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ANALYSIS OF THE EFFECTS OF ELECTROMAGNETIC FORCES ON THE RELATIVE MOTION OF A CHARGED SPACECRAFT FORMATION FLYING

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ABSTRACT. In recent years, studying Lorentz's force has become a possible good means to control the spacecraft to reduce the fuel cost by modulating spacecraft electrostatic charge (magnetic and electric fields). The generation of Lorentz force is finite by the natural magnetic field and the relative velocity of the spacecraft. Therefore, the Lorentz force cannot fully occur from conventional propulsion technologies. Previous studies are concerned with studying Lorentz's strength in the magnetic field only.

In this work, we developed a mathematical model for a new technique establishing a raise in the level of charging in the spacecraft surface that is moving in the Earth's magnetic field and provided by modulating spacecraft's electrostatic charge that induces acceleration via the Lorentz force. The acceleration will be used to find the relationship between capacitance and power required to minimize the consumption of control energy used in such cases or to replace the usual control thruster by Lorentz force.

Keywords: electromagnetic forces, formation flying, relative motion, a charged spacecraft

1. INTRODUCTION

Spacecraft charging is occurring in the space plasma when the spacecraft itself becomes the moving charged particle, creating a current along its orbital path; this is achieved by using electrical power to maintain an electrostatic charge on its body, which causes an interaction between the magnetic field and the vehicle in the form of the Lorentz force. The equations of motion that have been used widely in this field are known either as the Clohessy-Wiltshire equations (Clohessy, 1960) or the Hill equations (Hill, 1878).

Hill (1878) developed equations governing orbital perturbations concerning an unperturbed reference orbit and obtained an analytical solution for relative motion equations to study lunar motion. Clohessy and Wiltshire (1960) used constant-coefficient linear ordinary differential equations to describe the relative motion between spacecraft in the context of spacecraft rendezvous. These equations describe the motion of a deputy spacecraft concerning a chief spacecraft moving in a circular reference orbit, and without perturbation forces. Kechichian

(1998) derived the second-order nonlinear differential equations, which describe the spacecraft's relative motion that was placed in an elliptic orbit. These equations factor in the effects of both second zonal harmonic and atmospheric drag perturbations in a general elliptic orbit. Melton (2000) derived a state transition matrix for relative motion concerning an elliptical orbit of reference and used the nonlinear equations of motion up to second-order eccentricity. Early studies of spacecraft charging conclude that the natural spacecraft charging level may reach about 10^{-8} C/kg (Vokrouhlicky, 1989) Lorentz force with such charging level is insufficient to correct the drift in relative position due to perturb the orbit of satellite significantly. The charge to mass ratio required about 10^{-5} C/kg Lorentz force for orbital maneuvering in low Earth orbit (Pollock et al. 2011). Peck (2005) proposed a new concept of active application of charge when a spacecraft is introduced for artificial charging, which is referred to as Lorentz spacecraft. This mechanism generates a net charge on the surface of the spacecraft to induce Lorentz force via interaction with the Earth's magnetic field. Abdel-Aziz (2007) developed the variation in orbital elements of the satellite motion, under the effect of Lorentz force of a charged satellite in Earth's magnetic field, and used Lagrange planetary equations to derive periodic perturbations in the orbital elements of the Satellite Pollock et al. (2011) studied the charged spacecraft's relative motion under the Lorentz force perturbations because of the interactions with the planetary magnetosphere. Tsujii et al. (2013) derived a mathematical model of a charged satellite, considering the Lorentz force's effect, the applications applied on two elliptical and circular cases of formation flying satellite orbit. Huang et al. (2014) used the line-of-sight observations and gyro measurements to the developed orbital motion of Lorentz spacecraft for inclined low Earth orbit. Abdel-Aziz and Khalil (2014) studied the analytical expressions for the orbital motion of Lorentz spacecraft for inclined low Earth orbit. Abdel-Aziz and Shoaib (2015) studied the attitude dynamics of spacecraft, they studied the stability of the attitude orientation, and the regions of stability for various values of charge to mass ratio, their results confirm that the charge to mass ratio can be used as a semi-passive control for attitude. Peng and Gao (2017) investigated the periodic orbits under inter-satellite Lorentz force, and used a nonlinear dynamical model for the proposed relative motion, by the assumption that the first satellite generates a rotating magnetic dipole, while a constantly charging second satellite moves close to the artificial magnetic field of the chief satellite. Vepa (2018) developed nonlinear equations in terms of the varying true anomaly Tschauner-Hempel equations relative to a notional orbiting particle in a Keplerian orbit. In the present paper, the main idea is to install a small device (Ion collector) to increase the level of charging in the spacecraft surface.

If we get that the charging level is several orders of magnitude larger than the natural charging level, the induced Lorentz force could be electromagnetic propulsion to correct the drift in relative position due to perturb the orbit of the satellite. We developed nonlinear equations of motion using a numerical integrator 8th-order Runge-Kutta method to obtain the optimal charge to mass ratio q/m (C/kg) that can be used to keep the desired relative distances. The numerical results reviewed the effect of the Lorentz magnetic and electric forces for different values of charge to mass ratio.

2. NONLINEAR EQUATIONS OF RELATIVE MOTION

Suppose that we have two formation flying satellites, the first is the chief and the second is the deputy. The Hill-Clohessy-Wiltshire (C-W) formula is used for formation flying applications to determine the relative position and velocity of the deputy concerning the chief satellites.

In this section, we develop the relative motion equations using a Cartesian local-vertical local-horizontal (LVLH) coordinate frame attached to the chief satellite (see Figure1). The coordinate system (x, y, z), the origin is at the centroid of the chief satellite, the x-axis (radial direction) is directed from the center of the Earth towards the chief satellite, and the z-axis (cross-track) lies in the chief's orbital angular momentum direction, and the y-axis (along-track) completes the right-handed orthogonal triad.

The relation between the Earth-Centered inertial (ECI) frame $(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})$ and a local-vertical local-horizontal (LVLH) frame $(x, y, z, \dot{x}, \dot{y}, \dot{z})$ is given as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = Q_{Xx} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad Q_{Xx} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \cos i \sin\theta & \cos i \cos\theta & -\sin i \\ \sin i \sin\theta & \cos i \cos\theta & \cos i \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \dot{Q}_{Xx} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}, \quad \dot{Q}_{Xx} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \cos i \sin\theta & \cos i \cos\theta & -\sin i \\ \sin i \cos\theta & \sin i \sin\theta & \cos i \end{bmatrix} \tag{2}$$

where θ, i are the true anomaly and inclination of the chief satellite, respectively.

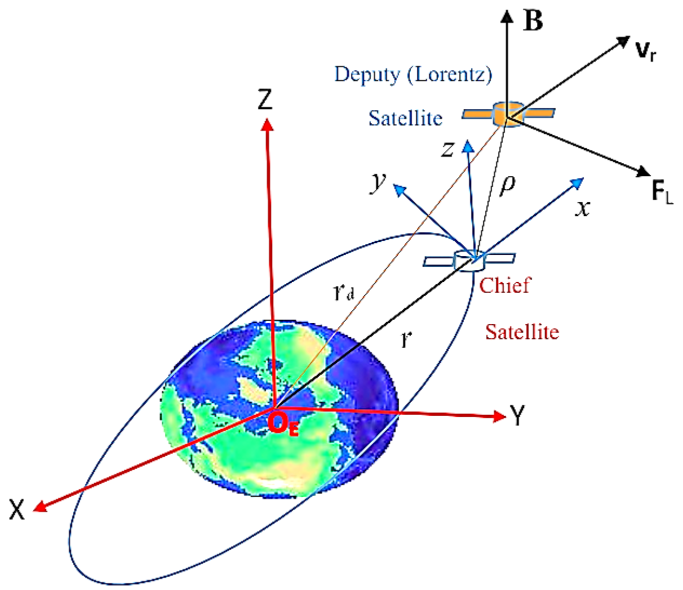

Figure 1. Local-vertical local-horizontal and Earth-Centered inertial coordinates

The position of deputy satellite is given by the following equation (Bakhtiari et al., 2017):

$$\mathbf{r}_d = \mathbf{r} + \boldsymbol{\rho} = (r + x)\hat{i} + y\hat{j} + z\hat{k} \tag{3}$$

where $\mathbf{r}_d$ $\mathbf{r}$ are the position vectors for the deputy and chief satellites and $\boldsymbol{\rho}$ is the relative position vector between the deputy and the chief satellites.

The angular rotational velocity vector $(\dot{\boldsymbol{\theta}})$ of the LVLH frame is as follows (Curtis, 2013)

$$\dot{\boldsymbol{\theta}} = \frac{\mathbf{r} \times \dot{\mathbf{r}}}{r^2}, \dot{\boldsymbol{\theta}} = [0 \quad 0 \quad \dot{\theta}]^T \tag{4}$$

where $\dot{\mathbf{r}}$ is the velocity vector for the chief satellite.

The angular acceleration vector $\left(\ddot{\boldsymbol{\theta}}\right)$ of the LVLH frame may be written as:

$$\ddot{\boldsymbol{\theta}} = -2\frac{\mathbf{r}.\dot{\mathbf{r}}}{r^2}\dot{\boldsymbol{\theta}} \tag{5}$$

The equation of motion for the deputy in the chief's frame is giving by the following equation:

$$\ddot{\mathbf{r}}_d = \ddot{\mathbf{r}} - \ddot{\boldsymbol{\theta}} \times \boldsymbol{\rho} - \dot{\boldsymbol{\theta}} \times \left(\dot{\boldsymbol{\theta}} \times \boldsymbol{\rho}\right) - 2\dot{\boldsymbol{\theta}} \times \dot{\boldsymbol{\rho}} - \ddot{\boldsymbol{\rho}} - \mathbf{a}_m - \mathbf{a}_e \tag{6}$$

where $\mathbf{a}_m$ and $\mathbf{a}_e$ are acceleration vectors for Lorentz geomagnetic and electric fields.

From Newton's second law and the law of universal gravitation, the chief's acceleration is given by:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r},\ \mathbf{r} = r\hat{\mathbf{i}} \tag{7}$$

where μ is the gravitational parameter.

Substitute (7) into (6):

$$\ddot{\mathbf{r}}_d = \left(-\frac{\mu}{r^2} - \ddot{\theta}y - \dot{\theta}^2 x - 2\dot{y}\dot{\theta} + \ddot{x} - a_{mx} - a_{ex}\right)\hat{\mathbf{i}} + \left(\ddot{\theta}x - \dot{\theta}^2 y + 2\dot{x}\dot{\theta} + \ddot{y} - a_{my} - a_{ey}\right)\hat{\mathbf{j}} + \left(\ddot{z} - a_{mz} - a_{ez}\right)\hat{\mathbf{k}} \tag{8}$$

Similarly, the acceleration of the deputy can be written from Equation (5) as the follows:

$$\ddot{\mathbf{r}}_d = -\frac{\mu}{r_d^3}\mathbf{r}_d,\ \mathbf{r}_d = \left[\left(x+r\right)\ y\ z\right]^T \tag{9}$$

The full nonlinear equations of relative motion by a substitute (7) into (6):

$$\begin{aligned}
\ddot{x} &= 2\dot{y}\dot{\theta} + \ddot{\theta}y + \dot{\theta}^2 x + \frac{\mu}{r^2} - \frac{\mu}{r_d^3}\left(x+r\right) + a_{mx} + a_{ex} \\
\ddot{y} &= -2\dot{x}\dot{\theta} - \ddot{\theta}x + \dot{\theta}^2 y - \frac{\mu}{r_d^3}y + a_{my} + a_{ey} \\
\ddot{z} &= -\frac{\mu}{r_d^3}z + a_{mz} + a_{ez}
\end{aligned} \tag{10}$$

3. ELECTROMAGNETIC FORCE (LORENTZ FORCE)

The Lorentz force induced by an ambient space plasma and magnetic field **B** on charged particles with a velocity $\mathbf{v}_r$ concerning that field (see Figure 1) can be written as (Huang et al., 2015)

$$\mathbf{F}_L = \mathbf{F}_m + \mathbf{F}_e = q\left(\mathbf{v_r} \times \mathbf{B}\right) + q\mathbf{E} = q\left[\mathbf{v_r} \times \mathbf{B} + \mathbf{E}\right] \tag{11}$$

Magnetic force ($\mathbf{F}_m$) is always orthogonal to the magnetic field, that is experienced by a particle moving through a magnetic field **B** with total electrostatic charge q (Coulombs); however, the electric force ($\mathbf{F}_e$) acts on a charged particle in the direction of the electric field, whether or not it is moving, and **E** is an electric field. Table 1 shows an estimate of the specific charge feasibility for Lorentz spacecraft (Pollock, 2010).

Table 1. Estimated Specific Charge Feasibility (Pollock, 2010)

Specific Charge, C/kg	Feasibility Assessment
10^{-8}	Natural spacecraft charge (peak, 2005)
10^{-4}	Near-term feasible
10^{-3}–10^{-2}	Possible with concerted development
10^{-1}	Very challenging
1	Futuristic
10^{11}	Upper limit: a single electron

3.1. Lorentz force experience with the magnetic field

Assume that the magnetic dipole is not tilted such that the field lines near the equatorial plane are in the $+\hat{z}$ direction (see figure 2). The magnetic field can be expressed as (Ulaby and Ravaioli, 2015)

$$\mathbf{B} = \begin{bmatrix} B_r & B_\Theta & B_\phi \end{bmatrix}^T = \frac{B_0}{r^3}\left[2\cos\Theta\hat{\mathbf{r}} + \sin\Theta\hat{\Theta} + 0\hat{\phi}\right] \tag{12}$$

where B_0 is the magnetic dipole moment of Earth (8×10^{15} T m^3).

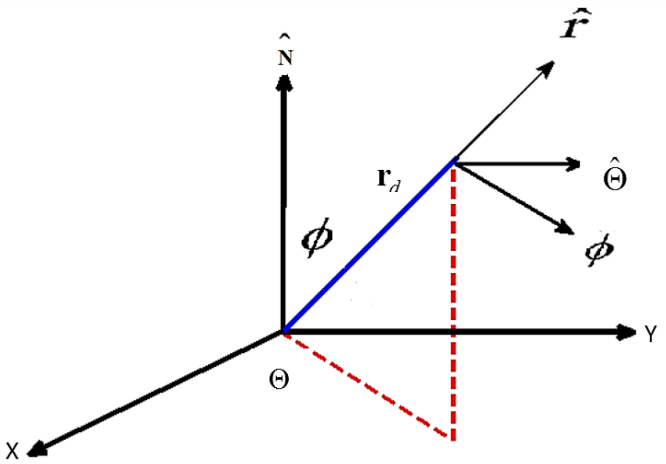

Figure 2. Spherical coordinates at the magnetic dipole are not tilted

The magnetic field in the Earth-Centered inertial coordinates (XYZ) is given by:

$$\mathbf{B} = \begin{bmatrix} B_x & B_y & B_z \end{bmatrix}^T = \frac{B_0}{r^5}\begin{bmatrix} 3XZ \\ 3YZ \\ 2Z^2 - X^2 - Y^2 \end{bmatrix} \tag{13}$$

The acceleration due to the Lorentz force that is experienced with the magnetic field is given by:

$$\mathbf{a}_m = \frac{\mathbf{F}_m}{m} = \frac{q}{m}\left(\mathbf{v_r} - \omega_E\hat{\mathbf{N}} \times \mathbf{r}\right) \times \mathbf{B} \tag{14}$$

where $\frac{q}{m}$ is the charge-to-mass ratio of the satellite in Coulombs per kilogram (C/kg), $\hat{\mathrm{N}}$ is a unit vector in the direction of the true north pole and ω_E is Earth's rotation rate.

The relative velocity can be written in local-vertical local-horizontal (LVLH) frame as:

$$\mathbf{v_r} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \dot{r} + \dot{x} - y\left(\dot{\theta}_z - \omega_E \cos i\right) - z\omega_E \cos\theta \sin i \\ \dot{y} - (r+x)\left(\dot{\theta}_z - \omega_E \cos i\right) + z\omega_E \sin\theta \sin i \\ \dot{z} - (r+x)\omega_E \cos\theta \sin i + y\omega_E \sin\theta \sin i \end{bmatrix} \tag{15}$$

Therefore, by substituting Eq. (1) and Eq. (2) into Eq. (13) and Eq. (15) and Eq. (13) into Eq. (14), the expressions of Lorentz magnetic acceleration in relative motion frame can be written as:

$$a_{mx} = \frac{qB_0}{mr^5}\begin{bmatrix} \left(\dot{y} - (r+x)\left(\dot{\theta} - \omega_E \cos i\right) + z\,\omega_E \sin\theta \sin i\right)\left(2z^2 - \left(x^2 + y^2\right)\right) - \\ \left(\dot{z} - (r+x)\omega_E \cos\theta \sin i + y\,\omega_E \sin\theta \sin i\right)3yz \end{bmatrix}$$

$$a_{my} = \frac{qB_0}{mr^5}\begin{bmatrix} \left(\dot{z} - (r+x)\omega_E \cos\theta \sin i + y\,\omega_E \sin\theta \sin i\right)3xz - \\ \left(\dot{r} + \dot{x} - y\left(\dot{\theta} - \omega_E \cos i\right) - z\,\omega_E \cos\theta \sin i\right)\left(2z^2 - \left(x^2 + y^2\right)\right) \end{bmatrix} \tag{16}$$

$$a_{mz} = \frac{qB_0}{mr^5}\begin{bmatrix} \left(\dot{r} + \dot{x} - y\left(\dot{\theta} - \omega_E \cos i\right) - z\,\omega_E \cos\theta \sin i\right)3yz - \\ \left(\dot{y} - (r+x)\left(\dot{\theta} - \omega_E \cos i\right) + z\,\omega_E \sin\theta \sin i\right)3xz \end{bmatrix}$$

In case the magnetic dipole field is tilted concerning Earth's rotation axis (Figure 3), we can write Lorentz accelerations from Eq. (13); however, the magnetic field (**B**) has the form as shown below (Pollock et al., 2011):

$$\mathbf{B} = \begin{bmatrix} B_x & B_y & B_z \end{bmatrix} = \frac{B_0}{r_d^3}\left[3\left(\hat{\mathbf{n}}.\hat{\mathbf{r}}_d\right)\hat{\mathbf{r}}_d - \hat{\mathbf{n}}\right] \tag{17}$$

where, $\hat{\mathbf{n}}$ is the unit vector in the direction of the magnetic dipole moment, and $\hat{\mathbf{r}}_d = \frac{1}{r_d}\left[r + x \quad y \quad z\right]^T$.

The expression of the magnetic dipole unit vector ($\hat{\mathbf{n}}$) in the LVLH frame is (Ulaby and Ravaioli, 2015):

$$\hat{\mathbf{n}} = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T = \begin{bmatrix} -(\cos\varepsilon\cos\theta + \sin\varepsilon\cos i \sin i)\sin\alpha - \sin i \sin\theta\cos\alpha \\ (\cos\varepsilon\sin\theta - \sin\varepsilon\cos i\cos\theta)\sin\alpha - \sin i\cos\theta\cos\alpha \\ \sin\varepsilon\sin i\sin\alpha - \cos i\cos\alpha \end{bmatrix} \tag{18}$$

where α is the tilt of the dipole angle between $\hat{Z}$ and $\hat{\mathbf{n}}$ for deputy satellite.

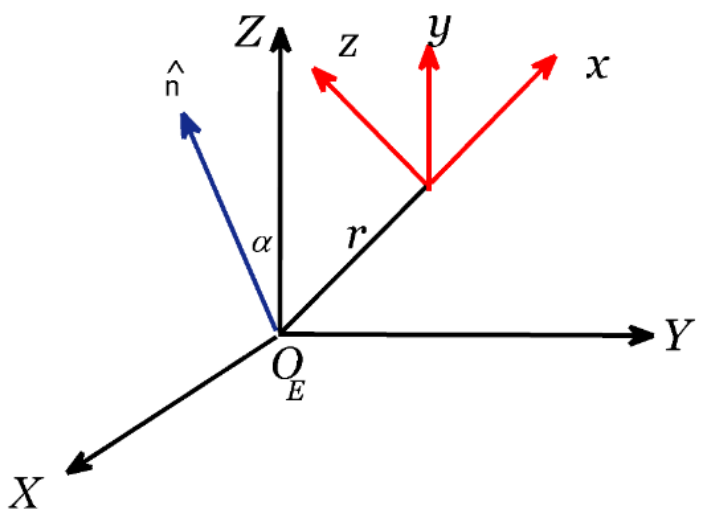

Figure 3. Earth magnetic with dipole angle (α)

The angle $\varepsilon = \Omega_m - \Omega$, with $\Omega_m = \omega_E\, t + \Omega_0$ is the inertial rotational angle of the magnetic dipole, with Ω being the right ascension of the ascending node of the chief, and Ω_0 is the initial rotation angle of the dipole.

The expressions of Lorentz magnetic acceleration in relative motion in the case of the tilt dipole magnetic can be derived as shown below:

$$\mathbf{a}_m = \frac{q}{m}\begin{bmatrix} a_{mx} \\ a_{my} \\ a_{mz} \end{bmatrix} = \frac{q}{m}\begin{bmatrix} v_x B_z - v_z B_y \\ v_z B_x - v_x B_z \\ v_x B_y - v_y B_x \end{bmatrix} \tag{19}$$

3.2. Lorentz force experience with the Electric field

The charge distribution in a material is discrete, that is, charge exists only where electrons and ions are and nowhere else (Ulaby, 2005). In case of differential charging on satellite surface, suppose two point charges of equal magnitude but opposite polarity, separated by a distance d, the electric dipole is consisted; so the electric potential V_e at any point P will be determined by applying the following equation:

$$V_e = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} + \frac{-q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right) \tag{20}$$

where $\varepsilon_0 = 8.85\times 10^{-12}\, C^2 / \left(N - m^2\right)$ is the permittivity of free space and $|r - r_i|$ is the distance between the observation point and the location of the charge q Coulombs.

Since d << r, in which case, the following approximations apply: $r_2 - r_1 \approx d\, \cos\Theta, r_2 r_1 \approx r^2$ (Huang et al., 2015):

$$V_e = \frac{qd\, \cos\Theta}{4\pi\varepsilon_0 r^2} \tag{21}$$

where d is the distance vector from the charge (+ q) to charge (- q) (Huang et al., 2015).

In spherical coordinates, we can write the electric force as mentioned below:

$$\mathbf{F}_e = -\nabla V_e = -\left(\frac{\partial v_e}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial v_e}{\partial \Theta}\hat{\Theta} + \frac{1}{r\sin\phi}\frac{\partial v_e}{\partial \phi}\hat{\phi} \right) \tag{22}$$

The Lorentz force experienced by an electric dipole moment in the presence of an electric field is:

$$\begin{aligned}\mathbf{F}_e &= \frac{qd}{4\pi\varepsilon_0 r^3}\left[2\cos\Theta\hat{\mathbf{r}} + \sin\Theta\hat{\Theta} + 0\hat{\phi}\right] \\ &= \frac{qd}{4\pi\varepsilon_0 r^3}\begin{bmatrix} \sin\Theta\cos\phi & \cos\Theta\cos\phi & -\sin\phi \\ \sin\Theta\sin\phi & \cos\Theta\sin\phi & \cos\Theta \\ \cos\Theta & -\sin\Theta & 0 \end{bmatrix}\begin{bmatrix} 2\cos\Theta \\ \sin\Theta \\ 0 \end{bmatrix}\end{aligned} \tag{23}$$

$$\mathbf{F}_e = \frac{qd}{4\pi\varepsilon_0 r^5}\begin{bmatrix} 3XZ \\ 3YZ \\ 2Z^2 - X^2 - Y^2 \end{bmatrix} \tag{24}$$

The Lorentz acceleration Expand by the electric field can be derived as:

$$\mathbf{a}_e = \begin{bmatrix} a_{ex} & a_{ey} & a_{ez} \end{bmatrix}^T = \frac{\mathbf{F}_e}{m} = \frac{q}{m}\frac{d}{4\pi\varepsilon_0 r^4}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{25}$$

$$\mathbf{a}_e = \begin{bmatrix} a_{ex} \\ a_{ey} \\ a_{ez} \end{bmatrix} = $$

$$\frac{q}{m}\frac{d}{4\pi\varepsilon_0 r^4}\begin{bmatrix} q_1(x^2 - y^2) + q_2 xy + xz\cos\theta\cos i - zy\sin\theta\cos i \\ \sin^2\theta + y^2\cos^2\theta - z^2 + 2yx\sin\theta\cos\theta + q_3(zy\cos\theta + zx\sin\theta) \\ -q_4 x^2 - q_5 y^2 - q_6 z^2 + 6q_7 zx + 6q_8 xy + 6q_9 zy \end{bmatrix} \tag{26}$$

where:

$$q_1 = \cos\theta\sin\theta\sin i, \quad q_2 = \sin i(\cos^2\theta - \sin^2\theta), \quad q_3 = (1 - 2\sin^2 i)\sin i\cos i,$$
$$q_6 = (1 - \cos^2 i) \quad , \quad q_5 = 1 - \cos^2\theta(1 + 4\sin^2 i), \quad q_4 = (1 - \cos^2\theta\sin^2 i),$$
$$q_7 = \sin\theta\cos i\sin i, \quad q_8 = \cos\theta\sin\theta\sin^2 i \quad \textit{and} \quad q_9 = \cos\theta\cos i\sin i$$

4. NUMERICAL RESULTS

In this section, we discuss the numerical simulations for verifying the effect of different acceleration on a relative position between two satellites due to Lorentz acceleration Expand by magnetic field (non-tilted dipole magnetic) Equations (16), tilted dipole magnetic field Equation (19) and Lorentz acceleration Expand by an electric field (26). We can apply those equations to get the perturbation in the separate magnetic and electric components of the Lorentz force. These numerical simulations were performed using MATLAB©. The nonlinear differential equations of motion were solved using the 8th order Runge-Kutta method. We applied our models for formation satellites. Assuming initial values of position and velocity chief satellite are ***r*** = [0 -7163.61171 0] km, v = [2.053396 0 -7.170630] km/s and deputy satellite $\mathbf{r}_d$ = [-0 -7163.51 0] km , v_d = [2.053596 -0.000000 -7.170668] km/s.

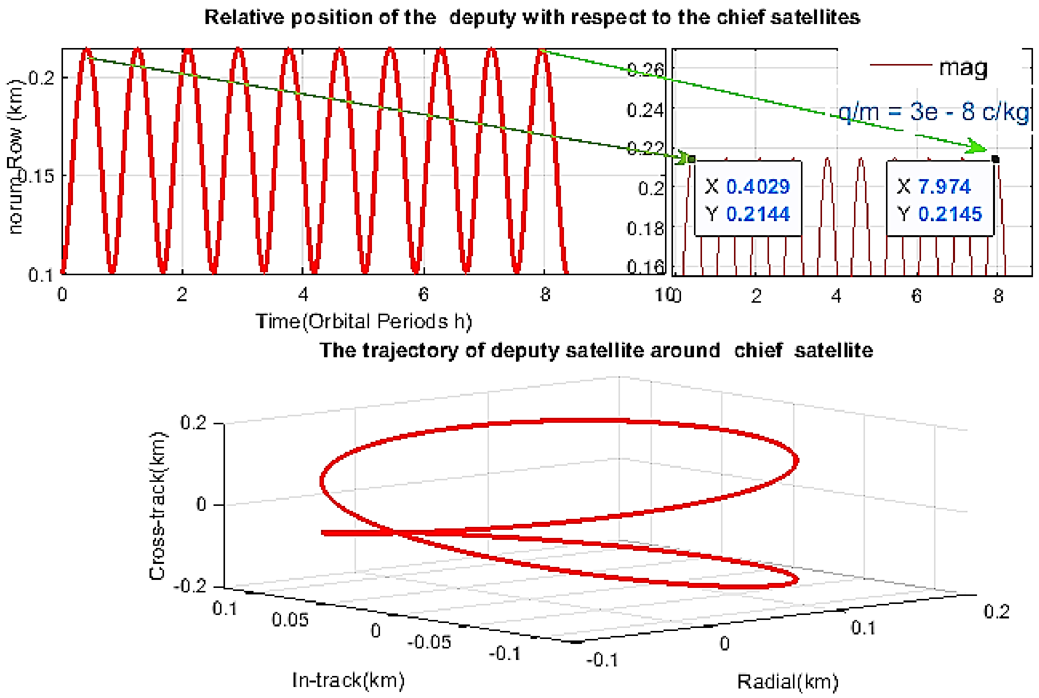

Figure 4. Relative position and trajectory for magnetic field at q/m = 3e-8 C/kg after 5 periods

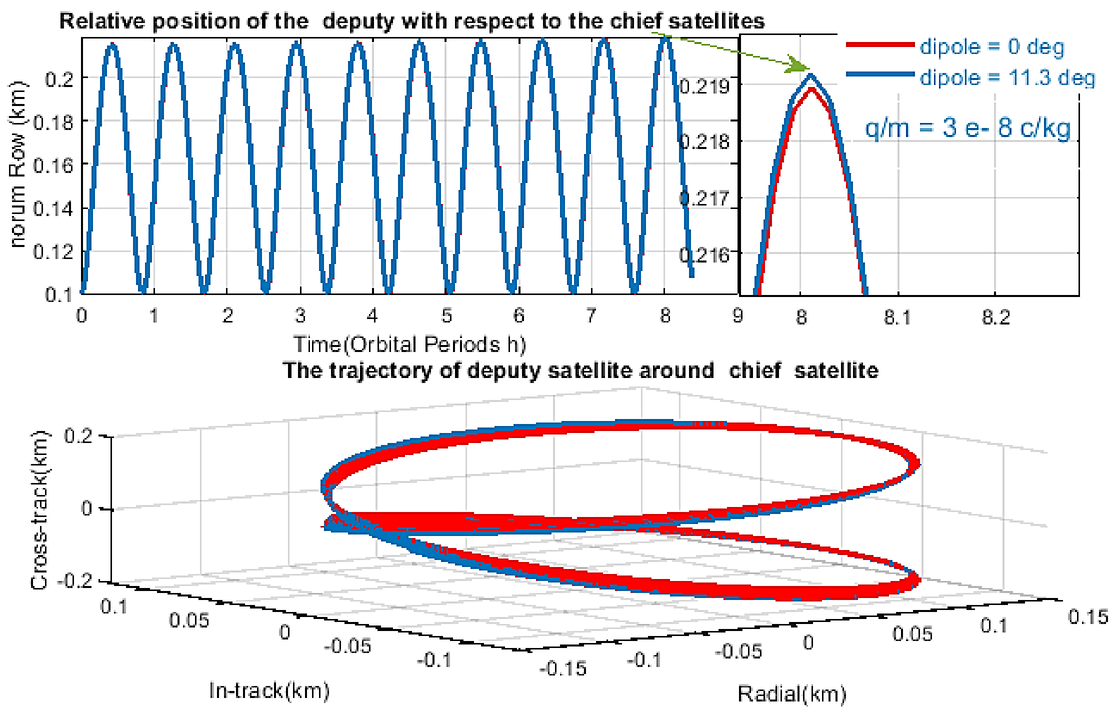

Figure 5. Relative position and trajectory considering the dipole magnetic angle ($\alpha = 0° - 11.3°$)

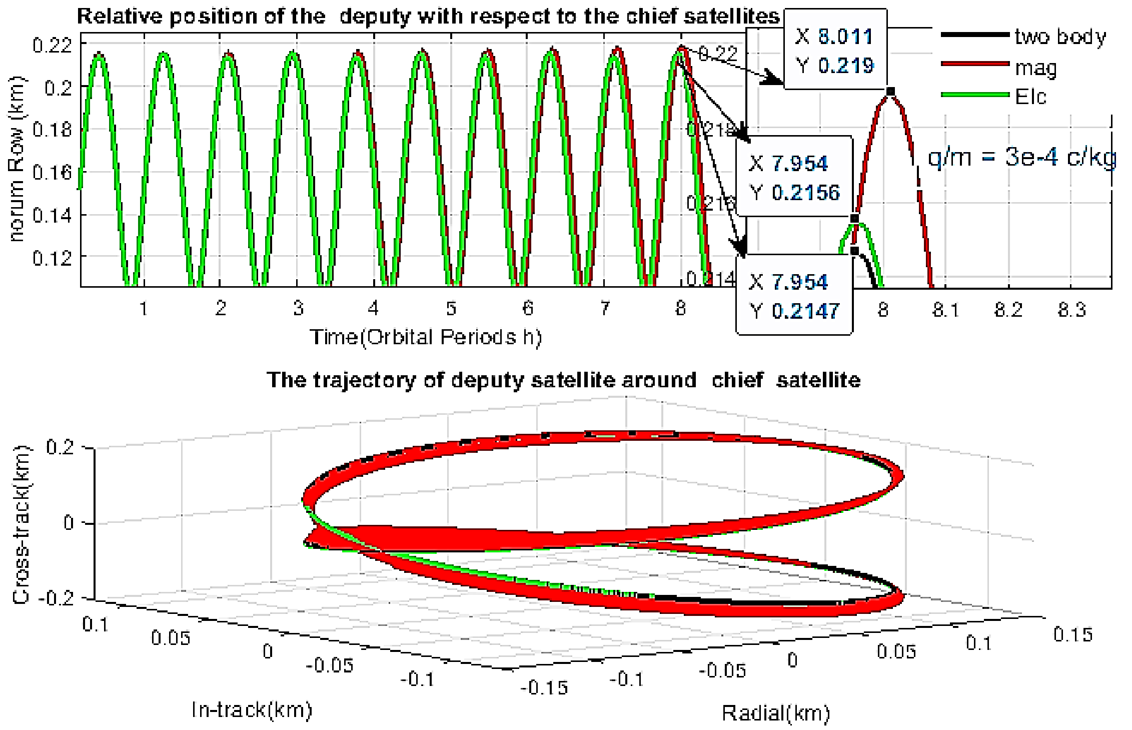

Figure 6. Relative position and trajectory for the magnetic and electric fields at q/m = 3 e-4 C/kg

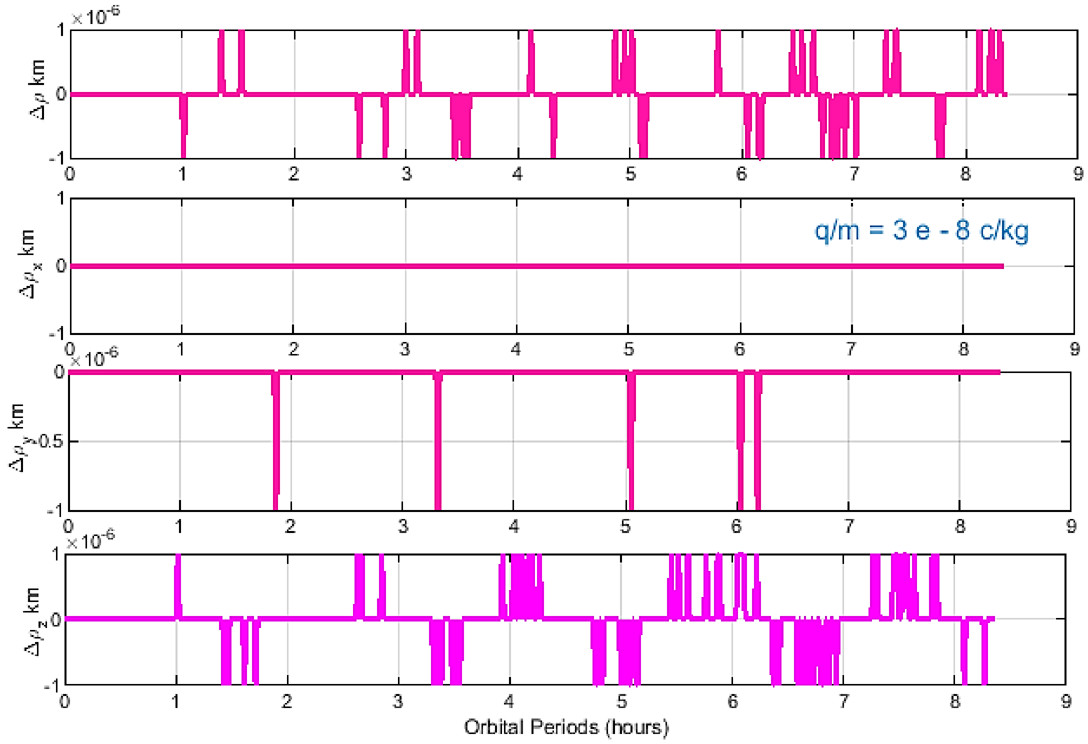

Figure 7. Error in relative position for electromagnetic force Satellite at q/m = 3e-8 C/kg

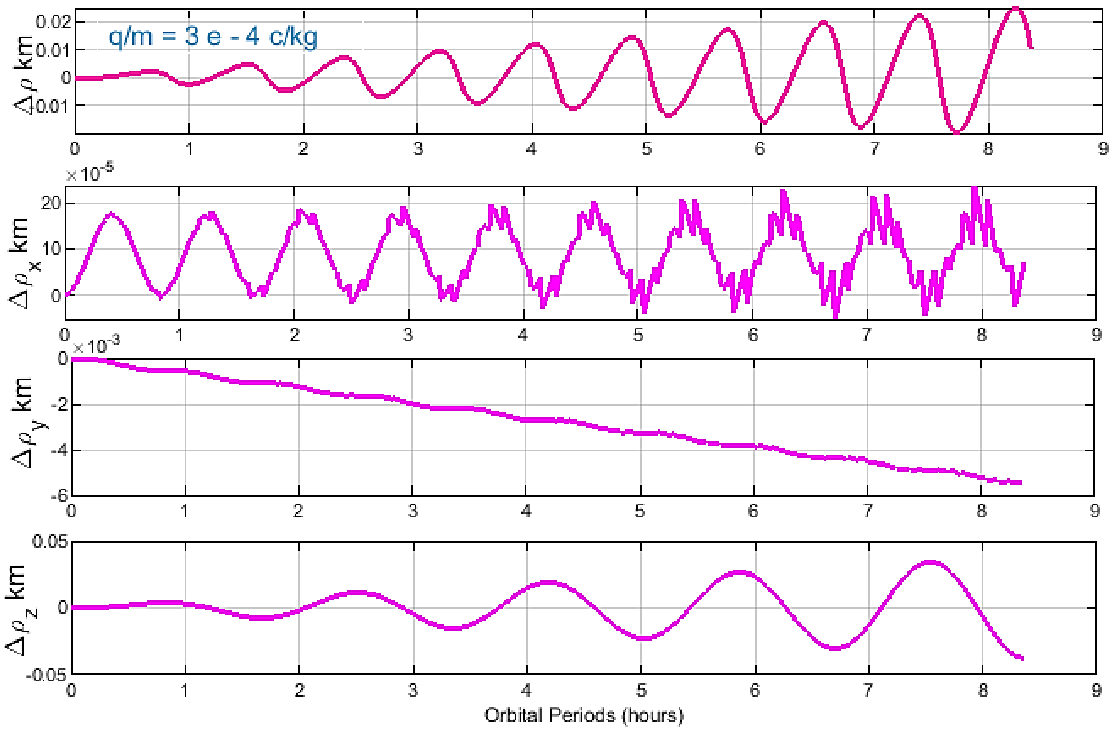

Figure 8. Error in relative position for electromagnetic force Satellite at q/m = 3e-4 C/kg electromagnetic

5. RESULTS AND DISCUSSION

Figure 4 shows that the change of norm relative position and trajectory of formation flying at natural of charge to mass ratio 3e-8 C/kg for Lorentz magnetic field after 5 periods. We can note that the rate of change in relative position is very low to cause a decrease in the total cost and increase the accuracy of the results in doing the required maneuvers. Figure 5 shows that the relative position with two cases different of Lorentz magnetic field values at natural of charge to mass ratio 3e-8 C/kg to study the effect of dipole magnetic, where the red curve indicates the effect of the Lorentz magnetic field with dipole magnetic is non-tilted ($\alpha = 0°$), and the Blue curve represents that the effect of the Lorentz magnetic field with dipole magnetic is tilted ($\alpha = 11.3°$). The results show that the change of relative position about ± 0.20 m after 5 periods when the dipole magnetic angle is calculated. Figure 6 shows that the first main target in this study for developing the equation of motion using the electric field. We assume that the level of charge in the surface of the spacecraft is increased by a small ion collector up to 3e-4 C/kg, where the black curve shows the norm relative and trajectory in the case of gravitational, while the red curve shows the effect of the magnetic field and the green curve shows the effect of the electric field, where the rate of change on a relative position about 4 meters after 5 periods when we take the magnetic force only and about ±1 m due to the effect of an electric field under the same conditions. Figures 7, 8 show that the error in norm relative position and the three axes after 5 periods, considering total Lorentz force (magnetic and electric). At two different values of charge to mass ratio q/m (C/kg): a) the natural of charge to mass ratio 3e-8 C/kg (see Figure 7), the error rate in relative position is very small about 10^{-3} m; b) at increasing the level of charging in the deputy satellite surface up to 3e-4 C/kg as shown in Figure 8, the error rate in relative position is about 20 m. The relative motion in the along-track direction is affected by the largest navigation errors, if the along-track compared with radial and cross-track, due to the uncertainties associated with the characteristics of the upper atmosphere. The satellite's orbit dynamics in along-track direction are highly coupled due to Kepler's equation. So, any maneuver execution errors and cross-coupling will cause a rapidly varying along-track motion with an offset that accumulates over time. The results show that the second condition will be sufficiently required to correct the drift in the relative position of formation flying due to the effect of perturbation forces.

6. CONCLUSIONS

In this work, we developed a new approach for formation flying satellites considering Electromagnetic force (Lorentz force). The Lorentz acceleration has been developed for two terms: a) the magnetic field in the case of absolute charging of the spacecraft, taking into account the effect of Earth's tilted magnetic dipole; b) the electric field in the case of differential charging of the spacecraft. The main idea was to install a small device (Ion collector) to increase the level of charging in the spacecraft surface to obtain an order of magnitude for the charge to mass ratio, which can be valid for orbital control. We have investigated the different values of charge to mass ratio in case of a magnetic part or electric part of Lorentz force, which can be useful for control and correct the drift in relative position. The numerical results have shown that the value of the charge to mass ratio in case of total Lorentz forces (magnetic + electric) is about ±3e-4 C/kg can be valid to correct drift in relative position after 5 periods. This means that an electric part is very important for decreasing the total cost by 25% and increase the accuracy of the results compared to the value of charge to mass ratio with Lorentz magnetic force only. In the future work, we are going to use feedback control for optimal control of spacecraft formation flying to correct the drift in the relative position of formation flying due to the effect of second zonal harmonic.

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