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SOME HYBRID MODELS OF SUBJECTIVE ANALYSIS IN THE THEORY OF ACTIVE SYSTEMS

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Abstract

The publication presents one of the ways the theoretical discussion of hybrid models of processes occurring in active systems. The new approach is based on a subjective analysis methods were compared with some conventional research methods of active systems. To compare the Kolmogorov equation models were used, as well as analogous the Boltzmann equation.
Keywords: subjective analysis, active system, preferences distribution, hybrid models, Markov process.

INTRODUCTION

We consider a hybrid theoretical schemes arising as a result of combining the principle of maximum subjective entropy with some known methods. The results obtained on the basis of hybrid schemes, reflect the influence of subjective factors.

MAXIMUM SUBJECTIVE ENTROPY

Extension of the research opportunities "subjective analysis" [1] is associated with the development of a number of hybrid methods based on a combination of subjective entropy maximum principle to known methods.

These methods include, in particular, the Euler-Lagrange variational calculus. The introduction of the modified functional allows us to develop an analogue of the classical calculus of variations and brings in the results obtained judgment.

Let $\vec{x} = (X_1, X_2, \ldots, X_m)$ – a vector of exogenous variables, and $\vec{\pi} = (\pi_{1,} \pi_2, \ldots, \pi_{N)}$ – vector indices preferences of alternatives $\sigma_i \in S_a$, which specifies the distribution of preferences in the set S_a. In the simplest case, the normalization condition $\sum_{i=1}^{N} \pi_1 = 1$. If you are using a non-additive measure, such a measure sugeno, the normalization condition has a different form.

An important case is the non-additive measure H when the normalization condition is given by:

$$\sum_{i=1}^{N} \pi_i + \lambda H_\pi = 1 \tag{1}$$

where $H_\pi = -\sum_{i=1}^{n} \pi_i ln\pi_i$ – Shannon entropy, but the built-probability relative weights of preferences π_i, λ – the exponent of non-additivity.

We introduce the functional:

$$\Phi_{\pi,\chi} = \Phi_{\pi,\chi} = \int_{t_1}^{t_2} \left[-\sum_{i=1}^{N} \pi_i \ln \pi_i \pm \beta \sum_{i=1}^{N} \pi_i F_i(\overrightarrow{X}, \dot{\overrightarrow{X}}, t) + \gamma \left(\sum_{i=1}^{n} \pi_i - 1 \right) \right] dt. \tag{2}$$

That in the classical variational calculus corresponds to the basic variational problem [2].

Denoting the integrand through F^*, we write the necessary optimality conditions (1):

$$\frac{\partial F^*}{\partial \pi_i} = 0; \ \frac{\partial F^*}{\partial \chi_s} - \frac{d}{dt}\left(\frac{\partial F^x}{\partial \dot{\chi}_s} \right) = 0. \tag{3}$$

In the integrand F^*no derivatives $\dot{\pi}_i$, so the first equation(2) is degenerate.

In the expanded form of the second equation (2) can be written as:

$$\sum_{i=1}^{n} \pi_i \left[\frac{\partial F_i}{\partial x_j} - \frac{d \ln \pi_i}{dt} \cdot \frac{\partial F_i}{\partial \dot{x}_j} - \sum_{s=1}^{N} \left(\frac{\partial^2 F_i}{\partial \dot{x}_j \partial \chi_s} \cdot \chi_s + \frac{\partial^2 F_i}{\partial \dot{x}_j \partial \dot{\chi}_s} \cdot \ddot{x}_s \right) + \frac{\partial^2 F_i}{\partial x \ \partial \dot{\chi}_j} \right] \tag{4}$$

$$\left(j \in \overline{1, N} \right)$$

where:

$$\dot{\pi}_i = \beta \pi_i \left(\dot{F}_i - \sum_{j=1}^{n} \pi_j \dot{F}_j \right), \tag{5}$$

$$\dot{F}_i = \frac{\partial F_i}{\partial t} + \sum_{s=1}^{m} \frac{\partial F_i}{\partial x_s} \cdot \dot{x}_s + \frac{\partial F_i}{\partial \dot{x}_s} \ddot{x}_s. \tag{6}$$

The canonical distribution π_i to be from the first equation (2). The first part of the integral (2) can be interpreted as the average in the segment $t_2 - t_1$, the degree of uncertainty.

As part of this approach of classical analytical mechanics: Hamilton equations, the modified Hamilton-Jacobi equation, the analogue of Noether's theorem, Cartan invariants, etc.

MARKOV CONTROL PROCESSES

The paper presents a model of some specific systems and results of mathematical modeling.

Feller equations and the Kolmogorov equations are a tool to determine the transition probabilities in the ordinary Markov process. For discontinuous Markov control processes Feller replaced equations Kolmogorov, which involve the probability density of random changes in the situation $q_j(t)$ which lead to a change in the transition probabilities $P_{jk}(t,t_1)$; $(t_1 > t)$ and the conditional probability $Q_{kj}(t)$ that the system that is in a situation of "k" at time t, as a result of the "jump" commit transition to the situation "j".

In the case where after the cause of the jump there is a set S_a of alternative strategies, and there is the problem of choice, the probability $Q_{kj}(t)$ can be calculated using the formula of total probability.

$$Q_{i,j}(t) = \sum_{m=1}^{L} P(\sigma_{m,i}(t)\big|i)Q_{i,j}(t\big|\sigma_{m,i}(t)) \tag{7}$$

where $P(\sigma|_{m,i}(t)|i)$ – the probability of choosing strategy $\sigma_m(t)$, $Q_{i,j}(t|\sigma_{m,i}(t)$ – the conditional probability of the transition $i \to J$, when selected strategy $\sigma_{m,i}(t)$. It is proposed to introduce the theory of the subjective factor, carrying out in (6) replacement $\rho(\sigma_m(t)|i) \to \pi(\sigma_m(t)|i)$, where $\pi(\sigma_m(t)|i)$ – index of the preferred alternative $\sigma_m(t)$. Which is determined from the variational principle of maximum entropy of subjective, the corresponding functional can be written as:

$$\begin{aligned}\Phi_{\pi_i} = -\sum_{m=1}^{L} \pi(\sigma_m(t)\,|\,i)\ln \pi(\sigma_m(t)\,|\,i) \pm \beta \sum_{m=1}^{L} \pi(\sigma_m(t)\,|\,i) F_m(\sigma_m(t)\,|\,i) \\ +\gamma\left(\sum_{m=1}^{L} \pi(\sigma_m(t)\,|\,i) - 1\right)\end{aligned} \tag{8}$$

(at unit normalization $\sum_{m=1}^{L} \pi_1 = 1$).

Instead of formula (6) we use the formula:

$$\begin{gathered} Q_{i,j}(t) = \sum_{m=1}^{L} \pi(\sigma_{m,i}(t)\big|t)Q_{i,j}(t\big|\sigma_{m,i}(t)) \\ \sigma_{m1}(t) \in S_{ai,}\,(m \in)\bar{1}, L) \end{gathered} \tag{9}$$

Kolmogorov equation in this case takes the form:

$$\begin{gathered} \frac{\partial P_{i,j}(t,t_{1)}}{\partial t} = -q_j(t)P_{i,j}(t,t_1) + \sum_{k=0}^{N(\infty)} q_k(t) \sum_{m=1}^{L_k} \pi(\sigma_{m,k}(t)\big|k) \\ Q_{k,j}(t\big|\sigma_{m,k}(t_1))P_{j,k}(t,t_1) \end{gathered} \tag{10}$$

$$\frac{\partial P_{i,j}(t,t_1)}{\partial t_1} = -q_j(t_1)\left[P_{i,j}(t,t_1)\right] - \sum_{k=0}^{N(\infty)}\sum_{m=1}^{L_i}\pi(\sigma_{m,i}(t)|i) \\ Q_{i,k}(t_1|\sigma_{m,i}(t_1))P_{k,j}(t,t_1) \tag{11}$$

the second term in the functional (8) can be expressed in terms of ”subjective” Bayesian risk, which also contains preferences and subjective probabilities [3].

As a special case of the model (10, 11) that ”subjective” – modified model equations of queuing theory applies, for example, to the theory of queues, ”taking into account the human factor”. The results of numerical simulation. Among the possible applications of subjective analysis highlighted problems associated with the study of certain social phenomena based mental factors: a theory of conflict, the theory of hierarchical organizational systems in the economy. Here we consider a variant of the description of social processes by means of the model equations analogous to the equations of the kinetic theory of gases (Boltzmann equation, for example), and the equation sociodynamics is: enter the distribution function, which depends on the distribution of preferences I kind of subject $\pi(\sigma_i)$, $\sigma_i \in S_a$.

And the distribution of preference rating type $|\zeta(j|u)$ (rating entity j in the eyes of the subject u), as well as their derivatives in time:

$$\dot{\pi}(\sigma_i); \dot{\xi}(j\text{I}\dot{k}); \xi(j\text{I}k) \in S_\xi$$

$$\int(\vec{\pi}, \dot{\pi}, \dot{\vec{\xi}}(\cdot|\cdot), \xi(\cdot\vec{|}\cdot), t)$$

The analogue of the Boltzmann equation:

$$\frac{\partial f}{\partial t} + \dot{\pi}\frac{df}{d\vec{\pi}} + \frac{\partial f}{\partial \dot{\pi}} \cdot \ddot{\vec{\pi}} + \frac{\partial f}{\partial \xi(\vec{\cdot|\cdot})} \cdot \dot{\vec{\xi}}(\cdot|\cdot) + \frac{\partial f}{\partial \dot{\vec{\xi}}(\cdot|\cdot)} \cdot \ddot{\vec{\xi}}(\cdot|\cdot) = S \tag{12}$$

Models of distribution $\vec{\pi}$ and $\xi(\cdot\vec{|}\cdot)$ is obtained from the corresponding maximum principle of subjective entropy. Thus the model is a hybrid.

S – ”collision term” taking into account the results of the ”information collision” subjects. In constructing the collision term has a number of significant differences from the corresponding problem for the Boltzmann equation: in particular, the absence of analogues of conservation, the need to consider not only the pair, but multiple collisions, as well as ”long-range” interaction.

The simplest model is similar to the model of the collision member Grad in the theory of gases. A model where the collision term is represented as the operator of the Fokker – Planck equation. In this case, we obtain a solution of the kinetic equation sociodynamics type ”source” similar solutions Chandrasekhara [4].

Some ways of modeling of control in a hierarchical active system on the basis of entropy paradigm we are shown in the publication [5].

CONCLUSIONS

Thus, in the paper the hybrid theoretical schemes arising as a result of combining the principle of maximum entropy subjective calculus of variations with the Euler-Lagrange equations.

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O NIEKTÓRYCH MODELACH HYBRYDOWYCH ANALIZY SUBIEKTYWNEJ W TEORII SYSTEMÓW AKTYWNYCH

Streszczenie

W publikacji przedstawiono jeden ze sposobów teoretycznych rozważań nad hybrydowymi modelami procesów zachodzących w systemach aktywnych. Nowe podejście oparte na metodach analizy subiektywnej porównano z niektórymi znanymi metodami badawczymi systemów aktywnych. Do porównania modeli wykorzystano równania Kołmogorowa, a także równania Boltzmanna.

Słowa kluczowe: analiza subiektywna, systemy aktywne, modele hybrydowe, procesy Markowa.