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Construction and Analysis of Mathematical Models of Hydrodynamic Forces and Moment on the Ship's Hull Using Multivariate Regression Analysis

O. Kryvyi & M. Miyusov
National University „Odessa Maritime Academy", Odessa, Ukraine

ABSTRACT: To analyse the existing mathematical models of hydrodynamic forces and moment on the ship's hull and build new effective ones, an approach based on multivariate regression analysis is suggested. As factors (regressors), various dimensionless ratios of the geometric parameters of the vessel, such as length, breadth, draught, and block coefficient, were taken. When analysing existing mathematical models of hydrodynamic derivatives and building new ones, the value of the multiple correlation coefficient R and the value of standard errors were estimated. The significance of the models and the significance of all factors (regressors) included in the model were assessed using Fisher's and Student's criteria. As a result, new adequate mathematical models have been obtained for hydrodynamic constants with a high degree of correlation and an excellent level of significance of regressors.

1 INTRODUCTION

When studying ship manoeuvres, such as circulation, Kempf's zigzag, safe passage, etc., the presence of adequate mathematical models of non-inertial forces and moment on the ship's hull plays a very important role. Many works are devoted to the construction of mathematical models of hydrodynamic forces and moment on the ship's hull [2–18][. At small drift angles $|\beta| < 15^{\circ}$, polynomial models are mainly used to describe hydrodynamic forces and moments. The numerical characteristics of such models are obtained, as a rule (see, for example, [14–16]), based on processing the data of full-scale and model tests in wind tunnels, experimental basins, on rotary installations and on planar mechanisms. A similar approach to the construction of models of hydrodynamic forces is also implemented in the MMG (Maneuvering Modelling Group) method, which is described in the works [2–6, 17, 18]. There are also expressions for hydrodynamic derivatives up to the fourth order for a large type of fishing ships, dry cargo ships and tankers. One of the key stages of the above-mentioned approaches is the choice of methods for analysis and processing of experimental data. So, in the works [4, 11], using the least squares method with respect to one explanatory variable (regressor), hydrodynamic force models for ships with the value of the block coefficient $C_b \in (0.49; 0.7)$. were constructed.

In the work [2] for ships with the value of the block coefficient mainly from the range $C_b \in (0.7; 0.9)$ models for the derivatives of longitudinal hydrodynamic forces were obtained using several regressors based on the minimum AIC (Akaike Information Criterion).

This paper suggests a unified approach to the construction of models of hydrodynamic forces and moment, based on multivariate regression analysis, using Fisher's and Student's criteria [1].

The analysis of existing models is carried out and adequate models of the derivatives of hydrodynamic forces and moment are obtained with a high level of significance both for the models as a whole and for each individual regressor for a wide range of values of the block coefficient: $C_b \in (0.49; 0.9)$.

2 GENERAL REPRESENTATIONS OF HYDRODYNAMIC FORCES

The projections X_h, Y_h of the hydrodynamic forces on the coordinate axis associated with the ship and the moment M_h around the axis are expressed as follows: $X_h = v^2 C_h^x$, $Y_h = v^2 C_h^y$, $M_h = v^2 C_h^m$, where $C_h^x = C_h^x(\beta,\omega)$, $C_h^y = C_h^y(\beta,\omega)$, $C_h^m = C_h^m(\beta,\omega)$ - the hydrodynamic characteristics of the ship's hull; $v, \beta, \omega -$ respectively, the magnitude of the resulting velocity, the drift angle and the angular velocity of the ship.

The solvability of the corresponding systems of differential equations of the ship's motion [7–10, 12], determines the sufficient smoothness of their right-hand sides, which gives grounds to assume the existence of Maclaurin series for the hydrodynamic characteristics of the ship $(p = \{x, y, m\})$

$$C_h^p = \sum_{j+k=0}^{\infty} C_{jk}^p \beta^j \omega^k, \; C_{jk}^p = \frac{1}{(j+k)!} \frac{\partial^{j+k} C_h^p}{\partial \beta^j \partial \omega^k}\bigg|_{\substack{\beta=0 \\ \omega=0}}. \tag{1}$$

C_{jk}^p are called hydrodynamic constants (or hydrodynamic derivatives) of forces and moment on the ship's hull. Representations (1) make it possible to approximate the hydrodynamic characteristics of the ship's hull by polynomials at small angles of drift and angular velocity.

For example, if we restrict ourselves in expansion (1) to terms of the order not higher than third one and take into account the features of hydrodynamic forces [14–16], and the equality resulting from them $C_{jk}^x = 0$ at $\{(j,k)\} \neq \{(0,0);(2,0);(1,1);\ (0,2);(0,4)\}$ and also equalities $C_{jk}^y = 0$, $C_{jk}^m = 0$ at $\{(j,k)\} \neq \{(1,0);(3,0);(0,1);(1,2);(2,1);(0,3)\}$, then we obtain the following representations

$$\begin{aligned} C_h^x &= -C_{x_0} + C_{20}^x \beta^2 + C_{11}^x \beta\omega + C_{02}^x \omega^2 + C_{40}^x \beta^4, \\ C_h^y &= C_{10}^y \beta + C_{01}^y \omega + C_{30}^y \beta^3 + C_{21}^y \beta^2 \omega + C_{12}^y \beta \omega^2 + C_{03}^y \omega^3, \\ C_h^y &= C_{10}^y \beta + C_{01}^y \omega + C_{30}^y \beta^3 + C_{21}^y \beta^2 \omega + C_{12}^y \beta \omega^2 + C_{03}^y \omega^3, \end{aligned} \tag{2}$$

where C_{x_0} is the coefficient of water resistance to the straight-line motion of a vessel.

The hydrodynamic constants in representations (2) are expressed through the geometric characteristics of the vessel by processing experimental data.

3 METHOD FOR DETERMINING HYDRODYNAMIC CONSTANTS

The following easily identifiable basic geometric characteristics of the ship are usually used to determine the hydrodynamic derivatives: $L -$ length on waterline, $B -$ breadth on current waterline, $T -$ the midship draught and block coefficient C_b. From these parameters, we compose the determining regressors (factors):

$$\eta_1 = C_b, \; \eta_2 = \frac{B}{L}, \; \eta_3 = \frac{T}{L}, \; \eta_4 = \frac{T}{B}. \tag{3}$$

We use factors (3) as basic ones in quasilinear polynomial models (linear in coefficients) of hydrodynamic derivatives. It should be noted that, as a rule, basic regressors are used to build models of hydrodynamic derivatives (3), this is primarily due to their simplicity and availability. As the defining regressors (explanatory parameters) of the models, we will use the basic regressors (3) or their multipliers (products of powers), i.e., we will look for the hydrodynamic constants in the following form $(\{p\} = \{x, y, z\})$

$$C_{jk}^p = \sum_{j=1}^{\kappa} \lambda_j \xi_j, \; \xi_j = \prod_{l=1}^{\kappa_j} \eta_l^{\varsigma_l}, \; (j,k = 0,\ldots,4). \tag{4}$$

Indicator κ, coefficients of the regression model λ_j and indicators κ_j, ς_j of the regressions are determined for each hydrodynamic constant C_{jk}^p.

When constructing dependencies (4), we will evaluate both the significance level of the model as a whole and the significance of each individual regressor. Consequently, the model will be considered adequate if the following criteria based on regression analysis and analysis of variance are met.

1. The maximum possible value of the multiple correlation coefficient R should be achieved:

$$R > \alpha_0, \; (0 < \alpha_0 < 1) \tag{5}$$

1 where parameter α_0 determines the level of connection (correlation) of hydrodynamic derivatives C_{jk}^p with regressors included in representations (4). Moreover, if $(0.5 < \alpha_0 < 0.7)$, the connection is considered average (satisfactory), if $(0.7 < \alpha_0 < 0.8)$, the connection turns out to be high (good) and if $(0.8 < \alpha_0 < 1)$, then the connection is considered very high (excellent). Otherwise, the connection cannot be considered acceptable.

2. The statistical overall significance in the whole of each model (4) will be determined based on the Fisher criterion:

$$F_c > F_{nk}(1 - \alpha_F, m-1, n-m), \tag{6}$$

1 where $F_c = \dfrac{R^2}{1-R^2} \dfrac{n-m}{m-1}$ is an observed statistics with the following Fisher-Snedecor distribution (*F*-distribution); $m -$ the number of non-zero coefficients of the models (4); $n -$ sample size; $F_{nk}(1 - \alpha_F, m-1, n-m)$ is a critical value of the *F*-

distribution for the significance level α_F. The less α_F is, at which inequality (3.4) is satisfied, the higher the overall statistical significance of the model is. Significance level $\alpha_F \le 0.05.$ is considered excellent.

3. The level of significance of statistics F_c will be determined using the probability $\gamma_F = P(F_c \le F_{nk}(1-\alpha_F, m-1, n-m))$. Moreover, the less γ_F is, the higher the level of significance of the statistics is. The level of significance can be considered acceptable if the following condition is fulfilled:

$$\gamma_F < \alpha_F. \tag{7}$$

4. Standard error σ_j of the regressor ξ_j must satisfy the condition:

$$\sigma_j < |\lambda_j|. \tag{8}$$

5. The statistical significance of each of the regression coefficients is determined based on the Student's t-test:

$$|t_j| > |t_{kr}|, \left(t_j = \lambda_j / \sigma_j \right) \tag{9}$$

1 where $t_j = t(1-\alpha_s, n-m)$ is a critical value of the Student's distribution for the level of significance α_s. The less of α_s, at which inequality (8) is satisfied, the higher is the overall statistical significance of the coefficients of the model is. The level of $\alpha_s \le 0.05.$ can be considered excellent.

6. The significance level of the model regressors is determined using the probability $\gamma_j = P(t_j \le t_{kr}(1-\alpha_s, n-m))$. In this case, the less γ_j is, the higher the level of significance of the corresponding regressor is. The level of significance can be considered acceptable if the following condition is fulfilled:

$$\gamma_j < \alpha_s. \tag{10}$$

7. The absence of multicollinearity of the obtained models, i.e., the absence of regressors with a high pairwise correlation:

$$\left| R_{\xi_j, \xi_l} \right| < \alpha_{mk}, \quad j \ne l, \tag{10*}$$

1 where R_{ξ_j, ξ_l} is a coefficient of pair correlation of regressors, α_{mk} is an indicator of the level of correlation of regressors. It is believed that multicollinearity is absent in the model if α_{mk} does not exceed $0.7 \div 0.8$.

When constructing models, the condition (5) is the key one. However, the obtained dependencies must be statistically significant with a sufficiently high level of significance, i.e. conditions (6) and (7) must be satisfied with a sufficiently small value of α_F. The condition (8) allows us to discard insignificant regressors, conditions (9) and (10), with a sufficiently small value of α_s, allow us to assess the significance and significance level of each regressor in the model respectively. Condition (10*) makes it possible to exclude regressors leading to multicollinearity of the obtained models.

To construct mathematical models of the derivatives of hydrodynamic forces, we will apply the procedure of adding regressors (explanatory parameters). In this regard, firstly, the most significant regressors of the model are determined (i.e., the regressors with the highest values of the pair correlation coefficients with the corresponding hydrodynamic derivative). Then, starting with some minimal regression model, with the most significant regressors, we add new defining regressors until criteria 1) - 6) are met. In this case, at each stage, we check the fulfilment of condition (10*).

It should be noted that several adequate regression models can be obtained in this way. In this case, we will select those models for which the values of α_F, α_s and κ is minimal, and the value of the multiple correlation coefficient R is the maximum possible.

4 ANALYSIS OF EXISTING MODELS OF HYDRODYNAMIC CONSTANTS

Using the above-mentioned approach, we will analyse the existing models of hydrodynamic forces and moment on the ship's hull. To determine the coefficients of the models, we will use the experimental databases for hydrodynamic derivatives of various types of vessels in deep water, given in the works [2, 4, 18].

In particular, using the experimental data of works [5, 18] (sample $V14 = \{n = 12; C_b \in (0.5; 0.7)\}$), and work [2] (sample $V15 = \{n = 18; C_b \in (0.5; 0.9)\}$), depending on the values of the block coefficient C_b, for the derivatives of the longitudinal hydrodynamic force, 3 samples (volume n) were compiled:

$$V11 = \{n = 30;\ C_b \in (0.5; 0.9)\}$$
$$V12 = \{n = 14;\ C_b \in (0.5; 0.7)\},$$
$$V13 = \{n = 16; C_b \in (0.7; 0.9)\}$$

In works [4, 18], using the regressor $\eta_0 = \eta_1 \eta_2$, models of longitudinal hydrodynamic forces (models A) were obtained. The coefficients of these models will be determined using samples *V14, V11, V12, V13*.

*Models **A**:*

$$C_{20}^x = \begin{Bmatrix} 1.15 \\ 1.01 \\ 1.23 \\ -0.62 \end{Bmatrix} \eta_0 - \begin{Bmatrix} 0.18 \\ 0.18 \\ 0.2 \\ -0.04 \end{Bmatrix},$$

$$C_{11}^x - m_y' = -\begin{Bmatrix} 1.91 \\ 1.6 \\ 1.77 \\ 0.1 \end{Bmatrix} \eta_0 + \begin{Bmatrix} 0.08 \\ 0.04 \\ 0.06 \\ -0.17 \end{Bmatrix},$$

$$C_{02}^x + x_G' m_y' = \begin{Bmatrix} -0.09 \\ 0.07 \\ 0.03 \\ 0.65 \end{Bmatrix} \eta_0 - \begin{Bmatrix} 0.008 \\ -0.017 \\ -0.012 \\ -0.098 \end{Bmatrix},$$

$$C_{40}^x = \begin{Bmatrix} -6.68 \\ -5.41 \\ -7.5 \\ 0.43 \end{Bmatrix} \eta_0 + \begin{Bmatrix} 1.1 \\ 1.11 \\ 1.18 \\ 0.43 \end{Bmatrix}$$

In the work [2], with the use of the minimum AIC (Akaike Information Criterion), models of longitudinal hydrodynamic forces (*models* ***B***) were obtained. The coefficients of these models are determined using samples *V15, V11, V12, V13*.

*Models **B**:*

$$C_{20}^x = \begin{Bmatrix} 7.14 \\ 0.63 \\ 1.41 \\ -15.67 \end{Bmatrix} \eta_1 + \begin{Bmatrix} 38.4 \\ 2.48 \\ 4.94 \\ -75.31 \end{Bmatrix} \eta_2 - \begin{Bmatrix} 46.6 \\ 3.1 \\ 7.35 \\ -91.19 \end{Bmatrix} \eta_1\eta_2 - \begin{Bmatrix} 5.94 \\ 0.55 \\ 1.01 \\ -12.91 \end{Bmatrix},$$

$$C_{40}^x = \begin{Bmatrix} 0.0182 \\ 0.0169 \\ 0.0159 \\ 0.0078 \end{Bmatrix} \eta_1^{-2} + \begin{Bmatrix} -0.0826 \\ -0.139 \\ -0.21 \\ 0.242 \end{Bmatrix}, \; C_{02}^x + x_G' m_y' = \begin{Bmatrix} 0.701 \\ 0.021 \\ 0.032 \\ 0.679 \end{Bmatrix} -$$

$$- \begin{Bmatrix} 5.2 \\ 0.087 \\ 0.075 \\ 5.139 \end{Bmatrix} \eta_0 - \begin{Bmatrix} 14.7 \\ 0.672 \\ 0.952 \\ 15.36 \end{Bmatrix} \eta_3 + \begin{Bmatrix} 107.8 \\ 3.093 \\ 3.972 \\ 113.94 \end{Bmatrix} \eta_0\eta_3,$$

The upper coefficients in *models* ***A*** and ***B*** were obtained, respectively, in the works [4, 18] and [2] based on the experimental data presented there (samples *V14, V15*, respectively). The second row of coefficients corresponds to the *V11* sample, the third to the *V12* sample and the fourth to the *V13* sample.

Tables 1 and 2 show the correlation characteristics of models ***A*** and ***B***, respectively.

Table 1. Analysis of the model ***A*** [18]

		R	α_F	Cond. (8)	α_s
C_{20}^x	*V14*	0.7	10^{-2}	+	0.01
	V11	0.55	$2\cdot10^{-3}$	+	$2\cdot10^{-3}$
	V12	0.72	$4\cdot10^{-3}$	+	$5\cdot10^{-3}$
	V13	0.16	0.55	-	0.8
$C_{11}^x - m_y'$	*V14*	0.9	10^{-4}	+	0.05
	V11	0.81	10^{-7}	+	0.2
	V12	0.91	10^{-5}	+	0.1
	V13	0.03	0.91	-	0.2
$C_{02}^x + x_G' m_y'$	*V14*	0.14	0.66	-	0.85
	V11	0.12	0.55	-	0.55
	V12	0.05	0.86	-	0.86
	V13	0.34	0.2	+	0.2
C_{40}^x	*V14*	0.7	0.00	+	0.013
	V11	0.41	0.03	+	0.025
	V12	0.73	$3\cdot10^{-3}$	+	$2\cdot10^{-3}$
	V13	0.01	0.96	-	0.96

Analysis of the data given in Table 1 shows that for the values of the block coefficient $C_b \in (0.5;0.7)$, model A establishes a good correlation between the hydrodynamic derivatives C_{20}^x and C_{40}^x with the regressor $(R > 0.7)$, and for the hydrodynamic derivative $C_{11}^x - m_y'$ this interconnection turns out to be completely excellent $(R > 0.9)$.

In all cases, there is a fairly high level of significance of the models and regressor. For the entire range of values of the block coefficient $C_b \in (0.5;0.9)$, the model for C_{20}^x establishes a satisfactory correlation with the regressor, for $C_{11}^x - m_y'$ - excellent.

However, the significance of the regressor for $C_{11}^x - m_y'$ is not high: $\alpha_s = 0.2$. In all other cases, model ***A*** turns out to be inadequate. In particular, for the hydrodynamic derivative $C_{02}^x + x_G' m_y'$ turns out to be inadequate for all ranges of variation of the block coefficient.

Analysis of the data given in Table 2 shows that for the hydrodynamic constants C_{20}^x and C_{40}^x models ***B*** are not adequate for all ranges of change in the values of the block coefficient. As for the hydrodynamic constant $C_{11}^x - m_y'$, a good correlation $(R = 0.71 \div 0.92)$, is observed for the range $C_b \in (0.5;0.9)$, for the *V15* sample, however, there is multicollinearity of the regressors: $\alpha_{mk} = 0.92$.

The latter leads to the fact that with an increase in the sample size *V11*, the model turns out to be inadequate with a low level of significance of the regressors. The same is observed for the ranges $C_b \in (0.7;0.9)$, and $C_b \in (0.5;0.7)$. As for the hydrodynamic constant $C_{02}^x + x_G' m_y'$, model ***A*** can only be used for $C_b \in (0.7;0.9)$.

Table 2. Analysis of the model ***B*** [2]

		R	α_F	Cond. (8)	α_s	α_{mk}
C_{20}^x	*V15*	0.59	0.12	+	0.05	0.9
	V11	0.63	0.01	-	0.4	0.79
	V12	0.75	0.07	-	0.5	0.98
	V13	0.53	0.25	+	0.23	0.93
$C_{11}^x - m_y'$	*V15*	0.71	0.05	+	0.05	0.92
	V11	0.81	5·10-7	-	0.48	0.78
	V12	0.61	0.06	+	0.05	0.9
	V13	0.92	3·10-5	+	0.25	0.89
$C_{02}^x + x_G' m_y'$	*V15*	0.52	0.03	+	0.03	0.64
	V11	0.23	0.7	-	0.76	0.81
	V12	0.33	0.75	-	0.93	0.99
	V13	0.75	0.02	+	0.04	0.42
C_{40}^x	*V15*	0.30	0.23	-	0.87	-
	V11	0.42	0.02	-	0.53	-
	V12	0.51	0.07	-	0.41	-
	V13	0.10	0.7	-	0.71	-

To analyse the existing models of the **derivatives of the transverse hydrodynamic forces**, we used the experimental data of the works [4, 18], from which, depending on the values of the block coefficient C_b, three samples were made for the derivatives of the transverse hydrodynamic forces:

$$V21 = \{n = 33; C_b \in (0.49;0.9)\}$$

$$V22 = \{n = 20; C_b \in (0.49;0.7)\}$$

$$V23 = \{n = 13; C_b \in (0.7;0.9)\}$$

In the work [4], using regressors (3), models of transverse hydrodynamic forces (*models* **C**) were obtained. We will calculate the coefficients of these models based on samples *V21, V22, V23*.

*Models **C***

$$C_{10}^{y} = \begin{Bmatrix} 1.90 \\ 3.33 \\ 1.93 \end{Bmatrix} \eta_1\eta_2 + \begin{Bmatrix} 0.11 \\ 0.02 \\ 0.10 \end{Bmatrix}, \quad C_{01}^{y} - m' - m'_x = -\begin{Bmatrix} 1.5 \\ 1.28 \\ 1.45 \end{Bmatrix} \eta_1\eta_2 + $$

$$+\begin{Bmatrix} 0.01 \\ -0.04 \\ -0.03 \end{Bmatrix}, \quad C_{12}^{y} = \begin{Bmatrix} 3.13 \\ -2.19 \\ 5.36 \end{Bmatrix} (1-\eta_1)\eta_4 + \begin{Bmatrix} 0.26 \\ 1.05 \\ 0.01 \end{Bmatrix}.$$

$$C_{30}^{y} = \begin{Bmatrix} 354 \\ 256 \\ -503 \end{Bmatrix} ((1-\eta_1)\eta_4)^2 + \begin{Bmatrix} 86.3 \\ 52.1 \\ -34.6 \end{Bmatrix} (1-\eta_1)\eta_4 + \begin{Bmatrix} -2.47 \\ 0.31 \\ 1.17 \end{Bmatrix},$$

$$C_{21}^{y} = -\begin{Bmatrix} 202.5 \\ 160.1 \\ -96.6 \end{Bmatrix} (\eta_1\eta_2)^2 + \begin{Bmatrix} 69.5 \\ 57 \\ -22.8 \end{Bmatrix} \eta_1\eta_2 - \begin{Bmatrix} 5.48 \\ 4.78 \\ -1.53 \end{Bmatrix},$$

The upper coefficients in models ***C*** and ***D*** correspond to the *V21* sample, the second and third respectively to the *V22, V23* samples.

On the same samples, the coefficients of the models of transverse hydrodynamic forces (*model **D***), proposed in the works [3–6, 18], were calculated:

*Models **D***

$$C_{10}^{y} = \begin{Bmatrix} 1.74 \\ 1.17 \\ 2.12 \end{Bmatrix} \eta_1\eta_2 + \begin{Bmatrix} 2.68 \\ 3.84 \\ 1.47 \end{Bmatrix} \eta_2, \quad C_{12}^{y} = \begin{Bmatrix} 5.16 \\ 5.12 \\ 5.48 \end{Bmatrix} (1-\eta_1)\eta_4,$$

$$C_{01}^{y} - m' - m'_x = -\begin{Bmatrix} 1.46 \\ 1.31 \\ 1.63 \end{Bmatrix} \eta_1\eta_2,$$

$$C_{30}^{y} = \begin{Bmatrix} 0.5 \\ 0.58 \\ 0.08 \end{Bmatrix} \eta_2^{-1} + \begin{Bmatrix} -0.98 \\ -0.87 \\ 0.52 \end{Bmatrix},$$

$$C_{21}^{y} = \begin{Bmatrix} 4.66 \\ 8.24 \\ -0.28 \end{Bmatrix} \eta_1\eta_2 - \begin{Bmatrix} 1.2 \\ 2.2 \\ -0.32 \end{Bmatrix}.$$

Tables 3 and 4 show the main correlation characteristics of the dependences ***C*** and ***D***.

Table 3. Analysis of the model ***C***

		R	α_F	Cond. (8)	α_s
C_{10}^{y}	*V21*	0.72	$3\cdot10^{-6}$	+	0.007
	V22	0.66	0.002	-	0.85
	V23	0.71	0.007	+	0.25
$C_{01}^{y} - m' - m'_x$	*V21*	0.78	10^{-16}	+	0.85
	V22	0.73	$2\cdot10^{-4}$	-	0.9
	V23	0.91	$1\cdot10^{-5}$	-	0.4
C_{30}^{y}	*V21*	0.58	$2\cdot10^{-3}$	+	$4\cdot10^{-2}$
	V22	0.44	0.16	-	0.94
	V23	0.86	$1\cdot10^{-3}$	+	0.29
C_{21}^{y}	*V21*	0.78	10^{-6}	+	$4\cdot10^{-4}$
	V22	0.68	$5\cdot10^{-3}$	+	0.06
	V23	0.67	$5\cdot10^{-2}$	-	0.49
C_{12}^{y}	*V21*	0.52	$2\cdot10^{-3}$	+	0.02
	V22	0.35	0.14	+	0.14
	V23	0.74	$5\cdot10^{-3}$	-	0.94

Analysis of models ***C*** shows that not all these models of the derivatives of transverse hydrodynamic forces are adequate.

Mathematical models have good correlation characteristics with good regression indicators only for constant C_{10}^{y} when $C_b \in (0.49; 0.9)$ and $C_b \in (0.7; 0.9)$, for constant C_{30}^{y} when $C_b \in (0.7; 0.9)$ and for constant C_{12}^{y} when $C_b \in (0.49; 0.9)$ and $C_b \in (0.49; 0.7)$.

Table 4. Analysis of the model ***D***

		R	α_F	Cond. (8)	α_s	α_{mk}
C_{10}^{y}	V21	0.99	$9\cdot10^{-29}$	+	$5\cdot10^{-7}$	0.27
	V22	0.99	$7\cdot10^{-17}$	+	0.01	0.64
	V23	0.99	$8\cdot10^{-11}$	+	0.07	0.4
$C_{01}^{y} - m' - m'_x$	V21	0.97	$2\cdot10^{-21}$	+	10^{-21}	-
	V22	0.95	$6\cdot10^{-11}$	+	10^{-16}	-
	V23	0.99	10^{-14}	+	10^{-15}	-
C_{30}^{y}	V21	0.4	0.02	-	0.41	-
	V22	0.52	0.02	-	0.51	-
	V23	0.1	0.78	-	0.8	-
C_{21}^{y}	V21	0.42	0.02	+	0.02	-
	V22	0.64	$2\cdot10^{-3}$	+	$2\cdot10^{-3}$	-
	V23	0.19	0.7	-	0.7	-
C_{12}^{y}	V21	0.91	$9\cdot10^{-14}$	+	10^{-13}	-
	V22	0.91	$3\cdot10^{-8}$	+	$2\cdot10^{-8}$	-
	V23	0.97	$4\cdot10^{-8}$	+	$1\cdot10^{-8}$	-

For models ***D*** mathematical models of constants C_{10}^{y}, $C_{01}^{y} - m' - m'_x$ and C_{12}^{y} have excellent regression indicators for the entire range of values of the block coefficient C_b. Mathematical models for the rest of the hydrodynamic constants are inadequate.

To analyse the mathematical models of ***the derivatives of the moment*** of hydrodynamic forces, we used the experimental data of the works [4, 18] and made three samples for the derivatives of the moment depending on the values of the block coefficient:

$$V31 = \{n = 33; C_b \in (0.49; 0.9)\}$$
$$V32 = \{n = 20; C_b \in (0.49; 0.7)\}$$
$$V33 = \{n = 13; C_b \in (0.7; 0.9)\}$$

In the work [4], using regressors (3), models of transverse hydrodynamic forces (*models **E***) were written. We will calculate the coefficients of these models based on samples *V31, V32, V33*.

*Models **E***

$$C_{10}^{m} = \begin{Bmatrix} 1.32 \\ 1.23 \\ 2.09 \end{Bmatrix} \eta_3, \quad C_{01}^{m} - x'_G m' = \begin{Bmatrix} 5.26 \\ 5.85 \\ 25.36 \end{Bmatrix} \eta_3^2 - \begin{Bmatrix} 1.14 \\ 1.25 \\ 2.13 \end{Bmatrix} \eta_3,$$

$$C_{21}^{m} = -\begin{Bmatrix} 52.39 \\ 3.24 \\ -21.37 \end{Bmatrix} (\eta_1\eta_2)^2 + \begin{Bmatrix} 16.02 \\ 1.24 \\ -7.75 \end{Bmatrix} \eta_1\eta_2 - \begin{Bmatrix} 1.46 \\ 0.58 \\ -0.50 \end{Bmatrix},$$

$$C_{03}^{m} = -\begin{Bmatrix} 3.91 \\ 3.17 \\ 60.6 \end{Bmatrix} (\eta_1\eta_2)^2 + \begin{Bmatrix} 1.36 \\ 1.18 \\ 19.32 \end{Bmatrix} \eta_1\eta_2 - \begin{Bmatrix} 0.13 \\ 0.12 \\ 0.71 \end{Bmatrix}.$$

The upper coefficients in the models ***E*** correspond to the *V21* sample, the second and third correspond to the *V22, V23* samples respectively.

On the same samples, the coefficients of the models of transverse hydrodynamic forces (*model **F***), suggested in the works [3–6, 18], were calculated:

Models B

$$C_{30}^{m} = -\begin{Bmatrix}0.9\\1.79\end{Bmatrix}\eta_1 - \begin{Bmatrix}0.72\\1.22\end{Bmatrix},$$

$$C_{21}^{m} = \begin{Bmatrix}1.51\\0.32\end{Bmatrix}\eta_1\eta_2 - \begin{Bmatrix}0.54\\0.52\end{Bmatrix},$$

$$C_{12}^{m} = \begin{Bmatrix}0.023\\0.32\end{Bmatrix}(1-\eta_1)\eta_2^{-1} + \begin{Bmatrix}-0.039\\0.013\end{Bmatrix},$$

$$C_{03}^{m} = \begin{Bmatrix}0.28\\0.28\end{Bmatrix}\eta_1\eta_2 - \begin{Bmatrix}0.056\\0.06\end{Bmatrix}.$$

The upper coefficients in the ***F*** model are obtained on the *V31* sample while the second ones are obtained on the *V32* sample.

When constructing models of hydrodynamic derivatives C_{03}^{y}, C_{30}^{m}, C_{12}^{m}, in the work [4], the parameter $\sigma_a = (1-C_{wa})(1-C_{pa})^{-1}$ is also used as the basic regressor, where C_{wa} and C_{pa} are the waterplane area coefficient and prismatic coefficient of aft half hull between APP and ship station five. The coefficients are calculated as: $C_{wa} = A_{wa}\cdot(L_a B_a)^{-1}$, $C_{pa} = \nabla_a (A_a L_a)^{-1}$, where A_{wa} is the water plane area of the aft section, A_a is the cross-sectional area equal to the largest underwater section of the aft hull, ∇_a is the displacement of the aft hull, B_a is the vessel's breadth of the aft hull and L_a is its length.

However, the analysis of these dependencies showed their poor correlation features. Moreover, the data for calculating the parameter σ_a are not always available in the reference literature.

Tables 5 and 6 show the correlation characteristics of models ***E*** and ***F***, respectively.

For models E mathematical models of hydrodynamic constants C_{10}^{m}, $C_{01}^{m} - x_G' m'$ and C_{03}^{m} have excellent regression indicators for the entire range of values of the block coefficient C_b. The mathematical model for C_{21}^{m} is inadequate.

Analysis of the ***F*** models shows that not all these models of the derivatives of the transverse hydrodynamic forces are adequate. Mathematical models only for the hydrodynamic constant C_{03}^{m} have quite good correlation characteristics and good regression indicators.

Table 5. Analysis of the model E

		R	α_F	Cond. (8)	α_s
C_{10}^{m}	*V21*	0.91	$2\cdot10^{-13}$	+	$8\cdot10^{-14}$
	V22	0.91	$2\cdot10^{-08}$	+	$9\cdot10^{-09}$
	V23	0.99	$1\cdot10^{-14}$	+	10^{-14}
$C_{01}^{m} - x_G' m'$	*V21*	0.88	$3\cdot10^{-9}$	+	$8\cdot10^{-5}$
	V22	0.99	$5\cdot10^{-14}$	+	$4\cdot10^{-10}$
	V23	0.77	$8\cdot10^{-3}$	+	0.07
C_{21}^{m}	*V21*	0.55	0.005	+	0.003
	V22	0.11	0.91	-	0.9
	V23	0.72	0.03	-	0.4
C_{03}^{m}	*V21*	0.84	10^{-8}	+	$5\cdot10^{-5}$
	V22	0.88	$4\cdot10^{-6}$	+	0.004
	V23	0.91	$2\cdot10^{-4}$	+	0.04

Table 6. Analysis of the model ***F***

		R	α_F	Cond. (8)	α_s
C_{10}^{m}	*V21*	0.4	0.02	+	0.02
	V22	0.34	0.14	+	0.14
$C_{01}^{m} - x_G' m'$	*V21*	0.28	0.11	+	$8\cdot10^{-5}$
	V22	0.1	0.67	-	0.67
C_{21}^{m}	*V21*	0.25	0.16	+	0.26
	V22	0.06	0.8	-	0.83
C_{03}^{m}	*V21*	0.7	$6\cdot10^{-6}$	+	$6\cdot10^{-6}$
	V22	0.79	$4\cdot10^{-5}$	+	$4\cdot10^{-5}$

Thus, the analysis of the existing models for the derivatives of hydrodynamic forces and moment shows that many of them cannot be used for the entire range of variation of the values of the block coefficient C_b . Only some of them can provided a fairly good correlation on limited ranges. Obviously, a univariate correlation analysis cannot provide the construction of adequate models with a high level of significance for the entire range of change in values C_b As for the approach of the work [2] for mathematical models of longitudinal hydrodynamic forces, then, it is obvious that the use of the minimum criterion AIC only cannot ensure the fulfilment of criteria 1) - 7).

5 CONSTRUCTION OF NEW MATHEMATICAL MODELS OF HYDRODYNAMIC FORCES AND MOMENTS

The analysis of the known models indicates that there is a need to build new adequate models of the derivatives of the longitudinal hydrodynamic forces on the ship's hull with a high level of significance that meet the criteria 1) - 7). The standard scheme of multivariate regression analysis [1], and the method described in the second section, made it possible to construct several new adequate models of the derivatives of longitudinal hydrodynamic forces and moment with high correlation indicators. Some of these models having the highest level of correlation and levels of significance as well as the standard errors of the regressors of which satisfy condition (7) are given below.

To construct the ***hydrodynamic derivatives of transverse forces***, we use samples *V11, V12, V13*.

In particular, for the constant C_{20}^{x} for the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ the following representations should be highlighted:

$$C_{20}^{x} = -0.086\eta_1 - 0.389(1-\eta_1)\eta_4 + 5.599\eta_1\eta_2\eta_3, \tag{11}$$

$$C_{20}^{x} = -0.173\eta_1 + 3.74\eta_1\eta_2\eta_3. \tag{12}$$

For the range of values of the block coefficient $C_b \in (0.5; 0.7)$ the following models provide excellent correlation:

$$C_{20}^{x} = -0.173\eta_1 + 4.855\eta_1\eta_2\eta_3, \tag{13}$$

$$C_{20}^{x} = -0.528\eta_1 + 6.088\eta_1\eta_2\eta_3, \quad (14)$$

$$C_{20}^{x} = 2.529(1-\eta_1)\eta_2 - 1.714(1-\eta_1)\eta_4, \quad (15)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$, the following model can be also used:

$$C_{20}^{x} = 2.529(1-\eta_1)\eta_4 - 15.086\eta_1\eta_2\eta_3. \quad (16)$$

Table 7 shows the correlation characteristics of models (11) - (16).

Table 7. Analysis of the model for C_{20}^{x}.

		R	α_F	α_s	α_{mk}
(11)	*V11*	0.8	$5\cdot10^{-5}$	0.02	0.5
(12)		0.75	$3\cdot10^{-4}$	$3\cdot10^{-4}$	0.5
(13)	*V12*	0.81	0.003	0.003	0.04
(14)		0.84	0.003	7·10-4	0.6
(15)		0.85	0.003	5·10-4	0.79
(16)	*V13*	0.8	0.004	0.14	0.59

For the constant $C_{11}^{x} - m_y'$ for the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ the following representations should be highlighted:

$$C_{11}^{x} - m_y' = -0.978\eta_0 - 0.603\eta_3, \quad (17)$$

$$C_{11}^{x} - m_y' = -0.504\eta_1\eta_4 - 3.086\eta_2\eta_3. \quad (18)$$

For the range of values of the coefficient of total completeness $C_b \in (0.5; 0.7)$ the following dependence also provides excellent correlation

$$C_{11}^{x} - m_y' = -0.396\eta_1\eta_4 - 3.634\eta_2\eta_3. \quad (19)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$ the following model can be also used

$$C_{11}^{x} - m_y' = -6.344\eta_0^2 - 3.634\eta_0^2\eta_3^2. \quad (20)$$

Table 8 shows the correlation characteristics of models (17) - (20).

Table 8. Analysis of the model for $C_{11}^{x} - m_y'$.

		R	α_F	α_s	α_{mk}
(17)	*V11*	0.98	10^{-17}	0.04	0.68
(18)		0.98	10^{-16}	$5\cdot10^{-5}$	0.14
(19)	*V12*	0.99	10^{-8}	$5\cdot10^{-5}$	0.56
(20)	*V13*	0.98	10^{-12}	0.14	0.38

For a constant $C_{02}^{x} + x_G' m_y'$ for the range of variation of the block coefficient $C_b \in (0.5; 0.7)$ the following representations should be highlighted:

$$C_{02}^{x} + x_G' m_y' = 0.07\eta_1 + 0.34(1-\eta_1)\eta_2 - 0.37\eta_1\eta_4, \quad (21)$$

$$C_{02}^{x} + x_G' m_y' = 0.303(1-\eta_1)\eta_2 - 0.166\eta_1\eta_4. \quad (22)$$

The following model is also adequate for the range $C_b \in (0.7; 0.9)$:

$$C_{02}^{x} + x_G' m_y' = 0.49\eta_1 + 9.68\eta_0\eta_1\eta_3 - 0.484. \quad (23)$$

Table 9 shows the correlation characteristics of models (21) - (23).

Table 9. Analysis of the model for $C_{02}^{x} + x_G' m_y'$

		R	α_F	α_s	α_{mk}
(21)	*V12*	0.68	0.07	0.24	0.586
(22)		0.68	0.05	0.08	0.378
(23)	*V13*	0.66	0.02	0.03	0.271

For the constant C_{40}^{x} for the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ the following representations should be highlighted:

$$C_{40}^{x} = 2.85\eta_1\eta_4 - 33.225\eta_1\eta_2\eta_3, \quad (24)$$

$$C_{40}^{x} = 0.899\eta_1 - 26.105\eta_1\eta_2\eta_3, \quad (25)$$

$$C_{40}^{x} = 4.89\eta_1\eta_4 - 17.463\eta_1\eta_3. \quad (26)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$ the following model provides excellent correlation:

$$C_{40}^{x} = 1.78\eta_1\eta_2\eta_3 - 35.83\eta_4, \quad (27)$$

$$C_{40}^{x} = 5\eta_1\eta_4 - 18.057\eta_1\eta_3. \quad (28)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$ the following model can be also used:

$$C_{40}^{x} = -26.973(1-\eta_1)\eta_2 + 167.485\eta_1\eta_2\eta_3. \quad (29)$$

Table 10 shows the correlation characteristics of models (24) - (29).

Table 10. Analysis of the model for C_{40}^{x}

		R	α_F	α_s	α_{mk}
(32)	*V11*	0.8	10^{-6}	10^{-4}	0.25
(33)		0.79	10^{-6}	0.002	0.28
(34)		0.79	10^{-6}	10^{-3}	0.55
(35)	*V12*	0.79	0.004	0.003	0.75
(36)		0.75	0.009	0.006	0.75
(37)	*V13*	0.86	10^{-4}	0.12	0.73

To construct the hydrodynamic derivatives of transverse forces, we use samples *V21, V22, V23*.

For the constant C_{10}^{y} for the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ the following representations should be highlighted:

$$C_{10}^{y} = 1.36\eta_2 + 16.79\eta_1\eta_2\eta_3. \quad (30)$$

$$C_{10}^{y} = 0.94(1-\eta_1)\eta_4 + 39.2\eta_1\eta_2\eta_3, \quad (31)$$

$$C_{10}^{y} = 0.33\eta_1 + 19.69\eta_1\eta_2\eta_3, \quad (32)$$

$$C_{10}^{y} = 0.13\eta_1 + 1.39\eta_2. \quad (33)$$

For the range of values of the block coefficient $C_b \in (0.5; 0.7)$ the following model provides excellent correlation:

$$C_{10}^{y} = 1.12\eta_2 + 24.15\eta_1\eta_2\eta_3, \quad (34)$$

$$C_{10}^{y} = 1.27(1-\eta_1)\eta_4 + 28.2\eta_1\eta_2\eta_3, \quad (35)$$

$$C_{10}^{y} = 0.24\eta_1 + 34.02\eta_1\eta_2\eta_3. \quad (36)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$ the following models can be also used:

$$C_{10}^{y} = 1.61\eta_2 + 11.23\eta_1\eta_2\eta_3, \quad (37)$$

$$C_{10}^{y} = 2.75(1-\eta_1)\eta_4 + 47.4\eta_1\eta_2\eta_3, \quad (38)$$

$$C_{10}^{y} = 0.38\eta_1 + 10.87\eta_1\eta_2\eta_3. \quad (39)$$

Table 11 shows the correlation characteristics of models (30) - (39).

Table 11. Analysis of the model for C_{10}^{y}

		R	α_F	α_s	α_{mk}
(30)	*V21*	0.99	10^{-28}	10^{-5}	0.65
(31)		0.97	10^{-19}	10^{-3}	0.06
(32)		0.99	10^{-24}	10^{-4}	0.55
(33)		0.99	10^{-24}	0.1	0.32
(34)	*V22*	0.99	10^{-16}	10^{-4}	0.72
(35)		0.99	10^{-15}	10^{-3}	0.57
(36)		0.98	10^{-14}	10^{-4}	0.34
(37)	*V21*	0.99	10^{-9}	0.04	0.51
(38)		0.97	10^{-6}	0.04	0.38
(39)		0.99	10^{-10}	0.02	0.72

For the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ the following representations should be highlighted

$$C_{01}^{y} - m' - m'_x = -0.21\eta_1 - 4.76\eta_1\eta_2\eta_3, \quad (40)$$

$$C_{01}^{y} - m' - m'_x = -0.21\eta_1 - 0.8\eta_3, \quad (41)$$

$$C_{01}^{y} - m' - m'_x = -0.12\eta_1 - 0.54\eta_2. \quad (42)$$

For the range of values of the block coefficient $C_b \in (0.5; 0.7)$ the following model provides excellent correlation:

$$C_{01}^{y} - m' - m'_x = -0.16\eta_1 - 5.85\eta_1\eta_2\eta_3, \quad (43)$$

$$C_{01}^{y} - m' - m'_x = -0.09\eta_1 - 1.37\eta_3. \quad (44)$$

Table 12 shows the correlation characteristics of models (40) - (44).

Table 12. Analysis of the model for $C_{01}^{y} - m' - m'_x$

		R	α_F	α_s	α_{mk}
(40)	*V21*	0.97	10^{-18}	10^{-4}	0.04
(41)		0.96	10^{-16}	10^{-2}	0.12
(42)		0.96	10^{-17}	10^{-2}	0.07
(43)	*V22*	0.96	10^{-9}	10^{-4}	0.15
(44)		0.95	10^{-17}	0.04	0.21

For the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ the following representation should be highlighted:

$$C_{30}^{y} = 24.09(1-\eta_1)\eta_4 - 99.72\eta_1\eta_2\eta_3. \quad (45)$$

For the range of values of the block coefficient $C_b \in (0.5; 0.7)$ the following models provide excellent correlation:

$$C_{30}^{y} = 24.82(1-\eta_1)\eta_4 - 116.47\eta_1\eta_2\eta_3, \quad (46)$$

$$C_{30}^{y} = 6.19\eta_1 - 17.62\eta_3. \quad (47)$$

For the range of values of the coefficient of total completeness $C_b \in (0.7; 0.9)$ the following models can be also used:

$$C_{30}^{y} = 10.82(1-\eta_1)\eta_4 + 64.99\eta_1\eta_2\eta_3, \quad (48)$$

$$C_{30}^{y} = 22.2\eta_3, \quad (49)$$

$$C_{30}^{y} = 64.43\eta_1\eta_2\eta_3 - 166.62((1-\eta_1)\eta_4)^2. \quad (50)$$

Table 13 shows the correlation characteristics of models (48) - (52).

Table 13. Analysis of the model for C_{30}^{y}.

		R	α_F	α_s	α_{mk}
(53)	*V21*	0.84	10^{-7}	0.005	0.57
(54)	*V22*	0.84	10^{-4}	0.02	0.77
(55)		0.88	10^{-5}	0.04	0.21
(56)	*V23*	0.96	10^{-5}	0.081	0.38
(57)		0.96	10^{-6}	10^{-06}	-
(58)		0.98	10^{-7}	0.004	0.36

For the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ the following representation should be highlighted:

$$C_{21}^{y} = 3.26\eta_1 + 8.92\eta_1\eta_2 - 3.52, \quad (51)$$

$$C_{21}^{y} = 4.87\eta_1 + 5.14\eta_2 - 4.44. \quad (52)$$

For the range of values of the block coefficient $C_b \in (0.5; 0.7)$ the following models provide excellent correlation:

$$C_{21}^{y} = -3.12\eta_1 + 10.3\eta_1\eta_2, \quad (53)$$

$$C_{21}^{y} = -4.83\eta_1 + 9.77\eta_1\eta_4. \quad (54)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$ the following models can be also used:

$$C_{21}^{y} = -0.93\eta_1 + 7.15\eta_1\eta_2, \quad (55)$$

$$C_{21}^{y} = 3.74\eta_1\eta_2 - 4.27(1-\eta_1)\eta_4. \quad (56)$$

Table 14 shows the correlation characteristics of models (51) - (56).

Table 14. Analysis of the model for C_{21}^{y}.

		R	α_F	α_s	α_{mk}
(51)	*V21*	0.75	10^{-5}	0.003	0.46
(52)		0.75	10^{-5}	0.005	0.08
(53)	*V22*	0.78	10^{-2}	0.004	0.63
(54)		0.78	10^{-2}	0.003	0.63
(55)	*V23*	0.91	10^{-2}	0.07	0.72
(56)		0.91	10^{-4}	0.04	0.12

For the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ the following representation should be highlighted:

$$C_{12}^{y} = -6.96\eta_3 + 9.07(1-\eta_1)\eta_4. \quad (57)$$

$$C_{12}^{y} = 0.52\eta_1 + 16.7(1-\eta_1)^2\eta_4^2. \quad (58)$$

For the range of values of the block coefficient $C_b \in (0.5; 0.7)$ the following models provide excellent correlation:

$$C_{12}^{y} = 1.78\eta_1 - 1.47\eta_2, \quad (59)$$

$$C_{12}^{y} = 1.98\eta_1 - 3.55\eta_1\eta_2. \quad (60)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$ the following models can be also used:

$$C_{12}^{y} = 1.55\eta_1 - 4.95\eta_2, \quad (61)$$

$$C_{12}^{y} = 1.25\eta_1 - 4.69\eta_1\eta_2. \quad (62)$$

Table 15 shows the correlation characteristics of models (57) - (62).

Table 15. Analysis of the model for C_{12}^{y}.

		R	α_F	α_s	α_{mk}
(65)	*V21*	0.95	10^{-15}	10^{-4}	0.71
(66)		0.89	10^{-10}	10^{-4}	0.62
(67)	*V22*	0.96	10^{-9}	10^{-2}	0.28
(68)		0.96	10^{-9}	0.07	0.03
(69)	*V23*	0.97	10^{-6}	0.01	0.46
(70)		0.96	10^{-5}	0.03	0.72

For this hydrodynamic derivative it was possible to obtain the following models with satisfactory statistical characteristics:

$$C_{03}^{y} = -\begin{Bmatrix} 0.1 \\ 0.23 \\ 0.07 \end{Bmatrix}\eta_1 + \begin{Bmatrix} 0.22 \\ 0.49 \\ 0.22 \end{Bmatrix}\eta_1\eta_4. \quad (63)$$

$$C_{03}^{y} = -\begin{Bmatrix} 0.6 \\ 0.86 \\ 0.31 \end{Bmatrix}\eta_1\eta_4 + \begin{Bmatrix} 2.06 \\ 3.07 \\ 1.12 \end{Bmatrix}(\eta_1\eta_4)^2. \quad (64)$$

The upper lines in the dependencies (63) and (64) were obtained for the *V21* sample; the second and third ones were obtained for the *V22* and *V23* samples respectively.

Table 16 shows the correlation characteristics of models (63) - (64).

Table 16. Analysis of the model for C_{03}^{y}.

		R	α_F	α_s	α_{mk}
(63)	*V21*	0.54	$5\cdot10^{-3}$	0.09	0.4
	V22	0.74	$9\cdot10^{-4}$	0.02	0.63
	V23	0.62	0.08	0.05	0.28
(64)	*V21*	0.65	$3\cdot10^{-4}$	$5\cdot10^{-4}$	-
	V22	0.71	$3\cdot10^{-3}$	$6\cdot10^{-3}$	-
	V23	0.64	0.06	0.03	-

To construct the hydrodynamic derivatives of the moments we will use the samples *V31, V32, V33*.

The following models have excellent regression characteristics for the constant C_{10}^{m} for all samples:

$$C_{10}^{m} = \begin{Bmatrix} 2.15 \\ 1.91 \\ 2.64 \end{Bmatrix}\eta_1\eta_3, \quad (65)$$

$$C_{10}^{m} = \begin{Bmatrix} 0.08 \\ 0.09 \\ 0.11 \end{Bmatrix}\eta_1 + \begin{Bmatrix} 0.66 \\ 0.59 \\ 1.96 \end{Bmatrix}\eta_3 - \begin{Bmatrix} 0 \\ 0 \\ 0.08 \end{Bmatrix}. \quad (66)$$

Table 17 shows the correlation characteristics of models (65) and (66).

Table 17. Analysis of the model for C_{10}^m

		R	α_F	α_s	α_{mk}
(65)	*V31*	0.96	10^{-18}	10^{-18}	-
	V32	0.93	10^{-8}	10^{-9}	-
	V33	0.99	10^{-12}	10^{-12}	-
(66)	*V31*	0.96	10^{-16}	10^{-4}	0.12
	V32	0.95	10^{-8}	$3\cdot10^{-3}$	0.22
	V33	0.99	10^{-8}	0.04	0.62

For the constant $C_{01}^m - x'_G m'$ for the entire range of variation of the block coefficient $C_b \in (0.5;0.9)$ the following representations have excellent correlation characteristics:

$$C_{01}^m - x'_G m'_x = -0.06\eta_1, \tag{67}$$

$$C_{01}^m - x'_G m' = -0.13\eta_4, \tag{68}$$

$$C_{01}^m - x'_G m' = -0.37(1-\eta_1)\eta_4, \tag{69}$$

$$C_{01}^m - x'_G m' = -3.14(1-\eta_1)\eta_3 + 34.93(1-\eta_1)^2\eta_3^2. \tag{70}$$

For the range of values of the block coefficient $C_b \in (0.5;0.7)$ the following models provide good correlation:

$$C_{01}^m - x'_G m' = -0.08\eta_1, \tag{71}$$

$$C_{01}^m - x'_G m' = -2.92(1-\eta_1)\eta_3 + 31.62(1-\eta_1)^2\eta_3^2. \tag{72}$$

For the range of values of the block coefficient $C_b \in (0.7;0.9)$ the following dependence can be also used:

$$C_{01}^m - x'_G m' = -0.05\eta_1. \tag{73}$$

Table 18 shows the correlation characteristics of the models (67) - (73).

Table 18. Analysis of the model for $C_{01}^m - x'_G m'$.

		R	α_F	α_s	α_{mk}
(67)	*V21*	0.86	10^{-9}	0.08	-
(68)		0.85	10^{-9}	10^{-9}	-
(69)		0.85	10^{-9}	10^{-9}	-
(70)		0.88	10^{-10}	10^{-5}	-
(71)	*V22*	0.97	10^{-12}	10^{-12}	-
(72)		0.98	10^{-9}	10^{-9}	-
(73)	*V23*	0.75	10^{-2}	10^{-2}	-

For the constant C_{30}^m for the entire range of variation of the block coefficient $C_b \in (0.5;0.9)$ and for values $C_b \in (0.5;0.7)$ the following models have good correlation features (the upper coefficients are the sample *V31*, the lower ones are *V32*):

$$C_{30}^m = \begin{Bmatrix} 2.27 \\ 2.45 \end{Bmatrix}(1-\eta_1)\eta_2, \tag{74}$$

$$C_{30}^m = \begin{Bmatrix} 1.33 \\ 1.42 \end{Bmatrix}(1-\eta_1)\eta_4, \tag{75}$$

$$C_{30}^m = \begin{Bmatrix} 5.68 \\ 5.83 \end{Bmatrix}(1-\eta_1)\eta_3, \tag{76}$$

$$C_{30}^m = \begin{Bmatrix} 1.83 \\ 3.33 \end{Bmatrix}\frac{(1-\eta_1)}{\eta_2\eta_4^{-1}} - \begin{Bmatrix} 1.59 \\ 2.6 \end{Bmatrix}\frac{(1-\eta_1)^2}{\eta_2^2\eta_4^{-2}} - \begin{Bmatrix} 0.48 \\ 1.37 \end{Bmatrix}\eta_1. \tag{77}$$

Table 19 shows the correlation characteristics of models (74) - (77).

Table 19. Analysis of the model for C_{30}^m.

		R	α_F	α_s	α_{mk}
(74)	*V21*	0.60	$2\cdot10^{-4}$	$2\cdot10^{-4}$	-
	V22	0.63	$3\cdot10^{-3}$	$3\cdot10^{-3}$	-
(75)	*V21*	0.55	$8\cdot10^{-4}$	$8\cdot10^{-4}$	-
	V22	0.57	$8\cdot10^{-3}$	$8\cdot10^{-3}$	-
(76)	*V21*	0.55	$9\cdot10^{-4}$	$9\cdot10^{-4}$	-
	V22	0.56	$9\cdot10^{-3}$	$9\cdot10^{-3}$	-
(77)	*V21*	0.64	10^{-3}	0.02	0.31
	V22	0.73	$5\cdot10^{-3}$	0.08	0.31

For the constant C_{12}^m for the entire range of variation of the block coefficient $C_b \in (0.5;0.9)$ the following representations have excellent correlation characteristics:

$$C_{21}^m = -3.33(1-\eta_1)\eta_4, \tag{78}$$

$$C_{21}^m = -4.14(1-\eta_1)\eta_4 + 9.8\eta_1\eta_2\eta_3. \tag{79}$$

For the range of values of the block coefficient $C_b \in (0.5;0.7)$ the following models provide good correlation:

$$C_{21}^m = -3.45(1-\eta_1)\eta_4, \tag{80}$$

$$C_{21}^m = -0.83\eta_1, \tag{81}$$

$$C_{21}^m = -0.59\frac{\eta_4}{\eta_2} + 0.17\frac{\eta_4^2}{\eta_2^2}. \tag{82}$$

For the range of values of the block coefficient $C_b \in (0.7;0.9)$ the following dependences should be also highlighted:

$$C_{21}^m = -0.2\eta_1, \tag{83}$$

$$C_{21}^m = -0.59\frac{\eta_4}{\eta_2} + 0.17\frac{\eta_4^2}{\eta_2^2}. \tag{84}$$

Table 20 shows the correlation characteristics of models (78) - (85).

Table 20. Analysis of the model for C_{21}^m.

		R	α_F	α_s	α_{mk}
(78)	*V21*	0.94	10^{-14}	10^{-15}	-
(79)		0.95	10^{-15}	$5\cdot10^{-3}$	0.57
(80)	*V22*	0.94	10^{-9}	10^{-9}	-
(81)		0.95	10^{-9}	10^{-9}	-
(82)		0.96	10^{-9}	0.02	0.77
(83)		0.97	10^{-10}	10^{-3}	-
(84)	*V23*	0.97	10^{-7}	10^{-7}	-
(85)		0.95	10^{-5}	0.01	-

For the constant C_{12}^m for the entire range of variation of the block coefficient $C_b \in (0.5; 0.9)$ and for the range $C_b \in (0.5; 0.7)$ the following model has satisfactory correlation characteristics:

$$C_{12}^m = \begin{Bmatrix} 0.76 \\ 1 \end{Bmatrix} (1-\eta_1)\eta_4 - \begin{Bmatrix} 0.34 \\ 0.51 \end{Bmatrix} \eta_1\eta_4. \quad (85)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$ the following dependence provide a satisfactory correlation:

$$C_{12}^m = -0.44\eta_1 + 2.26\eta_1\eta_4. \quad (86)$$

Table 21 shows the correlation characteristics of models (85), (86).

Table 21. Analysis of the model for C_{12}^m.

		R	α_F	α_s	α_{mk}
(85)	*V21*	0.5	$6\cdot10^{-3}$	$2\cdot10^{-2}$	0.3
	V22	0.54	$2\cdot10^{-2}$	0.2	0.72
(86)	*V23*	0.66	0.05	0.04	0.74

For the constant C_{03}^m for the entire range of variation of the block coefficient: $C_b \in (0.5; 0.9)$ the following representations have excellent correlation characteristics:

$$C_{03}^m = -0.35(1-\eta_1)\eta_4 + 1.47\eta_1\eta_2\eta_3, \quad (87)$$

$$C_{03}^m = -0.03\frac{\eta_4}{\eta_2} + 0.01\frac{\eta_4^2}{\eta_2^2}. \quad (88)$$

For the range of values of the block coefficient $C_b \in (0.5; 0.7)$ the following models provide good correlation:

$$C_{03}^m = -0.1\eta_1 + 0.27\eta_1\eta_2, \quad (89)$$

$$C_{03}^m = -0.05\frac{\eta_4}{\eta_2} + 0.02\frac{\eta_4^2}{\eta_2^2}, \quad (90)$$

$$C_{03}^m = -0.33(1-\eta_1)\eta_4 + 1.69\eta_1\eta_2\eta_3. \quad (91)$$

For the range of values of the block coefficient $C_b \in (0.7; 0.9)$ the following dependences should be also highlighted:

$$C_{03}^m = -0.1(1-\eta_1)\eta_4 - 0.96\eta_1\eta_2\eta_3, \quad (92)$$

$$C_{03}^m = -0.01\frac{\eta_4}{\eta_2}. \quad (93)$$

Table 22 shows the correlation characteristics of models (89) - (93).

Table 22. Analysis of the model for C_{03}^m.

		R	α_F	α_s	α_{mk}
(89)	V21	0.91	10^{-11}	10^{-4}	0.57
(90)		0.86	10^{-8}	10^{-3}	-
(91)	V22	0.93	10^{-7}	10^{-3}	0.31
(92)		0.93	10^{-7}	10^{-4}	-
(93)		0.93	10^{-7}	10^{-4}	0.77
(94)	V23	0.92	10^{-4}	0.2	0.38
(95)		0.92	10^{-5}	10^{-5}	-

6 CONCLUSIONS

The results shown in Tables 7-21 confirm that almost all the new models of hydrodynamic forces and moment on the hull which have been obtained, in contrast to the existing ones, establish a high degree of correlation with an excellent level of significance of the connection with regressors. The fact that there are several adequate models that meet criteria 1) - 7) for each hydrodynamic derivative allows to choose the optimal model. If the manoeuvre for vessels with a wide range of changes in the values of the block coefficient $C_b \in (0,5; 0,9)$ is studied, it is necessary to use models are based on the samples *V11, V21, V31*. For narrower ranges of change C_b, it is advisable to use models that are based on the samples *V12, V21, V32* or *V13, V23, V33*.

The suggested approach allows to obtain new adequate mathematical models of other non-inertial forces on the hull, which will allow to build more accurate mathematical models of the dynamics of the ship's propulsion complex.

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