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Dynamics of cellular rotor of asynchronous motor with deformable stator

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Abstract

The analysis of the dynamic stability of cellular rotor in asynchronous motor with deformable stator has been determined. The values of magnetic tension and angular velocities of the rotor under which the loss of stability is observed has been determined. A model of rotor with continuous mass distribution and changeable rigidity has been applied in the analysis. In order to estimate the stability of the rotor the equations of its transverse vibration has been formulated. This equations connecting the dynamic deflection of rotor with space and time. Then the differential equations has been solved. On the basis of the mentioned equations the values of magnetic tension and angular velocities of the rotor under which the loss stability is observed, have been determined.

Keywords: rotor, dynamic stability.

1. Introduction

Among in electric machines, the squirrel-cage asynchronous motors occupy a particular space. These motors have small value of the magnetic gap. For this reason, the basic problem encountered in the phase of construction of such machines is to estimate the stability of the rotors. The problem of stability rotors is in relation to the problem of vibration. On certain values of some quantities, such as rotational speed, magnetic tension, rigidity etc., the effect of unstability can take place. The assessment of the stability is of particular importance in the case of long rotors loaded by axial force, for example rotors of motors of deep-well pumps. Such pumps works in deep waters. Problem of estimation of stability of transverse motion of rotors collaborations in non-deformable stator are presented in the works [3, 4, 5, 6]. In this paper the influence of deformability of the stator on the dynamic stability of rotor has been determined.

2. Dynamic stability of rotor

The model of rotor accepted for calculations shown in Fig. 1.

In order to simplify the considerations a vertical position of the rotor has been assumed. The basis for describing the dynamic stability of the rotor is the differential equation of the centre line of the beam. The equation can be written as:

$$S\frac{\partial^4 y}{\partial x^4} = q_x \tag{1}$$

where:

S – flexural rigidity of the section 2,
y – deflection of the rotor,
q_x – load intensity.

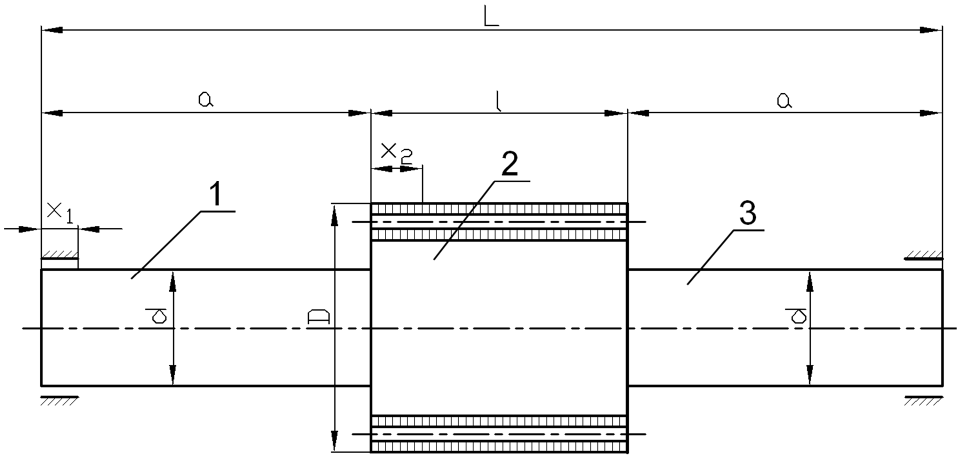

Figure 1. The model of rotor accepted for calculations; 1, 2, 3 – sections of the rotor

The load intensity q_x can be introduced in the form:

$$q_x = q_{1x} + q_{2x} \tag{2}$$

where:
q_{1x} – load intensity related to the influence of the forces of inertia,
q_{2x} – load intensity related to the influence of the magnetic tension.

Deflection of the rotor y(x) can be introduced in the form:

$$y(x) = y_1(x) + y_2(x) \tag{3}$$

where:
$y_1(x)$ – deflection of the rotor,
$y_2(x)$ – deflection of the stator.

The load intensity q_{1x} can be expressed as:

$$q_{1x} = -\mu \frac{\partial^2 y}{\partial t^2} \tag{4}$$

where:
μ – unit mass (per unit length) of the section 2 for rotor,
t – time.

The load intensity q_{2x} can be expressed as [1, 2, 3]:

$$q_{2x} = C \cdot y(x) \tag{5}$$

where:

C – coefficient of magnetic tension [1, 2, 3].

Based on equations (1) and (3) differential equations described vibrations of the rotor and of the stator has been introduced:

$$\beta_1{}^2 \frac{\partial^4 y_1}{\partial x_1{}^4} + \frac{\partial^2 y_1}{\partial t^2} - \gamma_1(y_1 + y_2) = 0 \quad (6)$$

where:

$$\beta_1{}^2 = \frac{S_1}{\mu_1},\ \gamma_1 = \frac{C}{\mu_1} \quad (7)$$

S_1 – flexural rigidity of the rotor (of the section 2),

μ_1 – unit mass (per unit length) of the section 2 for rotor.

$$\beta_2{}^2 \frac{\partial^4 y_2}{\partial x_2{}^4} + \frac{\partial^2 y_2}{\partial t^2} - \gamma_2(y_1 + y_2) = 0 \quad (8)$$

where:

$$\beta_2{}^2 = \frac{S_2}{\mu_2},\ \gamma_2 = \frac{C}{\mu_2} \quad (9)$$

S_2 – flexural rigidity of the stator (of the section 2),

μ_2 – unit mass (per unit length) of the section 2 for stator,

x – spatial variable.

The solutions of the equation (6) and (8) can be presented in the form an infinite series:

$$y_1(x,t) = \sum_{n=1}^{\infty} X_{n1}(x) T_{n1}(t),\ y_2(x,t) = \sum_{n=1}^{\infty} X_{n2}(x) T_{n2}(t) \quad (10)$$

After a separation of variables the following equation has been obtained:

$$\frac{d^4 T_{n1}}{dt^4} + a_n \frac{d^2 T_{n1}}{dt^2} + b_n T_{n1} = 0 \quad (11)$$

where:

$$a_n = \omega_{n1}{}^2 + \omega_{n2}{}^2 = \omega_{n1}{}^2(0) + \omega_{n1}{}^2(0) - (\gamma_1 + \gamma_2) \quad (12)$$

$$b_n = \omega_{n1}{}^2 \omega_{n2}{}^2 - \gamma_1 \gamma_2 = \omega_{n1}{}^2(0)\omega_{n2}{}^2(0) - \omega_{n1}{}^2(0)\gamma_2 - \omega_{n2}{}^2(0)\gamma_1 \quad (13)$$

ω_{n1} denotes the n-order frequency of free vibrations of rotor (with non-deformable stator),

ω_{n2} denotes the n-order frequency of free vibrations of stator (with non-deformable rotor),

$\omega_{n1}(0)$ and $\omega_{n2}(0)$ denotes n-order frequency of free vibrations of rotor and stator in which $\gamma_1 = \gamma_2 = 0$.

Based on the equation (11) the characteristic equation has been obtained:

$$\lambda_n{}^4 + a_n\lambda_n{}^2 + b_n = 0 \tag{14}$$

The solutions of the above equation can be presented in the following form:

$$\lambda_{1,2} = \pm\sqrt{\frac{-a_n-\sqrt{a_n{}^2-4b_n}}{2}} \tag{15}$$

$$\lambda_{3,4} = \pm\sqrt{\frac{-a_n+\sqrt{a_n{}^2-4b_n}}{2}} \tag{16}$$

Based on the equation (11) the following condition of instability has been obtained:

$$\omega_{n1}{}^2(0)\omega_{n2}{}^2(0) - \omega_{n1}{}^2(0)\gamma_2 - \omega_{n2}{}^2(0)\gamma_1 \le 0 \tag{17}$$

The above condition has been obtained:

$$C \ge \frac{\mu_1\mu_2\omega_{n1}{}^2(0)}{p\mu_1+\mu_2} \quad \text{or} \quad C \ge \frac{C_\infty}{pr+1} \tag{18}$$

where:

$$p = \frac{\omega_{n1}{}^2(0)}{\omega_{n2}{}^2(0)} \tag{19}$$

$$C_\infty = \mu_1\omega_{n1}{}^2(0) \tag{20}$$

$$r = \frac{\mu_1}{\mu_2} \tag{21}$$

C_∞ – coefficient of magnetic tension of the rotor with non-deformable stator.

Based on inequality (18) the values of magnetic tension under which the loss of stability is observed has been determined:

$$C_{kr} = \frac{C_\infty}{pr+1} \tag{22}$$

C_{kr} – coefficient of magnetic tension of rotor with deformable stator.

The following angular velocities of the rotor in which the loss of stability is observed has been determined:

$$\omega = \sqrt{\frac{1}{2}\left(a_n + \sqrt{a_n{}^2 - 4b_n}\right)} \quad \text{or} \quad \omega = \sqrt{\frac{1}{2}\left(a_n - \sqrt{a_n{}^2 - 4b_n}\right)} \tag{23}$$

3. Example of calculations

This chapter presents calculations of a rotor for following data: L = 0,7 m, l = 0,375 m, d = 0,05 m, D = 0,08 m. Dimensions of the stator:

$$D_z = 0{,}11\ m, \quad D_w = 0{,}1\ m$$

has been accepted. D_z and D_w the outside and inside diameters has been signified. The unit mass of the rotor:

$$\mu_1 = 39{,}2\ \frac{kg}{m}$$

and unit mass of the stator:

$$\mu_2 = 16{,}72\ \frac{kg}{m}$$

has been accepted.

$$\omega_{n1}(0) = 873{,}33\ s^{-1},\ \omega_{n2}(0) = 3370{,}8\ s^{-1}$$

$$C_\infty = 29{,}9\ MPa, \qquad C_{kr} = 25{,}83\ MPa.$$

Then angular velocities of the rotor in which the loss of stability occurs has been obtained.

In which $C = 19{,}62\ MPa, \omega = 451{,}7\ s^{-1}$ (with deformable stator).
In which $C = 19{,}62\ MPa, \omega = 512{,}12\ s^{-1}$ (with non-deformable stator).
In which $C = 24{,}525\ MPa, \omega = 370{,}37\ s^{-1}$ (with non-deformable stator).

4. Conclusions

1. The deformability of the stator of asynchronous motor decrease of the values of the magnetic tension at which the loss of stability occurs.
2. The deformability of the stator of asynchronous motor decrease of the values angular velocity at which the loss of stability occurs. The first velocity is less than natural frequency of free vibrations rotors in case of non-deformable stator. The second velocity is less than natural frequency of free vibrations stator in case of non-deformable rotor.

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