of the SPIM



a

b

Fig. 6. Examples of voltage characteristics of the main and starting windings at different angles of the initial phase of the network voltage (a – 0 degrees; b – 65 degrees)

Examples of the results of calculation experiments when changing the value of the starting capacitor, respectively, the resistance X_C, are presented in Fig. 4 –6.

The results of calculating the starting torque when starting a capacitor SPIM with a power of 1.5 kW, depending on the initial phase of the network voltage, are shown in Fig. 7, 8. If the initial phases of the voltage (φ_a=φ_b) supplied to the windings are equal, the dependence is periodic in nature, and the starting torque takes on the greatest value at a phase angle of 150 (330) degrees, and the smallest at 30 (210) degrees.

The highest value of the starting electromagnetic torque exceeds the value of the starting torque by 1.5-2 times, and the minimum value practically corresponds to the value of the starting torque determined by the static characteristic.

In Fig. 7 shows the dependences of the starting electromagnetic torque on the phase angles of the network voltage at fixed phase angles of the starting winding, which are also periodic in nature, but with a period equal to 2π. In this case, the phase angle of the voltage of the starting winding was set to fixed values of 0, 60, 120 electrical degrees. The phase angle of the voltage of the main winding varied from 0 to 360 electrical degrees. In this case, the maximum value of the torque occurs at the initial phase angle of the starting winding voltage equal to 0, and for the main winding – 330 degrees.

In Fig. 8 shows the dependences of the electromagnetic torque with fixed values of the initial phase angle of the voltage of the main winding. At an initial voltage angle of the main winding of 120 electrical degrees, in the range from 10 to 210 electrical degrees, the values of the electromagnetic torque are equal to or exceed the torque value when the windings are simultaneously connected to the network. Comparing the graphs, you can see that the effect of the initial phase angle of voltage on the electromagnetic torque is not the same along the stator axes of the windings.

The dynamic mechanical characteristics of the SPIM at startup are influenced by the following parameters: active resistance and inductive leakage resistance of the stator winding and the equivalent rotor winding, capacitor resistance.

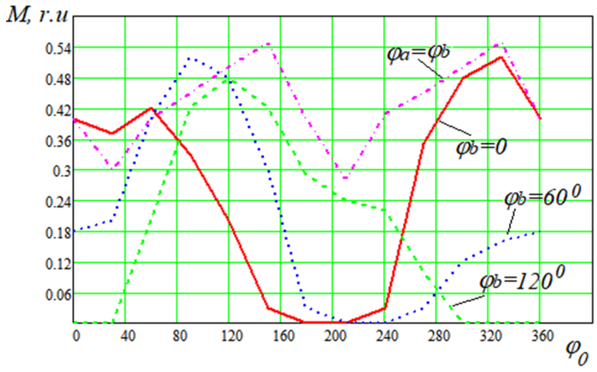


Fig. 7. Dependences of the starting electromagnetic torque M on the phase angles of the network voltage (φ_0) at fixed values of the initial phase angle of the starting winding (φ_b) and variable values of the phase angle of the main winding

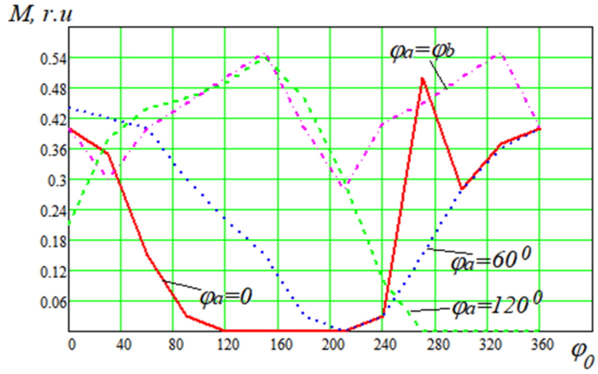


Fig. 8. Dependences of the starting electromagnetic torque M on the phase angles of the network voltage (φ_0) at fixed values of the initial phase angle of the main winding (φ_a) and variable values of the phase angle of the starting winding

Figures 9 – 12 show the dependences of the starting torque and starting current on the active and inductive resistance of the main stator winding, the inductive resistance of the capacitor and the inductive resistance of the rotor.

At the same time, studies have shown that changes in the active and inductive resistance of the starting winding have virtually no significant effect on the starting electromagnetic torque and starting current.

Starting torque and starting currents of windings with an increase in the inductive resistance of the stator starting winding from 1.1 to 1.8 relative units (r.u.) decrease by 2 times.

When the inductive resistance of the rotor winding increases from 1.1 to 2 r.u. the starting torque is halved, and the starting current has a minimum value at x = 1.5 r. u.

From the curves presented in Fig. 12 you can see that the greater the resistance of the capacitor, the lower the starting current and starting torque.

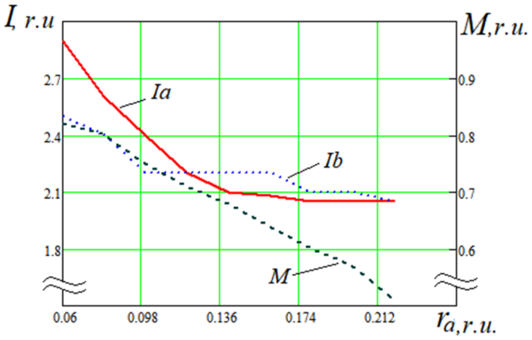


Fig. 9. Dependences of starting currents in the windings (Ia, Ib) and starting torque (M) on the active resistance of the main stator winding (r_a)

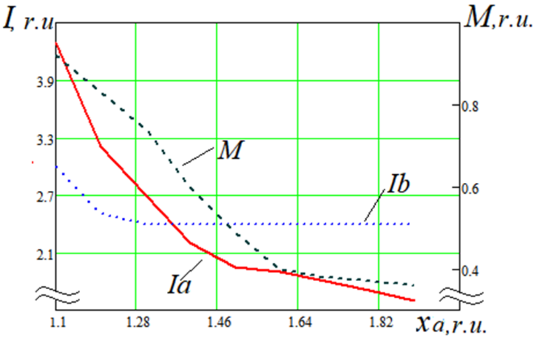


Fig. 10. Dependences of starting currents in the main and starting windings (Ia, Ib) and starting torque (M) on the inductive resistance of the main stator winding (x_a)

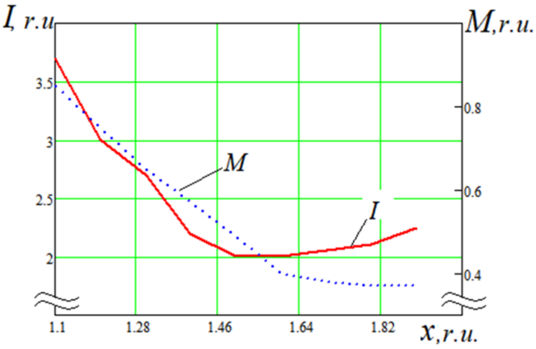


Fig. 11. Dependences of the starting current (I) and starting torque (M) on the inductive resistance of the rotor winding (x)

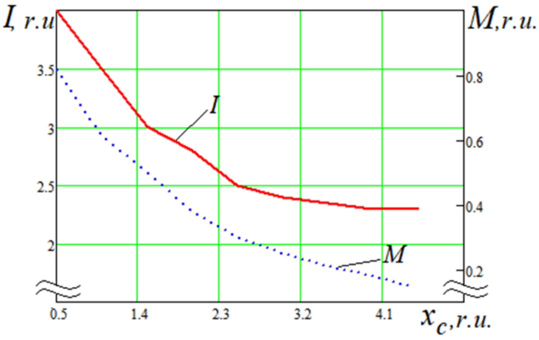


Fig. 12. Dependence of the starting current (I) and starting torque (M) on the inductive reactance of the capacitor (x_c)

3. Conclusions

Among the mathematical methods used to study asymmetrical machines, including SPIM, the most developed method is the method of symmetrical components. Therefore, this article uses this method as the basis for theoretical studies of SPIM and refinement of its mathematical model. A mathematical model of an electric drive with SPIM in a phase coordinate system has been developed, which, under appropriate assumptions, describes the operation of all types of SPIM operating from a single-phase network with adjustable voltage and frequency. The mathematical model of the electric drive consists of a system of seven nonlinear differential equations with periodic coefficients, the solution of which is possible only by numerous methods.

The quality of transient processes was assessed by changes in the start time, maximum values of the electromagnetic torque and stator current. As a result, it was determined that an increase in the active resistance of the main stator winding leads to a decrease in the starting torque and starting current and an increase in the starting time.

In turn, an increase in the inductive reactance of the main stator winding leads to a decrease in the starting torque and starting current and an increase in the starting time. The active resistance of the rotor windings has a significant impact on transient processes. An increase in the active resistance of the rotor windings leads to an increase in torque and a decrease in the starting current and starting time. An increase in the resistance of the capacitor leads to a significant decrease in the starting torque and starting current of the stator and an increase in the starting time. The starting currents and electromagnetic torque and the value of the stator current and torque in nominal mode are also reduced.

Theoretical studies made it possible to create a mathematical model of the SPIM in a coordinate system fixed relative to the stator, which is a system of nonlinear differential equations with constant coefficients, to develop a calculation method and to analyse the influence of parameters on transient electromechanical processes.

Created on the basis of the theoretical premises that are set out in the article, the calculation program in Mathcad allows for a preliminary assessment of the starting characteristics of the motor drive at the design stage without much time investment. After this, it is proposed to carry out final calculations in the Ansys Maxwell program using the finite element method, which require more time for calculations.

This algorithm for carrying out design calculations was used in the development of a AIE series 0.5kW – 2.2kW in *Ukrelectromash* OJSC (Kharkiv, Ukraine).

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Ph.D. Eng. Suad Omar Aldaikh
e-mail: suad.aldaikh@bau.edu.jo

Since 2007 she has been working at Al-Balqa Applied University, MAAN college University, Department of Technical Science as instructor for several courses related with Telecommunication and Electrical Engineering.
Her research interests are related to research in the field of smart grid, renewable energy and computer numerical control machines, Antenna, Circuit.

https://orcid.org/0000-0002-0664-892X

D.Sc. Eng. Mohannad O. Rawashdeh
e-mail: mohannad.rawashdeh@bau.edu.jo

Since 2015 he has been working at Al-Balqa Applied University, Faculty of Engineering Technology, Department of Mechanical Engineering. He has been a member of the Jordan Engineers Association (JEA).
His research interests include: related to research in the field of heat engines, electrical power engineering, smart grid.

https://orcid.org/0000-0001-7241-5000

Ph.D. Eng. Lina H. Hussienat
e-mail: lina.hsainat@bau.edu.jo

Lina H. Hussienat was from Irbid, Jordan. She is a member of IEEE, received a B.Sc. degree in Electrical Power and Control Engineering from Jordan University Science and Technology (JUST), Irbid, Jordan, in 2017 and an M.Sc. degree in Electrical Power Engineering from Yarmouk University, Irbid, Jordan, in 2021 with high honors GPA. Currently, she is a lecturer in the Faculty of Engineering at Al-Balqa Applied University, Al-Salt, Jordan.
Her research interests include Power and Control Electrical Engineering, Optimization of Distribution Networks, Renewable Energy Resources, and Smart Grid.

https://orcid.org/0000-0002-8276-4906

Ph.D. Eng. Mohamed Qawaqzeh
e-mail: qawaqzeh@bau.edu.jo

He received the M.Sc. degree in Electrical engineering from the Eastern Ukrainian State University, Ukraine, in 2001, and his Ph.D. degree in Electric Machinery and Apparatus from the Donetsk National Technical University, Donetsk, Ukraine, in 2006. He is currently working as an Associate Professor in Faculty of Engineering at Al Balqa Applied University, Jordan. His current research interests include power engineering, smart grids, power system automation. He has been a member of the Jordan Engineers Association (JEA) since 2002.
His research interests include: electrical power engineering, Renewable Energy Resources, Smart Grid.

https://orcid.org/0000-0001-7027-5577

Ph.D. Eng. Oleksiy Iegorov
e-mail: diaskk67@gmail.com

Since 2017 he has been working at O.M. Beketov National University of Urban Economy in Kharkiv as head of the Department of Alternative Energy. Oleksiy Iegorov conducts lectures and practical classes in the following courses: electrical machines, electrical networks and systems, transient processes in the energy sector, metrology. He is the author of 1 textbook, 3 monographs, 130 scientific articles, and has 10 patents for inventions and computer programs.
His research interests include: electrical machines, energy transients, renewable energy sources, energy modeling.

https://orcid.org/0000-0003-2599-1624

Ph.D. Eng. Olga Iegorova
e-mail: olgyu73@gmail.com

Works at the National Technical University National Technical University Kharkiv Polytechnic Institute as an associate professor at the Department of Heat engineering and energy-efficient technologies. Olga Iegorova conducts lectures and practical classes in the following courses: heat and mass transfer, mathematical and computer modeling of thermophysical processes, power supply systems and modern electric drives. Author of more than 120 scientific articles, 3 monographs, 2 utility model patents.
Her research interests include: electrical power engineering, lighting engineering, energy-saving technologies, energy modeling.

https://orcid.org/0000-0001-8593-1557

Ph.D. Mykola Kundenko
e-mail: n.p.kundenko@ukr.net

Works at the National Technical University National Technical University Kharkiv Polytechnic Institute as the head of the department of heat engineering and energy-efficient technologies. Mykola Kundenko conducts lectures on the courses: Non-traditional and renewable energy sources, automation of thermal power processes and installations, energy metering systems, design of energy supply systems, power equipment of installations with low-potential energy sources, electrified thermal technological processes and installations. Author of more than 150 scientific articles, 3 monographs, 6 patents.
His research interests include: electrical power engineering, biomedicine, biotechnology.

https://orcid.org/0000-0002-5841-4367

Ph.D Eng. Dmytro Danylchenko
e-mail: dmytro.danylchenko@khpi.edu.ua

He works at the National Technical University Kharkiv Polytechnic Institute as an associate professor of the Department of Electrical Energy Transmission. Dmytro Danylchenko gives lectures on the courses: Fundamentals of energy security, electrical systems and networks, Systems of transmission of electrical energy, electrical distribution networks. Head of scientific areas related to reliable and efficient power supply. Author of more than 150 scientific articles, 11 monographs, 2 patents.
Field of scientific interests: electric power engineering, renewable energy sources, modeling of electric power processes.

https://orcid.org/0000-0001-7912-1849

D.Sc. Eng. Oleksandr Miroshnyk
e-mail: omiroshnyk@btu.kharkiv.ua

Graduated from Kharkiv State Technical University of Agriculture, Kharkiv, Ukraine, in 2004, and was qualified as an Electrical Engineer. He received his Ph.D. in Electric Engineering (power stations, systems and networks) from the National Technical University Kharkiv Politechnic Institute, Ukraine, in 2009 and Doctor of Technical Sciences degree in 2016.
His research interests are related to research in the field of smart grid and renewable energy.

https://orcid.org/0000-0002-6144-7573

Ph.D. Eng. Taras Shchur
e-mail: shchurtg@gmail.com

Graduated from Kharkiv State Technical University of Agriculture, Kharkiv, Ukraine, in 2004, and was qualified as an Mechanical Engineer. He received his Ph.D. in Mechanical Engineering (Machines and means of mechanization of agricultural production) from the Kharkiv State Technical University of Agriculture, in 2009.
His research interests are related to research in the field of smart grid, renewable energy and computer numerical control machines.

https://orcid.org/0000-0003-0205-032X