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PERIDYNAMICS FOR DAMAGE MODELLING AND PROPAGATION VIA NUMERICAL SIMULATIONS

Adam MARTOWICZ
AGH University of Science and Technology, Department of Robotics and Mechatronics
al. A. Mickiewicza 30, 30-059 Krakow, Poland, adam.martowicz@agh.edu.pl

Abstract

The present work provides an overview on the selected applications of peridynamics to damage modelling and simulations of its propagation in mechanical structures. The theoretical fundamentals of the method are briefly described to highlight its advantages and the scope of practical use in the field of computational mechanics. Selected results of numerical analyses are provided to illustrate demanded capabilities. The perspectives of nonlocal and integral based problem formulations for dynamics are discussed.

Keywords: peridynamics, damage, nonlocal modelling, computational mechanics

PERYDYNAMIKA W MODELOWANIU USZKODZEŃ I SYMULACJI ICH PROPAGACJI W UJĘCIU NUMERYCZNYM

Streszczenie

Artykuł stanowi przegląd wybranych zastosowań perydynamiki w modelowaniu uszkodzeń i symulacji zjawisk ich propagacji w konstrukcjach mechanicznych. Przedstawiono podstawy teoretyczne metody ze szczególnym uwzględnieniem specyfiki zalet perydynamiki w zastosowaniach w mechanice komputerowej. Opisywane zalety metody są zilustrowane wybranymi przykładami analiz numerycznych. Artykuł przedstawia możliwości obliczeniowe nielokalnych sformułowań dla dynamiki bazujących na całkowych równaniach ruchu.

Słowa kluczowe: perydynamika, uszkodzenie, modelowanie nielokalne, mechanika komputerowa

1. INTRODUCTION

The classical, i.e. locally formulated spatial gradient based methods are widely used to solve dynamics problems within the scope of computational mechanics. However, the above-mentioned approaches fail when it comes to the conclusion that the physical matter, while downscaling, is not continues any more. Hence, spatial partial derivatives, which are required to be determined with regard to either stresses or strains, result in numerical inaccuracies. Similarly, macroscale engineering parameters, e.g. the elastic properties (Young's, Kirchhoff moduli, Poisson's ratio, etc.), do not lead to a realistic and, therefore, entirely reliable description of the material characteristics at micro- and nanoscale. Spatial partial derivatives do not simply stand for adequate means to solve problems in computational mechanics at these lengthscales properly. Nonlocal formulations have opened new promising perspectives to overcome the above inconveniences.

The idea of nonlocality for mechanics is not a new one. Elastic models with long-range interactions were introduced in the works by Eringen, Edelen, Kröner and Kunin already published in the 60thies and 70thies of the last century [1-3]. Similarly, complementary nonlocal damping models are proposed [4]. The early nonlocal models were proposed in reference to the results of experiments where dispersion curves were identified for phonons propagating in monocrystalline structures and ion-ion interactions. Nonlocal character of both the identified interactions and resultant elastic properties at nano- and microscale was experimentally proven, which inspired new approach of modelling. Similarly, van der Waals forces may be conveniently modelled using nonlocal formulation.

The unique property of the nonlocal modelling is the capability of keeping specific lengthscale, e.g. the distance between carbon atoms in graphene [5], and related physics through all modelled scales. This feature allows for hierarchical introduction of different physical properties and behaviour of the modelled material and structure, following the idea of multiscale modelling [6]. Hence, different factors in the constitutive equations, that are unveiled at consecutive geometric scales, can be conveniently considered and both continuous and granular character of matter can be reflected accurately.

Nonlocal models of damages (structural discontinuities, cracks) are also successfully introduced [7]. Integral based equations of motion are applied to model the behaviour of the damage, especially in the area of its tips, where differential description could fail [8]. Moreover, nonlocal approach may help to regularize the boundary value problems as reported in [7]. A desired property of the nonlocal models of damages is assuring more physical inference on the crack's propagation path, which is independent from the preferred directions of a mesh of nodes in numerical models. More spontaneous damage evolution can be observed [9].

Finally, nonlocality allows for convenient reduction of numerical dispersion [10,11]. Introduction of long-range interactions within chosen horizon suppresses the effect of both spatial and temporal discretization in numerical models, which inevitably leads to errors due to invalid dispersion properties of modelled media.

The objective of the paper is to focus on the properties and applications of a specific nonlocal modelling technique, namely peridynamics [12]. The following Sections 2 and 3 cover the fundamentals of peridynamics and its modelling capabilities for various types of analyses. Details of numerical modelling are introduced in Section 4, followed by the results of selected case studies for damage modelling and simulations of its propagation, presented in Sections 5 and 6. The last Section 7 summarizes the work and draws conclusions.

2. FUNDAMENTALS OF PERIDYNAMICS

A peridynamic model for a solid body considers nonlocal interactions between their pieces (referred to as particles) localized within given horizon H with respect to an actual central particle, which is localized at the position $\mathbf{x}$.

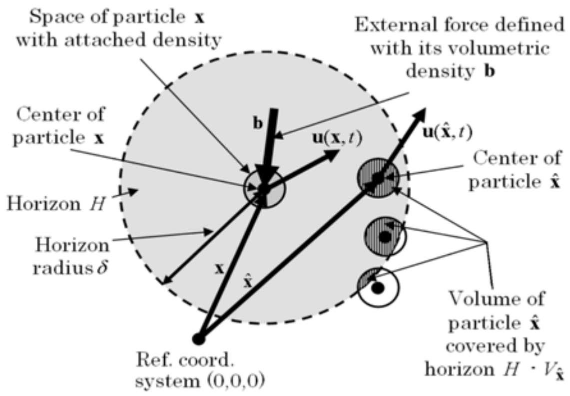

Fig. 1 Bond based peridynamics.

Spatial integral based governing equation takes the form [13,14]

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t)=\int_H \mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi})dV_{\hat{\mathbf{x}}}+\mathbf{b}(\mathbf{x},t) \tag{1}$$

where

$$\boldsymbol{\eta}=\mathbf{u}(\hat{\mathbf{x}},t)-\mathbf{u}(\mathbf{x},t) \tag{2}$$

$$\boldsymbol{\xi}=\hat{\mathbf{x}}-\mathbf{x} \tag{3}$$

denote the relative displacement and position, respectively, based on the coordinates of both the central ($\mathbf{x}$) and neighbouring particles ($\hat{\mathbf{x}}$) – defined in the reference coordinate system.

The material properties are defined by the mass density ρ and the pairwise function $\mathbf{f}$, which, in turn, contains elastic constants. Volumetric density of an external force acting on the body is derermined with the vector $\mathbf{b}$. Volumetric density of the resultant interaction force - including both local and nonlocal components - is found as the sum of product of the function $\mathbf{f}$ and volume portion $dV_{\hat{\mathbf{x}}}$ attached to the neighbouring particles.

Making a general reference of the reactions in the bond based peridynamic models to the axially deformed rods (or idealized springs governed solely by the stiffness coefficients), the function $\mathbf{f}$ is determined based on strain s and the micromodulus function c

$$\mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi})=\begin{cases} e(\boldsymbol{\eta},\boldsymbol{\xi})c(\boldsymbol{\xi})s \text{ if } \|\boldsymbol{\xi}\|\le\delta \\ 0, \text{otherwise}\end{cases} \tag{4}$$

e is the unit vector showing the reaction's direction. The function $\mathbf{f}$ takes non-zero values only if a neighbouring particls stays within the central particle's horizon of the radius δ. To provide examples of the definition for the micromodulus function c, in case of an isotropic and homogeneous material, it can be found for one- and two-dimensional models with the formulas

$$c=\frac{2E}{\delta^2 A} \tag{5}$$

$$c=\frac{6E}{\pi\delta^3(1-\nu)T} \tag{6}$$

Apart from the elastic properties - the Young's modulus E and the Poisson's ratio ν - geometry is defined either with the area of transverse cross-section A or the thickness T. In the following sections general characteristics of peridynamics is discussed, illustrated with the results of carried out numerical simulations.

3. CAPABILITIES OF PERIDYNAMICS

Due to nonlocal character of the governing equation for peridynamics (1), specific types of analyses are handled in a more convenient way in comparison to the classical local spatial partial derivative based approaches. Based on the theoretical assumptions presented in Section 2, it should be noted that - even for discretized solutions (case of numerical peridynamic models) – the path for the damage's growth reflects real evolutions of a crack. The modelled long-range interactions are similar to those observed at micro- and nanoscales. The key issue is that the growth of a modelled crack is physically determined, i.e. by breaking links between particles within considered horizon, and is not controlled by the properties of a structured mesh itself. Moreover no spatial partial derivatives are necessary to determine kinematic properties of the model. Hence, geometric discontinuities, e.g. fatigue cracks, can be fairly easily introduced to allow for reliable simulations in the field of NDT and SHM.

The work [8] briefly reports some of possible applications of peridynamics. This method was successfully used to assess the damage's size based on the reflection coefficient for the Lamb waves and rate of higher order harmonics generation. Moreover a crack can be identified based on the phenomenon of wave generation at its edges due to force excitation, that cyclically stretches and compresses the damaged structure. This behaviour is known as the clapping phenomenon. In this case, the mechanism of bilinear stiffness is complemented with generation of high frequency waves since the crack's faces hit while opening and closing.

In peridynamics various models of potential based relationships can be used to establish the function **f** in (1) [13]. This leads to different models of materials that can be adapted and may effectively reflect the physical properties. This property is allowed since the relationships between the stressess and strains can vary within given horizon H. It is feasible since there are known various formulations for the micromodulus function, not necessarily defined as constants – as shown in equations (5) and (6) [13]. This characteristics also references, the above-mentioned real behaviour observed at micro- at nanoscale, where long-range relationships govern the mechanisms of nucleation and growth of material disintegration, e.g. potential based ion-ion reaction forces and van der Waals forces. The above-mentioned nature of peridynamics, therefore, has opened new perspectives regarding modeling of a real crack's growth. What is even more promising, the fact that the peridynamic formulation incorporates macro scale material properties, e.g Young's, Kirchhoff moduli and Poisson's ratios, allows for very convenient modelling. Hence, peridynamics allows to effectively handle a real behaviour of material – e.g. crack's growth, which is an issue, especially at its tips – operating with approximate and engineering parameters.

The property of nonlocality in peridynamics also unveils interesting capabilities regarding multiscale simulations and related issues. As shown in [6], the material properties can be effectively preserved through subsequent geometric scales when applying peridynamics. This characteristics should be considered as the result of introduction the horizon in equation (1). The horizon determines a specific lengthscale of given order of spatial dimensions. This property also reflects the fact that the matter dramatically changes its properties at a very specific dimension scale, i.e. it may be effectively considered as continuous at macro- and mesoscale, whereas its granular nature should be taken into account when further downscaling. The mentioned feature is important as specific micro- or nanoscale phenomena may have dramatic influence on crack propagation, which is demanded to be kept at meso- or macroscale simulations. Similarly, due to the accessible lengthscale effects, different constitutive models, incorporating local phenomena via homogenization technique, can be effectively introduced as for orthotropic models of laminated composites shown in [9], where the paths for the damage's growth under different conditions were also studied. Finally, one may attempt to substitute the existing mesh of nodes with the one created for a peridynamic model, where the material or overall structural properties can be preserved via available nonlocal formulations and lengthscale. An adequate resultant numerical peridynamic model with a square mesh of nodes for a graphene sheet (of hexagonal mesh) can be found in [5].

A disadvantage of peridynamics, however, is an increase of computational costs, which results from more densely populated global system matrices and more demanding data processing that needs more interactions to be included into the discretized governing equations. Moreover, the arbitrariness related to the form of the function **f** leads to some ambiguity in the modeling of material properties. Theorethically, there is an infinite number of equivalent material models that can be applied, including various horizon radius for long-range interactions. On the other hand, however, this capability enables nonclassical models of elasticity or damping to be considered.

In the following the results obtained for the studies on damage propagation, using examples of numerical peridynamic cracked models, are shown.

4. NUMERICAL MODELING

The present section serves as the description of numerical procedure for peridynamic modelling and simulations. The following steps are performed by the author to simulate damage propagation:

- Model parameterization – setting geometric and material properties, including the ultimate stress, which is responsible for the level, at which the mechanisms of bonds cracking between particles is activated; setting the localization, length and orientation of cracks;
- Setting boundary conditions – external forces, that lead to model deformation;
- Setting initial conditions regarding nodal displacements and velocities;
- Setting the simulation parameters –the distances between particles, radius of horizon, integration time and total simulation time, maximum displacement error for iteration loop;
- Initial calculations to determine:
  - micromodulus function;
  - critical strain;
  - critical elongation for the bonds between particles;
  - volumes of particles within the horizon;

   - introduction of the crack via disbonding of selected links between particles;
   - initial values for the nodal quantities;
- Carrying out an iterative procedure to determine the nodal displacements and velocities for consecutive time steps - within the loop of:
   - calculation nodal parameters based on the explicit time integration algorithm;
   - check if already removed links between particles should be temporarily considered when disconnected particles touch;
   - check if additional links between particles must be removed since the relevant stains are exceeded;
- Postprocessing and data presentation.

When considering the boundary conditions for peridynamics, it must be noted that a special attention should be put on proper attachment of an external force. The peridynamic models tend to break in the regions of force application due to a specific distribution of long-range links between particles Fig. 2. references the issue.

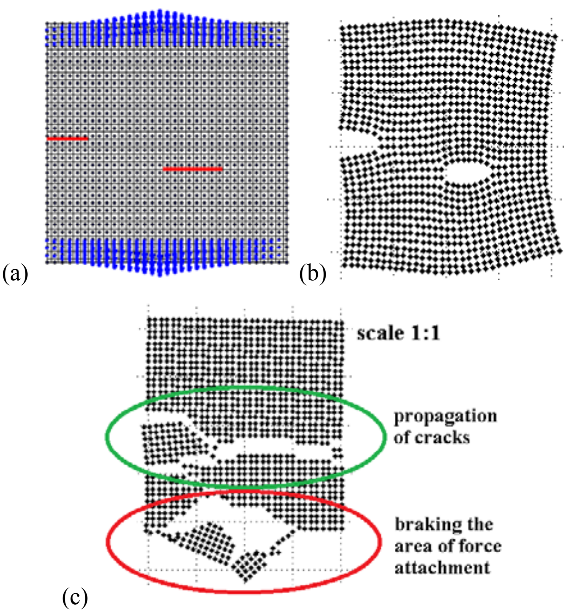

Fig. 2 Breaking the model due to improper force application: (a) localizations of stretching forces and initial notches (b) initial model deformation before the particles' links break (c) the model exhibits improper breaks as both the forces' amplitude rises too fast and the area of their attachment is too small.

There are two ways to solve the above-stated problem. One should either increase the stiffness of the links between the particles in the region where the forces are applied (more convenient but less physical approach) or enlarge the application region, which stands for a more physical solution as it spreads spatially the influence of external force acting on a modelled body. In the present work the latter approach is considered.

Another specific property of peridynamics - regarding boundary conditions – is its ability to operate without fixed displacements even for statics, quasi statics (under equilibrium of external forces) and transient studies, which refers to the cases studied in the present work. Eventually, if a peridynamic model does not consider any fixed displacements, and there is no equilibrium for the external loads, it undergoes both elastic deformation and change of its linear and angular position in the referential coordinate system. In the following examples of transient simulations a free-free boundary condition is considered.

5. AXIAL STRETCHING

First, numerical example for axial stretching of a cracked plate is discussed. A peridynamic model of a plate made of aluminium is shown in Fig. 3.

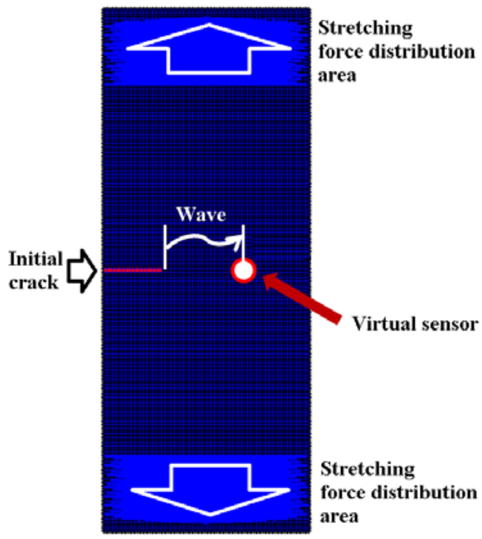

Fig. 3 A peridynamic model of a cracked plate.

The overall dimensions of the model equal 14 x 35.875 x 1 mm. The material properties are as follows: Young's modulus - 70GPa, Poisson's ratio - 0.3, mass density 2100kg/m3, and ultimate stress - 40MPa. The distance between particles is 0.125mm and the radius of horizon equals 0.5mm. The integration time is 2ns. A single 3.9375mm-long horizontal notch is introduced to initiate the processes of breaking the plate. The force is spatially distributed within two areas to allow for stretching the modelled sample. A virtual sensor is located in the area of the damage growth path in order to register the nodal oscillations that appear due to braking the plate. Fig. 4 and 5 present the plate deformation for various stages of damage progress. The average speed of crack propagation equals 834m/s.

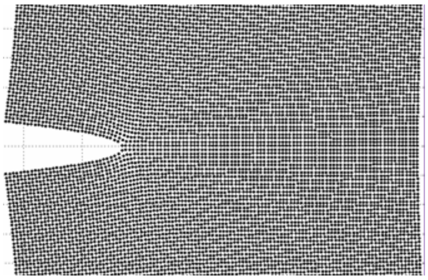

Fig. 4 First stage of the model deformation (scaled view) - before breaking the links between particles.

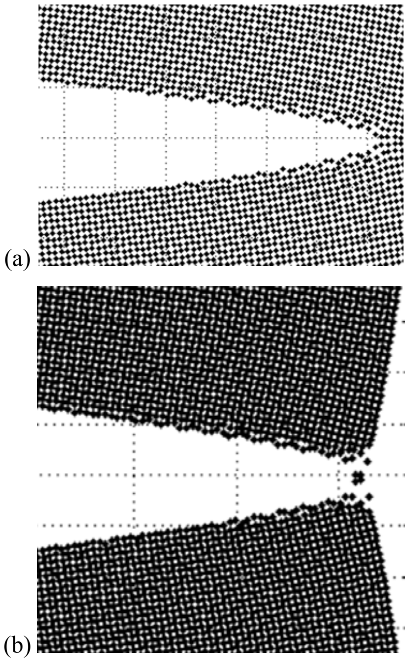

Fig. 5 Shear waves at the crack's edges: before (a) and after the modelled specimen was eventually broken into pieces (b).

Fig. 6 presents the frequency characteristics for the nodal displacements measured at the virtual sensor localization. The clearly recognized 3MHz displacement component refers to the wave propagating through the plate, which originates from moving tip of developing crack - subsequently braking links causes propagating model disturbances.

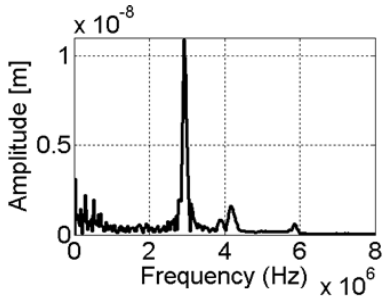

Fig. 6 Frequency characteristics for the nodal displacement in vicinity of growing crack. The slow motion trend is removed.

The identified 3MHz-wave component appears if the crack continues growing. Hence, it may be considered as an indicator showing damage progress. However, an inconvenience emerges regarding capability of effective measurements of the waves exhibiting amplitudes of the order of 10nm. Additionally, the generated waves may interfere with other cyclic displacements, including those ones resulting from external loads, which may eventually lead to data loss. In this case damage detection may be questionable. Nevertheless, peridynamics enables observation of the waves generated at the crack's tip while its growth.

6. FATIGUE

The second case study considers fatigue analysis. The model of a square aluminium plate of the dimensions 4 x 4.125 x 1 mm with a centrally localized 0.625mm-long crack undergoes cyclic stretching and compression. The amplitude and frequency of the external force are 33.75N and 200kHz, respectively. However, it should be mentioned that the amplitude of acting force is intentionally set at high level to assure fast crack's growth and register its rapid evolution even during subsequent stretching and compression cycles. The remaining parameters of the model are the same as those considered in the previous analysis. Fig. 7 presents selected stages of the crack progress appeared during the fatigue test.

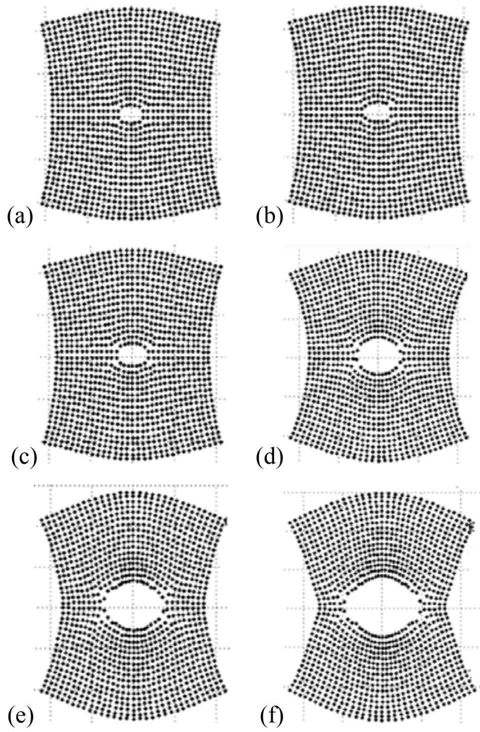

Fig. 7 Progress of damage in the plate for fatigue characterized by the crack's length (scaled view): (a) 0.625mm at 6.84us, (b) 0.75mm at 16.82us, (c) 1mm at 26.86us, (d) 1.5mm at 36.16us, (e) 2mm at 36.44us, (f) 3.125mm at 37.06us.

As referenced in Fig. 7, a gradual increase of the crack's length is observed. Subsequent cycles of the sinusoidal load causes continued model degradation. A spontaneous damage progress may be tracked for given value of ultimate stress. Additionally, regardless of the structure of particles mesh, different paths for growing cracks can be successfully simulated for various stretching and compression speeds, which is referenced in the other author's work [15].

7. CONCLUDING REMARKS

The paper discusses selected aspects of computational mechanics regarding applications of peridynamics for damage modelling and simulations of it propagation. As referenced in the work, peridynamics is a convenient analytical and numerical modelling tool for solving dynamics problems for cracked structures.

Considering an integral based formulation for governing equations, one can fairly easily handle crack propagation in numerical models. Spatial partial differential equations are excluded from problem description which, in turn, prevents from potential additional sources of computational errors.

Peridynamic models stand for a relatively reliable tool in the field of NDT and SHM. More physically induced mathematical description is provided for problem solution since a real path for propagating damage can by observed, i.e. not governed by the structure of particles mesh.

A slight inconvenience should be however mentioned when using peridynamics. As in the case of other nonlocal approaches an increased computational effort is required to handle all long-range interactions. However, this problem may be partially overcame via parallelization and computations on GPU. Due to specific - an integral based - formulation for peridynamics, the governing equation requires the sum of all reactions present within given horizon to determine new displacements for each degree of freedom. Hence, the kinematic properties for a peridynamic model can be calculated separately for each particle, running many threads simultaneously.

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Adam MARTOWICZ Ph.D. Since 2008, he is a postdoctoral researcher in the Department of Robotics and Mechatronics, AGH University in Krakow. His scientific interests focus on smart materials, computational mechanics, multiphysics and nonlocal modelling methods applied in the area of NDT and SHM. He has co-authored 150 publications, including reviewed journal papers, chapters in books and 2 patents.