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Quadrature Mapping, Saleh's Representation, and Memory Models

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***Abstract*—In the literature, Saleh's description of the AM/AM and AM/PM conversions occurring in communication power amplifiers (PAs) is classified as a representation without memory. We show here that this view must be revised. The need for such revision follows from the fact that the Saleh's representation is based on the quadrature mapping which, as we show here, can be expanded in a Volterra series different from an usual Taylor series. That is the resulting Volterra series possesses the nonlinear impulse responses in form of sums of ordinary functions and multidimensional Dirac impulses multiplied by coefficients being real numbers. This property can be also expressed, equivalently, by saying that the nonlinear transfer functions associated with the aforementioned Volterra series are complex-valued functions. In conclusion, the above means that the Saleh's representation incorporates memory effects.**

***Keywords*—AM/AM and AM/PM conversions, quadrature model, Saleh's representation, power amplifiers, Volterra series, memory effects.**

I. Introduction

SALEH'S description of the AM/AM and AM/PM conversions [1] occurring in communication power amplifiers (PAs) is viewed in the literature, see, for example, a tutorial [2], as a representation which does not incorporate memory effects. The objective of this paper is to show that this view must be revised because the Saleh's representation [1] is based on the quadrature mapping which, as we show here, can be expanded in a Volterra series different from an usual Taylor series. More precisely, we show here that the aforementioned Volterra series possesses the nonlinear impulse responses in form of sums of ordinary functions and multidimensional Dirac impulses multiplied by coefficients being real numbers. The latter means also, equivalently, that the nonlinear transfer functions associated with the above Volterra series are complex-valued functions.

In the above context, note that there is a related theory, for more details see, for example [3-6], which says that the memoryless systems (devices) described by a Volterra series are only such ones which possess the nonlinear impulse responses in form of pure multidimensional Dirac impulses multiplied by coefficients being real numbers. In the multidimensional frequency domains, this means equivalently that the corresponding nonlinear transfer functions of a given system (device) are real-valued constants.

In view of the above, the properties of the aforementioned Saleh's equivalent quadrature model show that it is not a memoryless one. In other words, it is a model with memory.

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II. General Model of AM/AM and AM/PM Conversions and Saleh's Representation

To begin, let us first recall the basics of modelling of the AM/AM and AM/PM characteristics of PAs (or other devices and systems) as it is done in the communications literature; as, for example, in [7]. Then, one applies the following bandpass signal at the PA input

$$x(t) = r(t)\cos\left(\omega_c t + \psi(t)\right) , \tag{1}$$

where $\omega_c = 2\pi f_c$ with f_c meaning the carrier frequency. Moreover, t in (1) denotes a time variable and $j = \sqrt{-1}$. Furthermore, the bandpass input signal $x(t)$ contains a slowly varying real-valued baseband signal $r(t)$ which modulates the carrier amplitude. And the carrier phase changes with the time according to a function $\psi(t)$. The latter function, similarly as $r(t)$, represents also a slowly varying baseband signal.

After [7], the PA output to (1) can be then expressed as

$$y(t) = A(r(t))\cos\left(\omega_c t + \psi(t) + \Phi(r(t))\right). \tag{2}$$

Note that $A(r(t))$ and $\psi(t) + \Phi(r(t))$ in (2) denote the carrier amplitude and its phase, respectively, at the amplifier output. The function $A(r(t))$ is assumed to be a nonlinear function of $r(t)$. That is $A(r(t)) \neq a \cdot r(t)$ holds, where a stands for a real-valued constant. Moreover, it is assumed that an additional phase component $\Phi(r(t))$ in (2) does not equal zero and depends upon the slowly varying baseband signal $r(t)$. This means that a kind of the amplitude modulation expressed by the nonlinear characteristic $A(r(t))$ as well as the phase modulation expressed by another function $\Phi(r(t))$ occur, and both are caused by the signal $r(t)$. Therefore, we refer to $A(r(t))$ as the AM/AM characteristic and to $\Phi(r(t))$ as the AM/PM characteristic.

The operation described by (2), which is performed by a PA on the signal $x(t)$ having a specific form given by (1) - together with the above accompanying explanations - is called here a general model of the AM/AM and AM/PM conversions. Note that all the other models published, as, for example, the Saleh's model [1], are its particularizations. Further, justification of its correctness was given, among others, in [7]. By the way, note that such a justification constitute also the derivations presented in [8] which were carried out with the use of the Volterra series method [9], [10].

A particular version of the above general model of the AM/AM and AM/PM conversions is the representation developed by Saleh in [1]. In this model, the functions $A(r(t))$ and $\Phi(r(t))$ are approximated in the following way

$$A(r(t)) = \frac{a_1 r(t)}{1+a_2 (r(t))^2}, \quad \Phi(r(t)) = \frac{b_1 (r(t))^2}{1+b_2 (r(t))^2}, \tag{3}$$

where the coefficients a_1 and a_2 as well as b_1 and b_2 are assumed to be real values and need adjustment to the measured data for a given amplifier.

III. Equivalent Quadrature Models

The operation which is used in the theory of communications for getting the so-called in-phase and quadrature components [7] is called here the quadrature mapping. Using it, one can represent the two models mentioned in the previous section in equivalent forms. Such equivalent forms have been already derived in [1], [11-13], and some other works. Here, we recall briefly these results. To do this, note that (2) can be rewritten in an equivalent form as

$$\begin{aligned} y(t) &= A(r(t))\cos(\Phi(r(t)))\cos(\omega_c t+\psi(t)) - \\ &- A(r(t))\sin(\Phi(r(t)))\sin(\omega_c t+\psi(t)) \ . \end{aligned} \tag{4}$$

From (4), it follows immediately that the in-phase $p(t)$ and quadrature $q(t)$ components of the amplifier output signal $y(t)$ are given by [1]

$$p(t) = P(r(t))\cos(\omega_c t+\psi(t)) \tag{5}$$

and

$$q(t) = -Q(r(t))\sin(\omega_c t+\psi(t)) \ , \tag{6}$$

respectively. Further, in the equivalent Saleh's version of this equivalent general model, the functions $P(r(t))$ and $Q(r(t))$ in (5) and (6), accordingly, follow from (4) and (3). That is we get then

$$\begin{aligned} P(r(t)) &= A(r(t))\cos(\Phi(r(t))) = \\ &= \frac{a_1 r(t)}{1+a_2 (r(t))^2} \cdot \cos\left(\frac{b_1 (r(t))^2}{1+b_2 (r(t))^2}\right) \end{aligned} \tag{7}$$

and

$$\begin{aligned} Q(r(t)) &= A(r(t))\sin(\Phi(r(t))) = \\ &= \frac{a_1 r(t)}{1+a_2 (r(t))^2} \cdot \sin\left(\frac{b_1 (r(t))^2}{1+b_2 (r(t))^2}\right) \ . \end{aligned} \tag{8}$$

Having these results, let us now formulate the following theorem.

Theorem 1. The necessary and sufficient condition for occurrence of the nonzero values of AM/PM conversion in the general model of the AM/AM and AM/PM conversions and in its version developed by Saleh [1] is the existence of the nonzero values of the quadrature component function $Q(r(t))$. (Note that we can express this shortly in such a way: the existence of "the quadrature path" is needed for the occurrence of the AM/PM conversion.)

Proof: Having in mind the explanations and results presented in this and in the previous section, the proof of this theorem is obvious because when $\Phi(r(t)) \equiv 0$, then $Q(r(t)) \equiv 0$ holds, according to (8), too. Further, if $Q(r(t))$ is not a function equal identically to zero, then also $\Phi(r(t))$ is not identically zero function (once again, according to (8)). □

Using the equivalent quadrature representations of the general model of the AM/AM and AM/PM conversions and of its Saleh's version, we will show in the next section that these representations constitute models incorporating memory effects. Moreover, we will show that these memory effects are closely related with the occurrence of the nonzero values of the AM/PM conversion.

IV. Quadrature Model is a Model with Memory

In what follows, we will use a version of the quadrature model which was utilized by Benedetto et al. in [14]. This model is convenient for us because its form allows, as we will see later, easy derivation of its Volterra series based nonlinear impulse responses, or equivalently, calculation of its nonlinear transfer functions.

Benedetto et al. [14] used the aforementioned model in their description of a nonlinear device (as, for example, a power amplifier working in its nonlinear region of operation) exhibiting the AM/AM and AM/PM conversions. Their model is shown in Fig. 1.

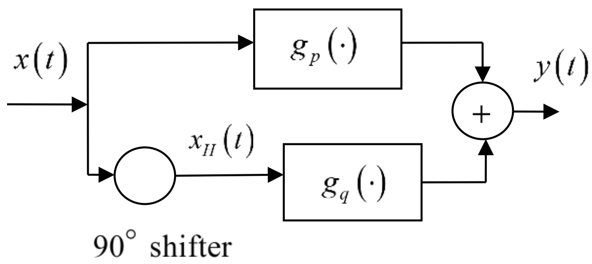

Fig. 1. The quadrature model of a nonlinear device exhibiting the AM/AM and AM/PM conversions.

The quadrature model of Fig. 1 consists of two parallel paths (branches). The upper one represents the model in-phase component and corresponds to relation (5). But, the lower path is its quadrature component and corresponds to relation (6), respectively. The first branch is described by a real-valued (memoryless) nonlinearity $g_p(\cdot)$, which can be represented, for example, by a power series with real-valued coefficients. Similarly, the another nonlinearity $g_q(\cdot)$ in the lower path is assumed to be real-valued (memoryless), too. Therefore, it can be also modelled by a power series with real-valued coefficients. The difference between the upper and lower paths in Fig. 1 lies in the fact that the latter nonlinearity is preceded by a 90 degree shifter. We will see that this makes a clear qualitative difference between the branches in Fig. 1.

Consider now the operation of shifting the signal $x(t)$ by 90 degrees in the lower branch of Fig. 1 to get another signal denoted as $x_H(t)$. This operation can be described in the time domain by the following convolution operation [15]

$$x_H(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau \ . \tag{9}$$

Note that the relation (9) describes the operation performed by the so-called Hilbert transformer, in the time domain. For more details regarding this topic, see, for example, [15].

Taking into account (9) and looking at Fig. 1, we see that the output signal $y(t)$ at the nonlinear device scheme of this figure is related with the input signal $x(t)$ by the relation

$$\begin{aligned} y(t) &= g_p(x(t)) + g_q(x_H(t)) = \\ &= g_p(x(t)) + g_q\left(\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\, d\tau\right) \ . \end{aligned} \tag{10}$$

In the next step, we expand the functions $g_p(\cdot)$ and $g_q(\cdot)$ in (10) in power series. That is we assume the following

$$g_p(x(t)) = c_1 x(t) + c_2 (x(t))^2 + c_3 (x(t))^3 + .. \tag{11}$$

and

$$\begin{aligned} g_q(x_H(t)) &= d_1 x_H(t) + d_2 (x_H(t))^2 + d_3 (x_H(t))^3 + .. = \\ &= \frac{d_1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\, d\tau + \frac{d_2}{\pi^2}\left(\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\, d\tau\right)^2 + \frac{d_3}{\pi^3}\left(\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\, d\tau\right)^3 + .. . \end{aligned} \tag{12}$$

where the coefficients $c_1,\ c_2,\ c_3,...$ in (11) and $d_1,\ d_2,\ d_3,...$ in (12) have the values being real numbers.

Note now that the successive components in (11) can be expressed in form of the multidimensional convolution integrals with the use of Dirac impulses $\delta(t)$ as

$$c_1 x(t) = \int_{-\infty}^{\infty} c_1\delta(\tau) x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} c_1\delta(t-\tau) x(\tau)\, d\tau \ , \tag{13a}$$

$$\begin{aligned} c_2(x(t))^2 &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_2\delta(\tau_1)\delta(\tau_2)\, x(t-\tau_1)\, x(t-\tau_2)\cdot \\ &\cdot d\tau_1 d\tau_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_2\delta(t-\tau_1)\delta(t-\tau_2)\, x(\tau_1)\, x(\tau_2)\cdot \\ &\cdot d\tau_1 d\tau_2 \ \ , \end{aligned} \tag{13b}$$

$$\begin{aligned} c_3(x(t))^3 &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_3\delta(\tau_1)\delta(\tau_2)\,\delta(\tau_3)\, x(t-\tau_1)\cdot \\ &\cdot x(t-\tau_2)\, x(t-\tau_3)\, d\tau_1 d\tau_2 d\tau_3 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_3\delta(t-\tau_1)\cdot \\ &\cdot\delta(t-\tau_2)\,\delta(t-\tau_3)\, x(\tau_1)\, x(\tau_2)\, x(\tau_3)\, d\tau_1 d\tau_2 d\tau_3 \ , \end{aligned} \tag{13c}$$

and so on. So, using (11) together (13) and (12) in (10), we can rewrite the latter in such a way

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty}\left(c_1\delta(t-\tau) + \frac{d_1}{\pi}\frac{1}{t-\tau}\right)x(\tau)\, d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(c_2\cdot \\ &\cdot\delta(t-\tau_1)\,\delta(t-\tau_2) + \frac{d_2}{\pi^2}\frac{1}{t-\tau_1}\frac{1}{t-\tau_2}\Big)x(\tau_1)\, x(\tau_2)\cdot \\ &\cdot d\tau_1 d\tau_2 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(c_3\delta(t-\tau_1)\,\delta(t-\tau_2)\,\delta(t-\tau_3) + \\ &+\frac{d_3}{\pi^3}\frac{1}{t-\tau_1}\frac{1}{t-\tau_2}\frac{1}{t-\tau_3}\Big)x(\tau_1)\, x(\tau_2) \\ &\cdot x(\tau_3)\, d\tau_1 d\tau_2 d\tau_3 + ... \ . \end{aligned} \tag{14}$$

The Volterra series used as description of a relation between the output and input signals of a mildly nonlinear device (system) is formulated as [9], [10]

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h^{(1)}(\tau) x(t-\tau) d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2)\cdot \\ &\cdot x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(3)}(\tau_1,\tau_2,\tau_3)\cdot \\ &\cdot x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + ... \ , \end{aligned} \tag{15}$$

where $h^{(1)}(\tau)$, $h^{(2)}(\tau_1,\tau_2)$, $h^{(3)}(\tau_1,\tau_2,\tau_3)$, and so on, are, respectively, the first order (linear), second order, third order, and so on, nonlinear impulse responses (Volterra kernels) of the device considered. Furthermore, it is easy to show that the Volterra series given by (15) can be also written in the following form

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h^{(1)}(t-\tau) x(\tau) d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(t-\tau_1, t-\tau_2)\cdot \\ &\cdot x(\tau_1)x(\tau_2)d\tau_1 d\tau_2 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(3)}(t-\tau_1, t-\tau_2, t-\tau_3)\cdot \\ &\cdot x(\tau_1)x(\tau_2)x(\tau_3)x(\tau_3)d\tau_1 d\tau_2 d\tau_3 + ... \ . \end{aligned} \tag{16}$$

Comparing now the components in the series (14) with the corresponding ones in (16) reveals the nonlinear impulse responses of the device modelled in Fig. 1. They are given by

$$h^{(1)}(\tau) = c_1\delta(\tau) + \frac{d_1}{\pi}\frac{1}{\tau} \ \ , \tag{17a}$$

$$h^{(2)}(\tau_1,\tau_2)=c_2\delta(\tau_1)\delta(\tau_2)+\frac{d_2}{\pi^2}\frac{1}{\tau_1}\frac{1}{\tau_2} \quad , \tag{17b}$$

$$h^{(3)}(\tau_1,\tau_2,\tau_2)=c_3\delta(\tau_1)\delta(\tau_2)\delta(\tau_3)+\frac{d_3}{\pi^3}\frac{1}{\tau_1}\frac{1}{\tau_2}\frac{1}{\tau_3} \quad , \tag{17c}$$

and so on. Note also that these nonlinear impulse responses can be transferred into the multifrequency domains using the following relation [9]

$$\begin{aligned} H^{(n)}(f_1,...,f_n) = \underbrace{\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}}_{n \text{ times}} h^{(n)}(\tau_1,...,,\tau_n)\exp(-j2\pi f_1\tau_1)\cdot \\ \cdots\exp(-j2\pi f_n\tau_n)\,d\tau_1\cdots d\tau_n \quad , \end{aligned} \tag{18}$$

where $n=1,2,3,...$, $H^{(n)}(f_1,...,f_n)$ means the n-dimensional Fourier transform of the function (distribution) $h^{(n)}(\tau_1,...,,\tau_n)$, and $f_1 = f,...,f_n$ are the frequencies in the n-dimensional frequency space. We mention also that $H^{(n)}(f_1,...,f_n)$ is called the system (device) nonlinear transfer function of the n-th order [9]. So, applying (18) to relations (17), we get

$$H^{(1)}(f)=c_1+d_1H_T(f) \ , \tag{19a}$$

$$H^{(2)}(f_1,f_2)=c_2+d_2H_T(f_1)H_T(f_2) \ , \tag{19b}$$

$$H^{(3)}(f_1,f_2,f_3)=c_3+d_3H_T(f_1)H_T(f_2)H_T(f_3) \ , \tag{19c}$$

$$\begin{aligned} H^{(4)}(f_1,f_2,f_3,f_4)=c_4+d_4H_T(f_1)\cdot \\ \cdot H_T(f_2)H_T(f_3)H_T(f_4) \ , \end{aligned} \tag{19d}$$

$$\begin{aligned} H^{(5)}(f_1,f_2,f_3,f_4,f_5)=c_5+d_5H_T(f_1)\cdot \\ \cdot H_T(f_2)H_T(f_3)H_T(f_4)H_T(f_5) \ , \end{aligned} \tag{19e}$$

and so on, where $H_T(f)$ is the transfer function of the Hilbert transformer. It is given by [15]

$$\begin{aligned} &H_T(f)=-j\,\mathrm{sgn}(f), \\ &\mathrm{sgn}(f)=\{1,\ f>0;\ 0,\ f=0;\ -1,\ f<0 \end{aligned} \quad . \tag{20}$$

Let us now substitute (20) in (19a). As a result, we get $H^{(1)}(f)=\{c_1-jd_1 \text{ for } f>0;\ c_1 \text{ for } f=0;\ c_1-jd_1 \text{ for } f<0$, which obviously is not a real-valued function. And observe that the same regards also $H^{(2)}(f_1,f_2)$, $H^{(3)}(f_1,f_2,f_3)$, and the next ones. All of them are not real-valued functions.

It is known [5], [6] that the mildly nonlinear systems or devices described by the Volterra series do not possess memory if their nonlinear impulse responses are pure multidimentional Dirac impulses $\delta(\tau)$, $\delta(\tau_1)\delta(\tau_2)$, $\delta(\tau_1)\delta(\tau_2)\delta(\tau_3)$, and so on, multiplied by some real numbers. Equivalently, in the multifrequency domains, the mildly nonlinear systems or devices described by the Volterra series exhibit memory effects only when their nonlinear transfer functions are complex-valued [5], [6].

Taking the above into account and looking at the relations (17) and (19), we conclude finally that these nonlinear impulse responses and nonlinear transfer functions describe a nonlinear device with memory. That is, in other words, the quadrature model of Fig. 1 is a model with memory.

V. AM/AM and AM/PM Conversions of PA via Its Nonlinear Transfer Functions

In [8] and [16], formulas for calculation of the AM/AM and AM/PM conversions of a PA with the use of its nonlinear transfer functions were derived. These formulas have the following form

$$A_V(r(t))=2\left|G(r(t),H^{(n)}(\chi_n^+(\pm f_c)),f_c,n,N)\right| \tag{21a}$$

and

$$\Phi_V(r(t))=\varphi_G\left(r(t),H^{(n)}(\chi_n^+(\pm f_c)),f_c,n,N\right), \tag{21b}$$

where the function $G(r(t),H^{(n)}(\chi_n^+(\pm f_c)),f_c,n,N)$ is given by

$$\begin{aligned} G(r(t),H^{(n)}(\chi_n^+(\pm f_c)),f_c,n,N) &= \sum_{n=1,\text{ odd}}^{N}\left(\frac{r(t)}{2}\right)^n\cdot \\ \cdot C(n,(n-1)/2)H^{(n)}(\chi_n^+(\pm f_c)) &= \left[\sum_{n=1,\text{ odd}}^{N}\left(\frac{r(t)}{2}\right)^n\cdot\right. \\ &\left.\cdot\, C(n,(n+1)/2)H^{(n)}(\chi_n^-(\pm f_c))\right]^* = \\ &= G^*(r(t),H^{(n)}(\chi_n^-(\pm f_c)),f_c,n,N) \quad . \end{aligned} \tag{21c}$$

In (21a) and (21b), $A_V(r(t))$ and $\Phi_V(r(t))$ mean the corresponding functions defined in (2) and calculated here with the use of the Volterra series [8], [16]. These functions are the magnitude and phase, as expressed by (21a) and (21b), respectively, of the function $G(r(t),H^{(n)}(\chi_n^+(\pm f_c)),f_c,n,N)$ given by (21c). The small letter n in (21a), (21b), and (21c) denotes a summation index in the latter, which is odd positive integer and changes from 1 to N. The number N means the order of approximation applied in (21c) and is equal to the highest odd order of the nonlinear transfer function of the PA used in this expression. Further, $\chi_n^+(\pm f_c)$ and $\chi_n^-(\pm f_c)$ in (21a), (21b), and (21c) denote such frequency sets $\{f_1,..,f_n\}$ whose elements $f_i,\ i=1,2,...,n,$ can take on only the values $+f_c$ or $-f_c$, and whose sums are equal to $+f_c$ or $-f_c$, respectively. Obviously, the definition of the coefficients

$C(n,(n-1)/2)$ and $C(n,(n+1)/2)$ occurring in (21c) is the following

$$C(n,m) = n!/(m!(n-m)!) \quad , \tag{22}$$

where n and m mean any nonnegative integers, and $n \geq m$ holds. And finally, the notation $(\cdot)^*$ in (21c) is used to denote the complex conjugate value of a given complex number.

Restricting ourselves to consideration of only first five nonlinear transfer functions of a PA in (21c) that is assuming the order of approximation $N = 5$ and substituting $H^{(i)}(\cdot),\ i = 1,3,5,$ given by (19a), (19c), and (19e), respectively, into (21c), we get

$$\begin{aligned} &G(r(t), H^{(n)}(\chi_n^+(\pm f_c)), f_c, n, N) \cong c_1 r(t) + \\ &+(3/4)c_3(r(t))^3 + (5/8)c_5(r(t))^5 + j\big(-d_1 r(t) \\ &+(3/4)d_3(r(t))^3 - (5/8)d_5(r(t))^5\big) \ . \end{aligned} \tag{23}$$

Observe then that by using this result in (21a), we arrive at

$$A_V(r(t)) = 2\sqrt{a^2 + b^2} \ , \tag{24a}$$

where

$$a \cong c_1 r(t) + (3/4)c_3(r(t))^3 + (5/8)c_5(r(t))^5 \tag{24b}$$

and

$$b \cong -d_1 r(t) + (3/4)d_3(r(t))^3 - (5/8)d_5(r(t))^5 . \tag{24c}$$

And further, applying also (23) in (21b), we obtain

$$\Phi_V(r(t)) \cong arctg\left(\frac{-d_1 + (3/4)d_3(r(t))^2 - (5/8)d_5(r(t))^4}{c_1 + (3/4)c_3(r(t))^2 + (5/8)c_5(r(t))^4}\right). \tag{25}$$

Note now that having (24a) and (25) a couple of very important conclusions can be drawn. The result given by (24a) with the coefficient b expressed by (24c) shows clearly the influence of the phase shifting occurring in a PA and of its nonlinear behaviour (parameters $d_i,\ i = 1,3,5$) on the values of the AM/AM conversion. In other words, the AM/AM conversion of the PA does not depend solely upon the parameters of the upper path of the model shown in Fig. 1. It is worth noting that this fact is not evident from the Saleh's formula on the left-hand side of (3).

Similarly, it follows from (25) that the AM/PM of the PA apart from the dependence upon the parameters $d_i,\ i = 1,3,5,$ associated with the lower path of the model shown in Fig. 1 depends also upon the parameters $c_i,\ i = 1,3,5,$ of the upper path of the above model. And once again, this fact is not evident from the Saleh's formula on the right-hand side of (3).

Further, note that it follows from (25) that if the coefficients $d_i = 0,\ i = 1,3,5$, then $\Phi_V(r(t)) = 0$. An vice versa, if $\Phi_V(r(t)) \neq 0$, then at least some of the coefficients d_i in (21) must differ from zero. This means of course that at least some of the nonlinear transfer functions of the PA given by (19a), (19c), and (19e) (with taking into account (20)) are complex numbers. On the other hand, we know that such nonlinear transfer functions describe systems (devices) possessing memory [3], [9], [10]. Therefore, the conclusion is the following: the nonzero values of the AM/PM conversion ($\Phi_V(r(t)) \neq 0$) are inherently linked to the memory properties of a system (device) considered. In view of this, the model of the AM/AM and AM/PM conversions derived here and given by (24) and (25) is a model with memory.

Obviously, the Saleh's model given by (3) and our model associated with the expressions (24) and (25) are conceptually identical (by basic equations (1) and (2)). So, taking into account this and the conclusion formulated above, we see that the Saleh's model cannot be considered as a memoryless as considered, for example, in [2].

It is also worth noting the fact that the AM/AM and AM/PM conversions expressed by (24) and (25) do not depend upon the carrier frequency f_c.

VI. Concluding Remark

The results and conclusions presented in the previous section show that our analytical model of the AM/AM and AM/PM conversions, which was developed in this paper, seems to be more powerful than the Saleh's measurement-based one [1].

Obviously, after getting this analytical model and carrying out its theoretical analysis, further practical investigations should follow. For example, such ones which would lead to obtaining the values of the coefficients $c_i,\ i = 1,3,5,...,$ and $d_i,\ i = 1,3,5,...,$ for concrete PAs. Also, it would be interesting to investigate differences for these PAs in the functions modelling their behaviour according to (3) and according to the expressions (24) and (25) for different values of r. These will be tasks for further investigations.

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