Metrol. Meas. Syst., Vol. XX (2013), No. 4, pp. 667-676

PAN POLSKA AKADEMIA NAUK

METROLOGY AND MEASUREMENT SYSTEMS

Index 330930, ISSN 0860-8229

www.metrology.pg.gda.pl

M&MS

A HIGH PRESSURE RESONATOR TRANSDUCER WITH A PROGRAMMED CORRECTION OF STATIC CHARACTERISTICS

Krzysztof Tomczuk

Institute of Aviation, Processing and Power Machines Engineering, Wroclaw University of Technology, Wybrzeze Wyspianskiego 27, 50-370 Wroclaw, Poland (✉ krzysztof.tomczuk@pwr.wroc.pl, +4871 320 23 48)

Abstract

A high pressure resonator transducer (0 to 100 MPa) devised by the author has been described. The elastic element of the converter consists of a cylinder with an offset arranged axis hole. Quartz resonators were used for the measurement of deformations of the pipe. Based upon the results of the transducer testing, a new algorithmic method for the minimizalizsation of the temperature error, that eliminates the need for a temperature gauge has been worked out.

Keywords: pressure transducer, quartz resonator, algorithmic correction.

1. Introduction

Presently the measurement of high pressure in advanced processes is difficult, especially in cases where enhanced accuracy is required [1, 2, 3]. In tensometric high pressure transducers currently used for this purpose the elastic element consists of a thick-walled measuring cylinder. Its strains, which are proportional to the inner pressure, are measured with the aid of strain gauges glued onto the cylinder surface.

The distribution of stresses in thick-walled cylinders is known to be non-uniform. The maximum stress (strain) is present at the inner surface of the cylinder, and the minimum stress at the outer surface, where strain gauges are glued. Fig. 1 shows the graphical relation between the ratio of the tangential stress on the outer surface vs. the equivalent stress on the inner surface and the ratio of the external (r_1) vs. internal (r_2) cylinder radii.

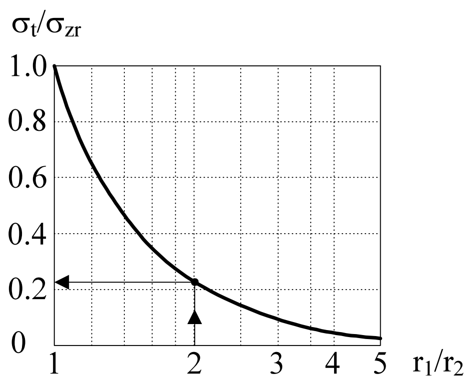


Fig. 1. Relation between tangential stress to equivalent stress ratio σ_t/σ_{zr} and external to internal cylinder radii ratio r_1/r_2.

Article history: received on Jan. 23, 2013; accepted on Sep. 15, 2013; available online on Dec. 10, 2013; DOI: 10.2478/mms-2013-0057.

With the increasing thickness of a cylinder (higher values of r_1/r_2), the tangential stress on its outer surface decreases as compared to the equivalent stress present on the inner surface. If, for example, the thickness of the wall is equal to the inner radius of the cylinder ($r_1/r_2 = 2$) then the tangential stress on its surface is equal to one quarter of the equivalent stress present on the inner surface.
Based on the above analysis a conclusion can be drawn that thick-walled measuring cylinders in the standard form are relatively poor in terms of sensitivity. In view of this fact, research projects of new designs of cylinder-based pressure transducers aimed to enhance their sensitivity and accuracy have recently been carried out [4–7].

2. The design of a high pressure transducer

The main aim of the present paper is to report the design and developmental details and measurement results of a high pressure resonator transducer (Fig. 2) based upon a PL 178676 patent [8] with a new method of temperature compensation.

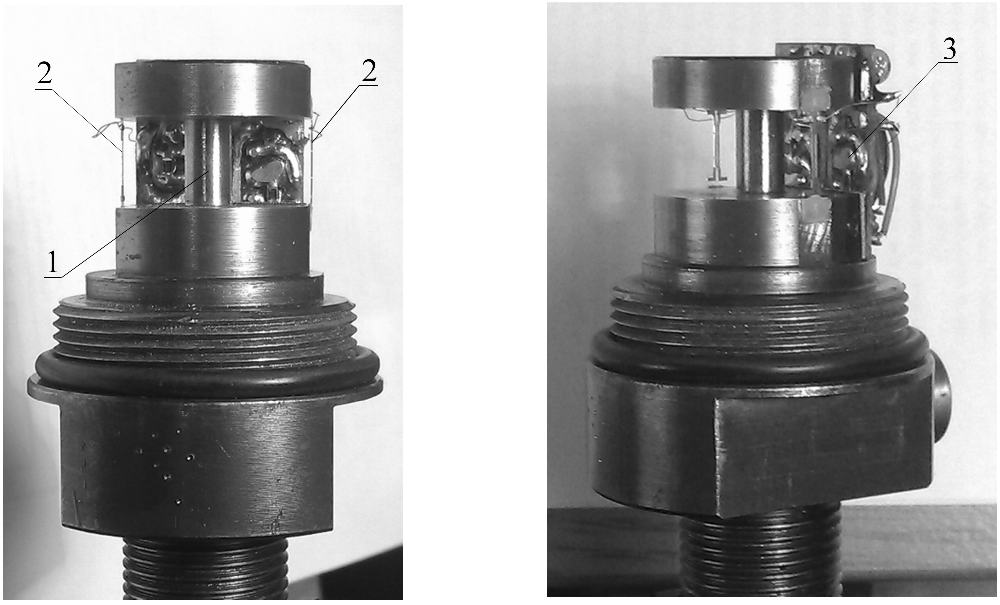


Fig. 2. View of the transducer without casing.
1 – measuring pipe, 2 – quartz resonators, 3 – electronic circuits.

Fig. 3 shows the design of the developed transducer (0 to 100 MPa range). The main subassembly of the transducer is a steel measuring cylinder (1) with a non-axial cylinder hole (2). The upper end of the cylinder hole (2) is blind, whilst the lower end is directly connected to the source of the measured pressure *p*. Due to the offset arrangement of the test hole, the wall thickness of the measuring cylinder is non-uniform. On the side where the thickness is minimal, the body was deep milled forming two flat surfaces (3). On the opposite side, where the cylinder thickness is maximal, the milling depth is smaller forming two flat surfaces (4). Two resonators (5) and (6) are glued onto surfaces (3) and (4) [9, 10]. Both resonators are connected to two independent generators (7), the electrical frequencies of which are controlled by the frequency of mechanical vibrations of the respective resonators.
The above-mentioned design features underline an important property of the transducer, namely the measuring cylinder 1 gradually bends when the measured internal pressure *p* is increased. The direction of this deformation is defined by the orientation of the torque M (Fig. 3). As a result, resonator 5 is axially stretched and resonator 6 is axially compressed

which means that the natural frequency of resonator 5 is increased and that of resonator 6 is decreased. The difference between the frequencies of both resonators (a difference of components f_1 and f_2) forms an output frequency signal Δf of the transducer (the output signal), which is proportional to the measured pressure p.

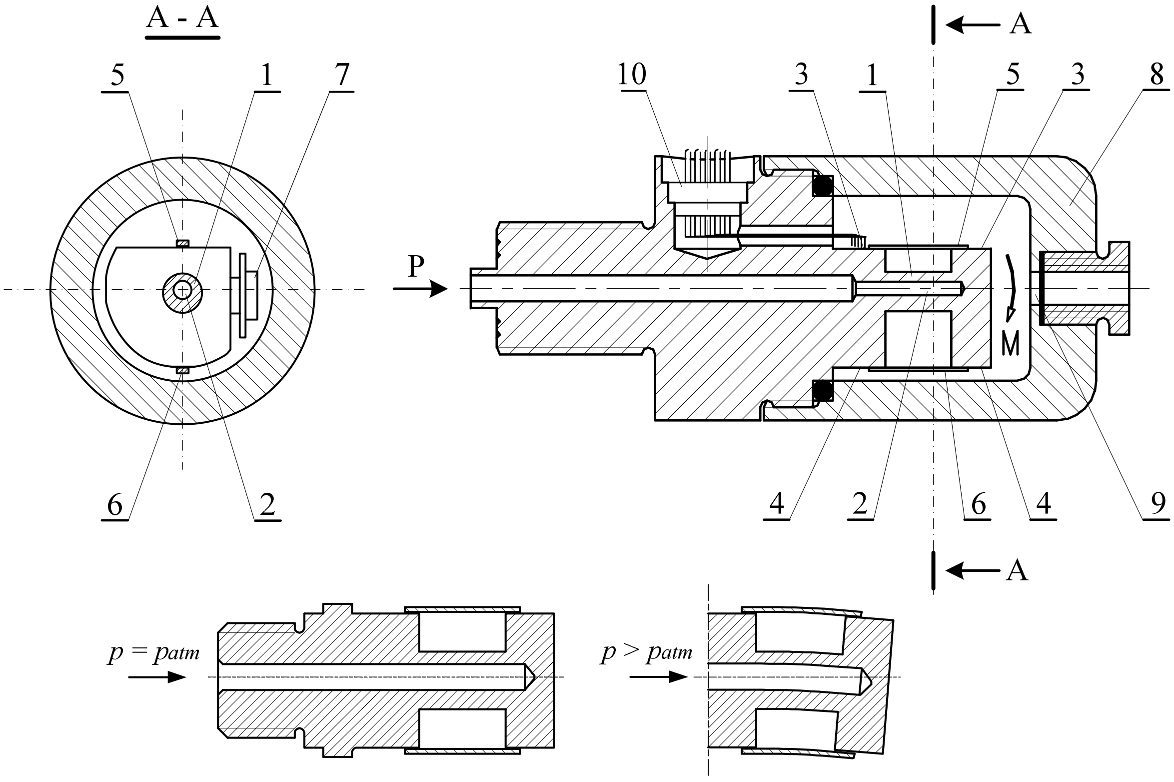


Fig. 3. Transducer design and its principle of operation.

1 – measuring cylinder, 2 – non-axial cylinder hole, 3, 4 – flat surfaces, 5, 6 – quartz resonators, 7 – generators, 8 – hermetic case, 9 – auxiliary hole, 10 – electrical connection.

The different milling depth of flat surfaces 3 and 4 is necessary to amplify the values of forces acting on the compressed resonator: the influence of the test pressure results in additional deformation of the test cylinder, which, apart from bending, also elongates limiting the range of stresses in the compressed cylinder.

The measuring cylinder is encapsulated inside a hermetic case (8). The casing has a hole (9) used for the removal of contained air during assembly. Such an operation should prevent the possible condensation of water vapours on the resonators.

Due to the application in the transducer of two separate quartz resonators operating in a differential configuration, its maximum temperature error is limited to 0.04% (experimentally confirmed in the temperature range from 0˚ C to 40˚C). This value is only acceptable in industrial transducers with an accuracy class of one or higher. In enhanced accuracy transducers, as in the aforementioned high pressure resonator transducer, the temperature error should not exceed 0.01 – 0.02 %/K. Therefore, it is obvious that there is a need for further minimization of the temperature error.

This task could be easily accomplished using an algorithmic (program) compensation method described in [11]. This compensation procedure enables the introduction of temperature characteristics of the transducer to the memory of the compensating system $\Delta f = f\,(p,\ t)$. Based upon the output signal of the transducer Δf and actual temperature t (a value given in an additional measurement) the measured pressure p is determined.

A certain inconvenience of this method is caused by the necessity of utilization of an additional temperature sensor necessary for evaluation of the temperature of the body of the pressure transducer. In the case of the discussed transducer, this deficiency can be eliminated by determining the temperature in an indirect way, without the use of a dedicated temperature sensor.

3. A new method of the algorithmic compensation of the transducer temperature error

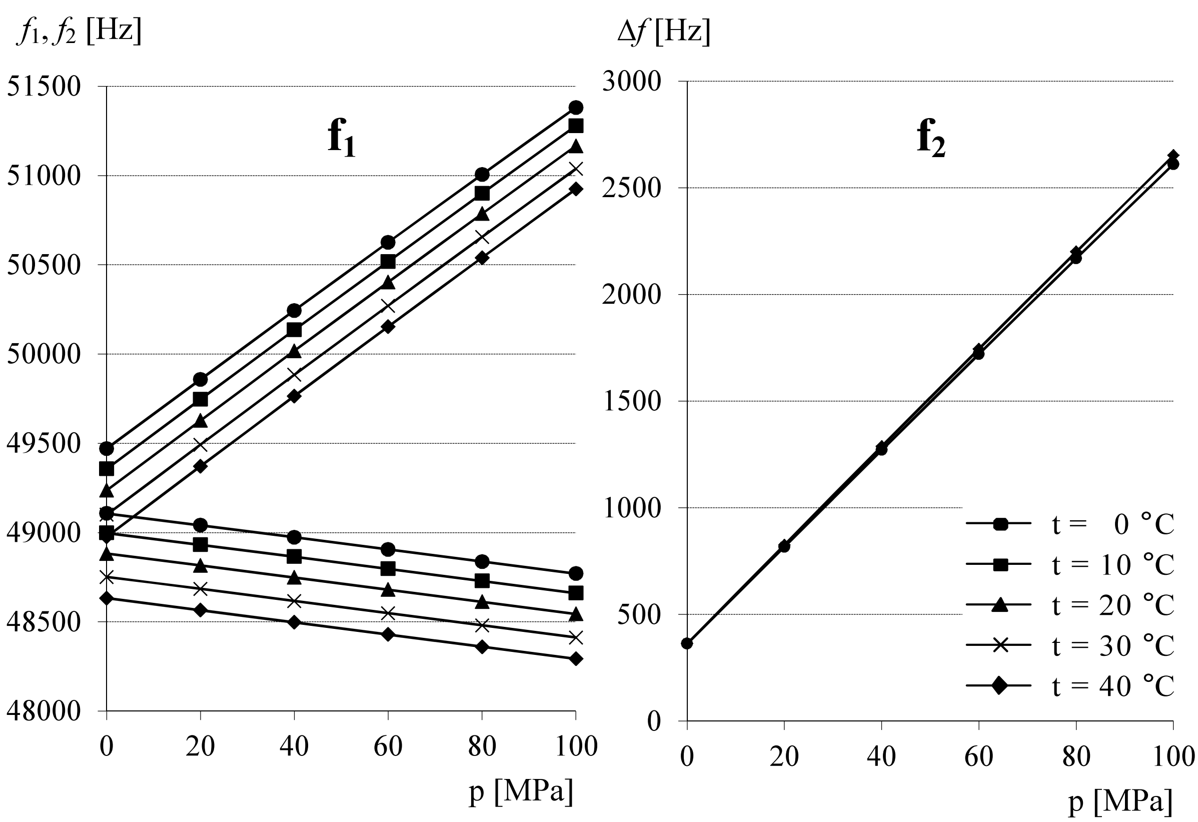
Fig. 4 shows experimental static characteristics of a transducer with two quartz resonators which were obtained based upon a five-element set of measuring points.

The very idea of the method, algorithm I, is based on equations describing the dependence of frequencies f_1 and f_2 on pressure p and temperature t (Fig. 4a):

$$f_1 = g_1(p, t), \tag{1}$$

$$f_2 = g_2(p, t), \tag{2}$$

where g_1 and g_2 are functions describing a group of relevant characteristics.



(a) (b)

Fig. 4. Static characteristics of the high pressure resonator transducer.
a) dependence of vibration frequency f_1 (marked f1) and f_2 (marked f2) on pressure p at different temperatures t,
b) dependence of the resonators frequency difference Δf (the output signal) on pressure p at different temperatures t.

If functions (1) and (2) together with the values of the measured frequencies f_1, f_2 are known, then a set of two equations with two unknowns is given: pressure p and temperature t. By solving this set of equations, values of the measured pressure p and temperature t are obtained where the values of the pressure are temperature corrected (to the actual temperature).
A condition for the successful implementation of the presented method can be seen to be the knowledge of the adequately precise analytical form of equations (1) and (2). The fulfilment of this condition would allow satisfactory results and termination of the compensation procedure without the need for the use of the calculated temperature t.

A well grounded suspicion is that the accuracy of calculations of pressure p from equations (1) and (2), when confronted with the target value of 0.02 – 0.01 %/K, might be insufficient. In view of the above, an extension of the presented method for the determination of pressure p (algorithm II) seems to be promising. In order to achieve this purpose, the characteristic of the frequency difference Δf obtained under varying temperature t (Fig. 4b) can be written similarly to equations (1) and (2):

$$\Delta f = g\,(p, t), \tag{3}$$

where: g – is a function describing a group of characteristics.

If the form of function (3) and frequency Δf (a difference of f_2 and f_1) is known then equation (3) can be solved (temperature t is found from equations (1) and (2)). The obtained value of pressure p, with a sufficiently adequate description of the characteristics with function (3) and sufficiently accurate calculation of temperature t, shall be satisfactory in terms of accuracy.
In the case of algorithm I, a few models of experimental characteristics of the transducer have been found and evaluated:

- with linear functions

$$p = \mathrm{a}_0 + \mathrm{a}_1 t + \mathrm{a}_2 f_1, \tag{4}$$

$$p = \mathrm{b}_0 + \mathrm{b}_1 t + \mathrm{b}_2 f_2, \tag{5}$$

- with linear functions in the developed form

$$p = \mathrm{a}_0 + \mathrm{a}_1 t + (\mathrm{a}_2 + \mathrm{a}_3 t)\, f_1, \tag{6}$$

$$p = \mathrm{b}_0 + \mathrm{b}_1 t + (\mathrm{b}_2 + \mathrm{b}_3 t)\, f_2, \tag{7}$$

- with polynomials of the second degree

$$p = \mathrm{a}_0 + \mathrm{a}_1 t + \mathrm{a}_2 t^2 + \mathrm{a}_3 f_1 + \mathrm{a}_4 f_1^2, \tag{8}$$

$$p = \mathrm{b}_0 + \mathrm{b}_1 t + \mathrm{b}_2 t^2 + \mathrm{b}_3 f_2 + \mathrm{b}_4 f_2^2, \tag{9}$$

- with polynomials of the second degree in the developed form

$$p = a_0 + a_1 t + a_2 t^2 + (a_3 + a_4 t + a_5 t^2)\, f_1 + (a_6 + a_7 t + a_8 t^2)\, f_1^{\,2}, \tag{10}$$

$$p = b_0 + b_1 t + b_2 t^2 + (b_3 + b_4 t + b_5 t^2)\, f_2 + (b_6 + b_7 t + b_8 t^2)\, f_2^{\,2}, \tag{11}$$

- with polynomials of the third degree in the developed form

$$p = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + (a_4 + a_5 t + a_6 t^2 + a_7 t^3)\, f_1 + (a_8 + a_9 t + a_{10} t^2 + a_{11} t^3)\, f_1^{\,2} + \\ + (a_{12} + a_{13} t + a_{14} t^2 + a_{15} t^3)\, f_1^{\,3}, \tag{12}$$

$$p = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + (b_4 + b_5 t + b_6 t^2 + b_7 t^3)\, f_2 + (b_8 + b_9 t + b_{10} t^2 + b_{11} t^3)\, f_2^{\,2} + \\ + (b_{12} + b_{13} t + b_{14} t^2 + b_{15} t^3)\, f_2^{\,3}, \tag{13}$$

where a_0 to a_{15}, b_0 to b_{15} are constant coefficients.

Also in the case of algorithm II, a few models of experimental characteristics of the transducer, based upon approximation and interpolation of these characteristics, have been found and evaluated:

- with linear function

$$p = a_0 + a_1 t + a_2 \Delta f, \tag{14}$$

- with linear function in the developed from

$$p = a_0 + a_1 t + (a_2 + a_3 t)\Delta f, \tag{15}$$

- with the second degree polynomial in the developed form

$$p = a_0 + a_1 t + a_2 t^2 + (a_3 + a_4 t + a_5 t^2)\Delta f + (a_6 + a_7 t + a_8 t^2)\Delta f^{\,2}, \tag{16}$$

- with the third degree polynomial in the developed form

$$p = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + (a_4 + a_5 t + a_6 t^2 + a_7 t^3)\Delta f + (a_8 + a_9 t + a_{10} t^2 + a_{11} t^3)\Delta f^{\,2} + \\ + (a_{12} + a_{13} t + a_{14} t^2 + a_{15} t^3)\Delta f^{\,3}, \tag{17}$$

- with the fourth degree polynomial in the developed form

$$p = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + (a_5 + a_6 t + a_7 t^2 + a_8 t^3 + a_9 t^4)\Delta f + (a_{10} + a_{11} t + a_{12} t^2 + a_{13} t^3 + \\ a_{14} t^4)\Delta f^{\,2} + (a_{15} + a_{16} t + a_{17} t^2 + a_{18} t^3 + a_{19} t^4)\Delta f^{\,3} + (a_{20} + a_{21} t + a_{22} t^2 + a_{23} t^3 + a_{24} t^4)\Delta f^{\,4}, \tag{18}$$

- with spline functions

$$p = \mathrm{a}_0 + \mathrm{a}_1\Delta f + \mathrm{a}_2\Delta f^2 + \mathrm{a}_3\Delta f^3 + \beta_1(\Delta f - \Delta f_2)_+^{\,3} + \beta_2(\Delta f - \Delta f_3)_+^{\,3} + \beta_3(\Delta f - \Delta f_4)_+^{\,3} + \beta_4(\Delta f - \Delta f_5)_+^{\,3}, \tag{19}$$

where a_0 to a_{24}, β_1 to β_4 are constant coefficients

$$\left(\Delta f - \Delta f_i\right)_+ = \begin{cases} 0 & \Delta f < \Delta f_i \\ \Delta f_i & \Delta f \geq \Delta f_i \end{cases}, \quad i = 2...5.$$

The current temperature t was found using the linear model of algorithm I.

4. Evaluation of metrological parameters of the transducer

Had the presented method of temperature compensation employed measurement error-free points of the transducer experimental characteristics, then the uncertainty of the pressure measurement, due to ambient temperature variations and nonlinearity of the transducer characteristics, would have been predominantly a result of the inaccuracy of approximation of the experimental characteristics with the model.

Unfortunately, procedures involved in the determination of the experimental characteristics result in additional measurement inaccuracies (result spread, instrument class [12, 13]). Each of the experimentally obtained measurement points of the transducer characteristic, which constitutes the model and base for algorithmic compensation, is loaded with a certain inaccuracy of measurement of pressure, frequency and temperature. Moreover, models of the characteristics of a transducer constructed for algorithmic compensation are barely an approximation of those characteristics, and the resulting errors are defined by the approximation error.

In the present paper, the above mentioned errors, apart from those stemming from the approximation and interpolation of characteristics, are a result of using the following instruments during experimentation: a manometer, class 0.2; frequency meters (0.05) and a thermometer (0.5).

Tables 1 and 2 show the total uncertainties u_{cp} of the measurement of pressure with their components included for the considered models of characteristics, both for algorithm I and II.

Table 1. Total uncertainties for the models of characteristics $f_1 = f\,(p, t)$ and $f_2 = f\,(p, t)$.

Tools used to describe the characteristic		Total uncertainties [MPa]			
		u_{cp1}	u_{cp2}	u_{cp3}	u_{cp}
Approximation	Linear	0.20	0.16	0.69	1.05
	Linear (d.f.)*		0.21	0.30	0.71
	Polynomial of the 2nd degree		0.26	0.24	0.70
	Polynomial of the 2nd degree (d.f.)		0.56	0.16	0.92
	Polynomial of the 3rd degree (d.f.)		0.80	0.10	1.10
Interpo-lation	Polynomial of the 2nd degree (d.f.)		0.54	0.14	0.88
	Polynomial of the 3rd degree (d.f.)		0.95	0.20	1.35

u_{cp1} is the measurement uncertainty of model pressures measured with a digital manometer.
u_{cp2} is the measurement uncertainty of pressure calculation resulting from the law of uncertainty propagation.
u_{cp3} is the measurement uncertainty of the analytical description of the experimental characteristics of the transducer.
$u_{cp} = u_{cp1} + u_{cp2} + u_{cp3}$ is the overall combined uncertainty in pressure.

* the developed form.

Table 2. Total uncertainties for the models of characteristics $\Delta f = f(p, t)$.

<table>
<tr><th colspan="2" rowspan="2">Tools used for the description of the characteristics</th><th colspan="4">Total uncertainties [MPa]</th></tr>
<tr><th>u_{cp1}</th><th>u_{cp2}</th><th>u_{cp3}</th><th>u_{cp}</th></tr>
<tr><td rowspan="4">Approximation</td><td>Linear</td><td rowspan="8">0.20</td><td>0.03</td><td>0.69</td><td>0.92</td></tr>
<tr><td>Linear (d.f.)</td><td>0.04</td><td>0.28</td><td>0.52</td></tr>
<tr><td>Polynomial of the 2nd degree (d.f.)</td><td>0.01</td><td>0.14</td><td>0.35</td></tr>
<tr><td>Polynomial of the 3rd degree (d.f.)</td><td>0.78</td><td>0.13</td><td>1.11</td></tr>
<tr><td rowspan="4">Interpolation</td><td>Polynomial of the 2nd degree (d.f.)</td><td>0.03</td><td>0.11</td><td>0.34</td></tr>
<tr><td>Polynomial of the 3rd degree (d.f.)</td><td>0.61</td><td>0.14</td><td>0.95</td></tr>
<tr><td>Polynomial of the 4th degree (d.f.)</td><td>0.94</td><td>0.09</td><td>1.23</td></tr>
<tr><td>Spline function of the 3rd degree</td><td>0.05</td><td>0.00</td><td>0.25</td></tr>
</table>

Fig. 5 presents the overall combined uncertainty plot as a function of measured pressure in case of the most accurate model obtained by means of spline functions.

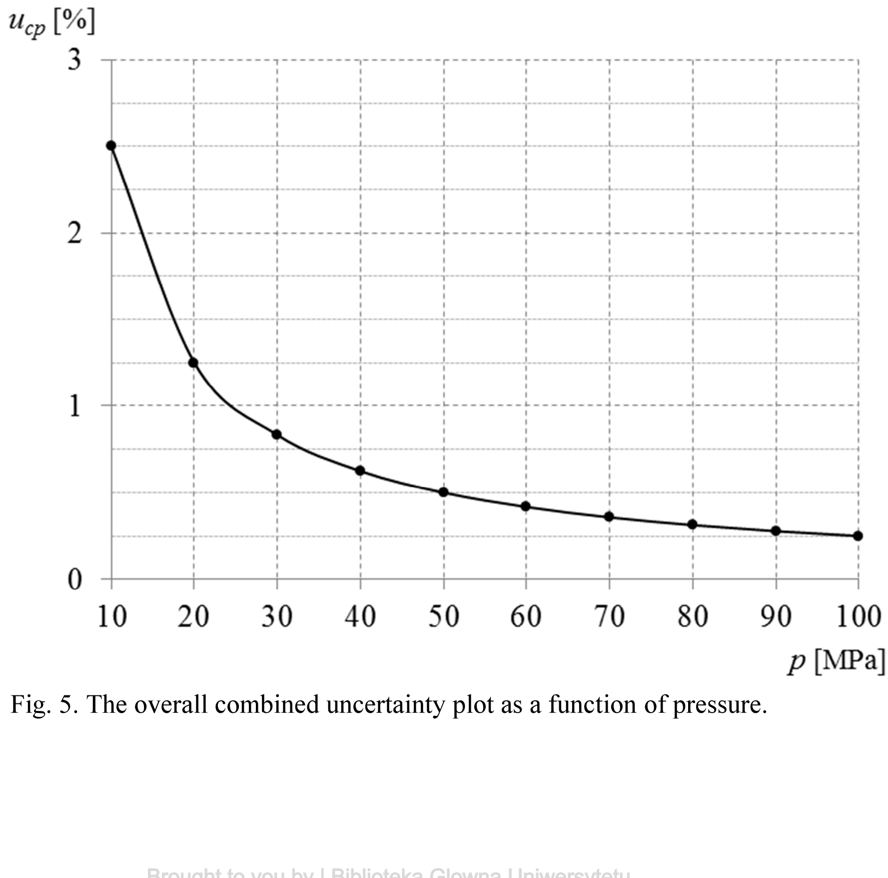


Fig. 5. The overall combined uncertainty plot as a function of pressure.

5. Summary and conclusions

A new method of pressure measurement error compensation (influence of the ambient temperature) for a high pressure resonator transducer has been proposed. The following conclusions have been formulated:

- The application of the algorithmic compensation based solely on mathematical models of characteristics $f_1 = f(p, t)$ and $f_2 = f(p, t)$ (algorithm I) makes it possible to limit the error that is associated with the measurement of pressure to a value of 0.03%/K. This is due to the influence of the ambient temperature and this value cannot be considered as satisfactory.
- Compensation based upon an extension to algorithm I (the use of mathematical models of characteristics $\Delta f = f(p, t)$ (algorithm II) makes it possible to obtain significantly better results than in the case of algorithm I, mostly due to the application of spline functions (the uncertainty of pressure measurement is at a level of 0.01%/K). In view of the criteria set for this transducer, its performance can be considered as satisfactory.
- The most significant disadvantageous influence on overall combined uncertainty u_{cp} (0.25) was caused by the digital manometer (with class of 0.2) used as a reference device. It can be supposed that achieved results could be improved in case of application of a more precise pressure reference measuring instrument.
- Some of the considered models, despite their high complexity, have not provided satisfactory results; increasing of polynomials degree resulted in a lowering of models accuracy.

Acknowledgements

The work was financed by a statutory activity subsidy from the Polish Ministry of Science and Higher Education for the Faculty of Mechanical and Power Engineering of Wroclaw University of Technology.

References

[1] Newhall, D. H., Abbot, L. H. (1961). High-Pressure Measurement. *Instruments & Control Systems*.

[2] Wisniewski, R., Molinar, G. (1999). Pressure standards and sensors up to 3 GPa, actual state and developmenttrends. *Physica B: Condensed Matter*, 265(1-4), 246–253.

[3] Gillum, D. R. (2009). Industrial Pressure, Level, and Density Measurement (2nd Edition). *International Societyof Automation*.

[4] Kobata, T. (2005). Characterization of quartz Bourdon-type high-pressure transducers. *Metrologia*, 42(6), S235S238.

[5] Kocas, I., Bergoglio, M. (December 2012). An investigation of quartz type pressure transducer behavior undercontinuous pressure conditions and metrological characterization. *Measurement: Journal of theInternational Measurement Confederation*, 45(10), 2486–2489.

[6] Jia, R. (1999). Principle of pressure-temperature transducer of quartz crystal resonator. *Proceedings of theAnnual IEEE International Frequency Control Symposium*, 1054–1057.

[7] EerNisse, E. P. (1987). Theoretical modeling of quartz resonator pressure transducers, *Proceedings of the AnnualIEEE International Frequency Control Symposium*, 339–343.

[8] Werszko, M., Werszko, R. Rezonatorowy przetwornik cisnienia (A resonator pressure transducer), Patent PL 178676.

[9] Karrer, E., Ward, R. (1977). A low-range quartz resonator pressure transducer, *ISA Transactions*, 16(2), 90–98.

[10] Cheshmehdoost, A., Jones, B. E. (June 1995). Design and performance characteristics of an integrated highcapacity DETF-based force sensor, *The 8th International Conference on Solid State Sensors andActuators, and Eurosensors IX*, Stockholm, Sweden, 52(1-3), 99–102.

[11] Bryzek, J. (July 1994). Signal conditioning for smart sensors and transducers, *Sensors* (Peterborough, NH),11(7), 151–160.

[12] Gleser, L. J. (1998). Assessing uncertainty in measurement, *Statistical Science*, 13(3).

[13] Yadav, S., Gupta, V.K., Bandyopadhyay, A.K. (2010). Investigations on measurement uncertainty and stabilityof pressure dial gauges and transducers, *Measurement Science Review*, 10(4), 130–135.