RECONSTRUCTION OF THE COLLISION OF MULTIPLE CARS CONSIDERED AS A SYSTEM OF MATERIAL PARTICLES

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Summary

In the situation of high intensity of motor vehicle traffic, collisions or accidents at road intersections often have the form of a collision of multiple cars. The reconstruction of such a road incident is often carried out as an attempt to estimate the velocities of individual cars, especially the velocity of the vehicle that caused the collision. In the analysis of motion of a larger number of vehicles, it may be sufficient to assume that they can be adequately represented by a system of material points with their mass values being known. In such a case, it may be possible to determine the velocity values necessary to assess the safety level and to estimate the degree of responsibility of individual drivers on the grounds of some elementary equations describing a system of material points. The objective of this article is to show, with a collision of three cars being taken as an example, that the determining of the velocity of the vehicle that caused the collision comes down to solving a few simple algebraic equations.
Before the reconstruction of a road incident is commenced, data are collected from the incident site. These data constitute the basic sources of evidence. Unfortunately, they may be burdened with a wide uncertainty margin. Therefore, adequate reliability of such data is often decisive for the obtaining of a correct result of the reconstruction. The article shows a possibility of improving the reliability of such data by using relative sensitivity factors to identify the parameters of the road incident under consideration that have the strongest influence on the final calculation result. This means that the values of such parameters should be estimated with the greatest possible accuracy because they have the strongest impact on the uncertainty of assessment of the traffic situation being examined.

Keywords: collision of cars; system of material points; relative sensitivity factors

1. Introduction

Many serious accidents, not to mention minor collisions, occur at road intersections. One of the main factors that have an influence on road traffic safety is high traffic intensity, in result of which collisions between more than two motorcars occur quite often.

In the analysis of motion of a larger number of vehicles, it may be sufficient to assume that they can be adequately represented by a system of material particles with their mass values being known. As an example, elementary equations of motion of material particles

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may be used to determine certain quantities necessary to assess the safety level and to estimate the degree of responsibility of individual drivers.

The article shows, with using an example selected, a possible way to estimate the speed of the vehicle whose driver caused the collision under analysis. The mathematical apparatus used for this purpose does not go beyond the elementary relations applicable to a system of material particles. Such relations can be easily found in textbooks on classical Newtonian mechanics; therefore, they do not have to be discussed here in greater detail.

An interest may be aroused by the issues of error and uncertainty of calculation results, addressed in numerous works. In Polish, these terms are often used interchangeably, which is not in conformity with ISO standards. The "measurement error" is a random variable, whose realizations xi assigned to the measured quantity x show the dispersion around the true value. Based on this dispersion, an interval that includes the unknown true value may be estimated. A quantitative measure of this dispersion, which characterizes the inaccuracies of the measurement, is "measurement uncertainty". This notion is defined as a parameter related to the measurement result, characterizing the dispersion of the values measured, which may be reasonably assigned to the measurand. Thus, the uncertainty is a non-random quantity, unlike the error, which is a random quantity [11].

For the effectiveness of examination of road incidents to be improved, which is an issue addressed in numerous studies, the uncertainty of estimation of values of the quantities that are important for the assessment of a road incident should be known. The authors of publication [1] have proposed the use of the Finite Difference and error analysis method, which does not require a high level of mathematical competence but gives good results. A similar approach has been presented in publication [2], where a stress has been put on estimating the uncertainty of marks left on the road surface and on using post-accident vehicle photographs. A practical approach to the assessment of uncertainty of the input variables has been undertaken by the authors of publication [3]. One of the techniques used in calculations is the substitution of upper and lower limit values of all the input parameters, which makes it possible to estimate the level of uncertainty of the calculation results. Another approach is the use of statistical information about the input data. The uncertainty issues have been summarized and further developed in article [5], where the author has carried out a review of different Finite Difference Analysis (FDA) methods used for the reconstruction of motor vehicle accidents.

As regards Polish authors, the works that are worth mentioning are those reported by M. Guzek and Z. Lozia, who have presented in many articles (e.g. [7], [8], or [9]) an approach to the error and uncertainty problems based on, inter alia, the "black box" idea and an analysis of the output quantities. The velocity and trajectory of the centre of mass of the moving vehicle have been described as simulated "black box" outputs. The simulation method used makes it possible to compare the simulated accurate values of the velocity and trajectory of the centre of vehicle mass with the readouts obtained by integration of the output simulations.

Many works ([12]-[16]) were carried out that were dedicated to the issues related to the development and application of the sensitivity theory.

This article presents a method of using relative sensitivity coefficients to identify the parameters that have the strongest influence on the final calculation result and, in consequence, whose values should be estimated with the greatest care. These coefficients have been chosen because they are dimensionless, thanks to which the influence of input parameters expressed in different units of measure may be evaluated. Direct use of the relative sensitivity coefficients in an analysis of issues related to agricultural engineering has been presented in publication [4].

2. Elementary relations that hold in a system of material particles

In the schematic drawing below (Fig. 1), the position of every material particle m_i is defined by position vector $\vec{r}_i$, $(i = 1, ..., N)$. The position of the centre of vehicle mass C, where the total mass of the system of material particles is concentrated, is defined by position vector $\vec{r}_c$.

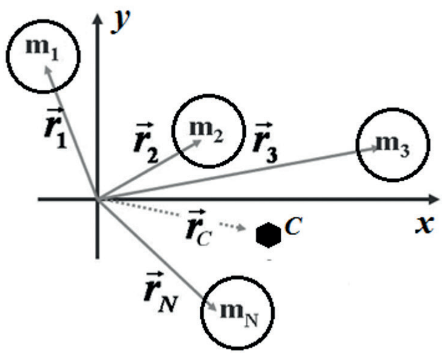

Fig. 1. A system of material particles

Here, the following relations hold:

$$m = \sum_i m_i \text{ and } m\vec{r}_C = \sum_i m_i \vec{r}_i \qquad (1)$$

By differentiating the latter of the two expressions above with respect to time, we will obtain:

$$\frac{d}{dt}\sum_i m_i \vec{r}_i = \sum_i m_i \vec{v}_i = \frac{d}{dt}\left(m\vec{r}_C\right)$$

Hence:

$$dt\sum_i m_i \vec{v}_i = d\left(m\vec{r}_C\right)$$

By changing to a finite time interval ($dt \rightarrow \Delta t$), the above may be written in the form:

$$\Delta t \sum_i m_i \vec{v}_i = \Delta\left(m \vec{r}_c\right) \tag{2}$$

3. Collision of three cars

Let us consider a road intersection with the traffic being controlled by traffic lights (Fig. 2) where the traffic incident to be analysed here took place.

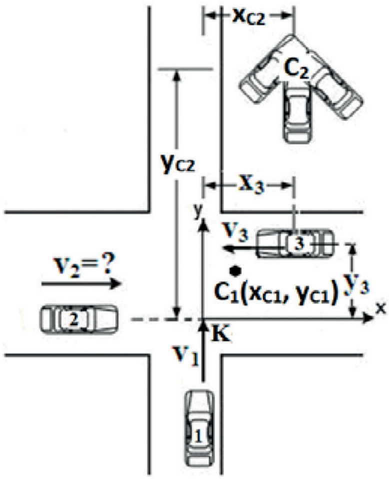

Fig. 2. Schematic diagram of a collision of three cars

Having noticed the yellow traffic light going on, the driver of car 2, which was moving with velocity v_2, estimated the time available as sufficient for him to cross the intersection within the current traffic light cycle and did not stop the car before entering the dangerous area. Unfortunately, it was actually too late for car 2 to enter the intersection; in consequence, car 2 crashed at point K into car 1, which was moving with velocity v1 without braking because the traffic lights have just turned green for it. Car 3 came to the intersection and stopped because its traffic light turned red. The description of the situation shows that the collision was unquestionably caused by the driver of car 2. For the road incident to be analysed correctly, it is important that velocity v_2 of car 2 when hitting car 1 should be known.

To start the analysis, a Cartesian coordinate system (x, y) with its origin at point K (Fig. 2) has to be adopted as a reference frame.

In result of the first impact, cars 1 and 2 moved and hit car 3, which was standing at the lights; afterwards, all the three cars moved to point C_2.

The cars will be treated as a system of three material particles with masses m_1, m_2, and m_3, respectively. The position of mass m_3 representing car 3 is defined by coordinates (x_3, y_3). At the instant of the first impact, the centre of mass of system (m_1, m_2, m_3) was situated at point C_1 with coordinates (x_{C1}, y_{C1}). If motor vehicle collisions may be treated as plastic, then point C_2 with coordinates (x_{C2}, y_{C2}) may be considered as a new position of the centre of mass of the system of three material particles (m_1, m_2, m_3).

4. Determining the velocity of the collision perpetrator

Based on equation (2), with (1) being taken into account, the following may be formulated:

$$\Delta t\left(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3\right) = (m_1 + m_2 + m_3)\Delta\vec{r}_{C1,C2} \tag{3}$$

where $\Delta\vec{r}_{C1,C2}$, is the distance between successive positions of the centre of mass of the system of material particles (m_1, m_2, m_3), i.e. between points C_1 and C_2.

The velocity to be found (v_2) is parallel to axis x of the reference frame (x, y); therefore, $v_2 = v_{2x}$. It may be determined by formulating an expression that would represent the x-axis component of vector equation (3). Thus, the following will be obtained:

$$x \rightarrow \quad \Delta t(m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x}) = (m_1 + m_2 + m_3)(x_{C2} - x_{C1})$$

The x-axis component of the velocity of car 1 are equal to zero and car 3 is at a standstill at the instant of the first impact; therefore, the equation above may be rewritten in the form as follows:

$$\Delta t(m_1 \cdot 0 + m_2 \cdot v_2 + m_3 \cdot 0) = (m_1 + m_2 + m_3)(x_{C2} - x_{C1})$$

Hence, the quantity to be found, i.e. the velocity of car 2, will be:

$$v_2 = \frac{(m_1 + m_2 + m_3)(x_{C2} - x_{C1})}{m_2 \Delta t} \tag{4}$$

The unknown value of parameter Δt, present in equation (4), may be determined with using the y-axis component of vector equation (3). In result of this, the following will be obtained:

$$y \uparrow \quad \Delta t(m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y}) = (m_1 + m_2 + m_3)(y_{C2} - y_{C1})$$

$$\Delta t(m_1 \cdot v_1 + m_2 \cdot 0 + m_3 \cdot 0) = (m_1 + m_2 + m_3)(y_{C2} - y_{C1})$$

$$\Delta t = \frac{(m_1 + m_2 + m_3)(y_{C2} - y_{C1})}{m_1 v_1} \tag{5}$$

The values of car masses m_1, m_2, and m_3 can be easily determined. The values of coordinates x_{C2} and y_{C2} defining the vehicle positions after the second impact as well as velocity v_1 of car 1 and position of car 3 may be estimated on the grounds of visual inspection of the incident site and analysis of camera records or witnesses' testimony. The values of coordinates x_{C1} and y_{C1} defining the position of the centre of mass of the system of material particles (m_1, m_2, m_3) at the instant of the first impact may be determined by using the x- and y-axis components of the vector equations (1):

$$x \rightarrow \quad m_1 \cdot 0 + m_2 \cdot 0 + m_3 \cdot x_3 = (m_1 + m_2 + m_3) x_{C1}$$

$$x_{c1} = x_3 \frac{m_3}{(m_1 + m_2 + m_3)} \tag{6}$$

$$y \uparrow \quad m_1 \cdot 0 + m_2 \cdot 0 + m_3 \cdot y_3 = (m_1 + m_2 + m_3) y_{C1}$$

$$y_{c1} = y_3 \frac{m_3}{(m_1 + m_2 + m_3)} \tag{7}$$

5. Relative sensitivity coefficients

Relative sensitivity coefficients (logarithmic, normalized) enable quantitative evaluation of the influence of changes in individual parameters on the value of the primitive function.

For a function $y = f(x_i)$, $i = 1, \ldots, r$, the relative sensitivity function may be defined as follows [1]:

$$WW_{xi}^{y} = \frac{\partial \ln y}{\partial \ln x_i} = \frac{\partial y / y}{\partial x_i / x_i} = \frac{\partial y}{\partial x_i} \cdot \frac{x_i}{y} \tag{8}$$

The relative sensitivity function shows the percentage by which the value of function y will change if the value of parameter x_i changes by 1 %. The higher the value WW_{xi}^{y}, the stronger the influence of parameter x_i on the result of calculations. A negative value of WW_{xi}^{y} means that an increase in the value of parameter x_i will result in a decrease in the value of function y.

The amount of the influence of parameter xi on the value of function y is different at different places in this function. Therefore, the numerical results of calculations of the relative sensitivity are applicable to a specific point in function y defined by substitution of the estimated nominal values of parameters of this function. For another point of function y, different WW_{xi}^{y} values may be obtained.

6. Analysis of the influence of parameters on the assessment of the collision of three cars

Based on equations (4), (5), (6), and (7), the following may be written, after some transformations:

$$v_2 = v_1 \frac{m_1\left(x_{C2} - x_3 \frac{m_3}{m_1 + m_2 + m_3}\right)}{m_2\left(y_{C2} - y_3 \frac{m_3}{m_1 + m_2 + m_3}\right)} \tag{9}$$

Equation (9) shows that velocity v_2 of the car that caused the collision at the road intersection depends on the accuracies of estimation of velocity v_1 of car 1, masses m_1, m_2, and m_3 of the cars involved in the collision, as well as geometrical quantities x_{C2}, y_{C2}, which define the position of the centre of mass of the system after the second impact, and x_3, y_3, which define the position of the centre of mass of car 3. Some of these values, e.g. the mass values, may be estimated quite precisely, but the accuracy of estimation of some others may be much worse.

The following estimates of the values of individual parameters were adopted for the analysis:

v_1 = 50 km/h; m_1 = 1 400 kg; m_2 = 800 kg; m_3 = 1 000 kg;

x_{C2} = 12 m; y_{C2} = 20 m; x_3 = 9 m; y_3 = 5 m.

Based on formula (9) and the parameter values assumed as above, the velocity of the car driven by the collision perpetrator was found to be:

v_2 = 43.6 km/h = 12.11 m/s.

To assess which of the parameters have the strongest influence on the final calculation result and, therefore, should be estimated more carefully than the others are, the corresponding values of the relative sensitivity coefficients were determined with using formulas (8) and (9). These parameters have been presented in Table 1, arranged in descending order of their influence on the calculated value of velocity v_2.

Table 1. Parameters taken for the analysis, arranged according to their influence on the calculated value of velocity v_2

Parameter	x_{C2}	m_1	y_{C2}	v_1	m_2	x_3	m_3	y_3
Parameter value	12 m	1 400 kg	20 m	50 km/h	800 kg	9 m	1 000 kg	5 m
Sensitivity function	WW_{xC2}^{v2}	WW_{m1}^{v2}	WW_{yc2}^{v2}	WW_{v1}^{v2}	WW_{m2}^{v2}	WW_{x3}^{v2}	WW_{m3}^{v2}	WW_{y3}^{v2}
Sensitivity function value	1.31	1.1	−1.08	1.0	−0.94	−0.31	−0.15	−0.08

The data presented in Table 1 show that the strongest influence on the estimated value of velocity v_2 of the car that caused the collision is exerted by the accuracy of determining the position of point C_2, i.e. the centre of mass of the system of three cars after the end of the second impact. If the second impact had the features of a plastic collision, the cars might be expected to remain close to point C_2; in such a case, the estimation of the values of coordinates x_{C2} and y_{C2} would not give rise to any considerable difficulties. If, however, the collision were not sufficiently plastic, the cars might happen to move away from point C_2 and the estimation of the values of coordinates x_{C2} and y_{C2} would not be easy. This is a weak point of the reconstruction of the collision under consideration.

A strong influence on the value of velocity v_2 is exerted by mass m_1. This is because this mass is much higher than masses m_2 and m_3. On the other hand, the determining of car masses is not particularly difficult.

The influence of the position of car 3 on the value of velocity v_2 is quite small. This may be explained by the fact that this car "joined" the system of material particles under analysis as late as after the end of the first impact.

For parameters y_{C2}, m_2, x_3, and m_3, the values of their sensitivity functions are negative. This means that higher values of these parameters result in lower values of the velocity v_2 to be found.

7. Conclusions

1. The elementary relations applicable to a system of material particles can be used for the reconstruction of a collision of multiple vehicles in road traffic.

2. The information obtained from an analysis of relative sensitivity coefficients makes it possible to improve the accuracy of estimation of the uncertainty of results of the road accident reconstruction.

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