ACOUSTIC STREAMING IN FOCAL AND POST FOCAL REGION

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The paper presents concise theoretical description of mechanism of acoustic streaming in the field of finite amplitude wave. Special attention was concentrated on acoustic streaming in focal area into a beam produced by a bowl shaped source. A set up used in experimental investigation is also shown. The measurement was carried out at fixed conditions. Its result was compared to the result of calculation based on theory derived from the reasoning presented by W. Nyborg et al. {1].

INTRODUCTION

Acoustic waves of finite amplitude cause movement of medium within the area of beam of wave. It results from the fact that both pressure and velocity averaged in time are not equal to zero. First mentions about the movement of gas caused by oscillating plates go back to the nineteenth century (Faraday, 1831 [2]). As regards the contemporary works one should mention publications made by Lighthill [3], Rudenko and Soluyan[4], Nyborg et al. [1]. All those works describe certain cases of appearing of areas, in which movement of medium caused by finite amplitude wave is observed. The important factor in this particular matter is the viscosity of the medium (Reynolds tensor of viscous stresses), that is rarely taken into account for infinitesimal amplitude waves when describing the mechanism of wave propagation. It is only considered in the case of loss of wave energy.

1. THE THEORETICAL CHARACTERISTICS OF THE STREAMING PHENOMENON CAUSED BY FINITE AMPLITUDE WAVE

Initially, it is assumed that compressible and viscous as well as isotropic medium is considered. Later, the influence of compressibility will be omitted because it is a factor of less importance in the case of liquid streaming.

Physical quantities that will be used todescribe the phenomenon occurring in the medium are velocity $\bar{u}(x,y,z,t)$, pressure $p(x,y,z,t)$ and density $\rho(x,y,z,t)$.

The equation of mass conservation called the continuity equation and the equation of motion of the medium (the Navier-Stokes equation) are classical equations describing the medium. The equations are given in the following form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{1}$$

$$\frac{\partial (\rho \vec{u})}{\partial t} = -\nabla P + (4/3\mu + \xi)\nabla\nabla \cdot \vec{v} - \mu \nabla \times \nabla \times \vec{u} + \rho(\vec{u} \cdot \nabla)\vec{u} + \vec{u}\nabla \cdot (\rho \vec{u}) \tag{2}$$

where

$\vec{u}()$ - velocity of streaming of the medium,

$\rho()$ - density of the medium,

$P()$ - internal pressure of the medium,

∇ - Hamiltonian,

$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial j} + \vec{k}\frac{\partial}{\partial z}$ $\{\vec{i}, \vec{j}, \vec{k}\}$ - versors of axis

$\nabla \times$ - vector product of Hamiltonian and vector function (rotation).

Taking additionally the following denotation:

$$F' = -\rho(\vec{u} \cdot \nabla)\vec{u} + \vec{u}\nabla \cdot \rho \vec{u} \tag{3}$$

assuming steady state and $\rho = const$ (the incompressible medium) we obtain:

$$\frac{\partial}{\partial t}(\rho \vec{u}) = 0 \tag{4}$$

Moreover lack of mass sources inside examined medium is assumed which means that:

$$\nabla \cdot \vec{u} = 0 \tag{5}$$

Taking advantage of above simplification we can obtain the following formula describing external forces in the case of irrotational streaming (laminar $\nabla \times \vec{u} = 0$):

$$\vec{F} = \nabla P + \mu \nabla^2 \vec{u} \tag{6}$$

F - volume density of force.

The solution of the above equation in the case of bounded medium is given in the following form:

$$\vec{u} = \frac{1}{8\pi\mu} \int_V \left(\frac{\vec{F}}{r} + \frac{(\vec{F} \cdot \vec{r})\vec{r}}{r^3} \right) dV \tag{7}$$

If only one component of force for instance as direction of X-axis is taken into consideration, velocity could be determined as follows:

$$\vec{u} = \frac{1}{8\pi\mu} \left(\frac{iF_x}{r} + \frac{(\vec{F} \cdot \vec{r})}{r^3} \right) dV \tag{8}$$

where: $\vec{i}$ – X-axis versor. The formula is obtained for local coordinate in point r, around which element of the volume dV is concentrated. Along the X-axis the component of the velocity is given as the formula:

$$u = F_x \frac{dV}{4\pi\mu|x|} \tag{9}$$

If the volume dV is assumed to be a cylindrical one with the radius r and height h, the volume of cylinder equals to $dV = \pi a^2 \cdot h$. So the component of velocity could be given as:

$$u = \frac{F_x \cdot a^2 \cdot h}{4\mu r'} \qquad r' = \sqrt{a^2 + x^2} \tag{10}$$

2. ACOUSTIC STREAMING IN FOCUSED BEAM IN THE FOCAL REGION

As it has already been mentioned above averaged values of velocity and other variable quantities are not equal to zero, so the finite value of velocity exists. The values of velocity that will create flows called acoustic streaming, will be treated with special interest. If we assume, as it is usually done for nonlinear quantities, that functions of pressure, density and velocity could be given in form of series, they will be presented as following expressions:

$$p - p_o = p_1 + p_2 + \ldots \tag{11}$$

$$\rho - \rho_o = \rho_1 + \rho_2 + \ldots \tag{12}$$

$$\vec{u} = \vec{u}_1 + \vec{u}_2 + \ldots \tag{13}$$

where appropriately $p_1, p_2 \ldots$, $\rho_1, \rho_2 \ldots$, $\vec{u}_1, \vec{u}_2 \ldots$ are successive approximations of functions of pressure, density and velocity.

Basing on relation given previously strength density $\vec{F}_2$ is given in the form:

$$\vec{F}_2 = \nabla p_2 - \mu \nabla^2 \vec{u} \; (14) \tag{14}$$

where: $F_2 = -\rho_o < (\vec{u}_1 \cdot \nabla)\vec{u}_1 + \vec{u}(\nabla \cdot u_1) >$

$< \;\; >$ denotes values averaged in time.

If one-dimensional form of an elastic wave is assumed propagating in the positive direction of the X-axis, velocity vector *u(x,t)* describing it is given in the form:

$$u(x,t) = u_o e^{-\alpha x} e^{j(\omega t - kx)} \tag{15}$$

where:

$k = \frac{\omega}{c_o}$ is a wave number,

ω - angular frequency,

c_o - speed of sound in linear approximation,

u_o - amplitude of wave.

The component of the force density acting in the direction of X-axis has the form:

$$F_{2x} = -2\rho_o < u_1 \frac{\partial u_1}{\partial x} > = 2\alpha\rho_o u_o^2 e^{-2\alpha x} \tag{16}$$

Using relation connecting the component of the velocity u_2 and the component of the force density F_{2x} we obtained:

$$u_2(x) = \frac{\alpha\rho_o u_o^2 e^{-2\alpha x} h a^2}{4\mu r'} \tag{17}$$

Introducing symbol describing the averaged power of wave crossing the area πa^2 as:

$$W = \frac{\pi a^2 \rho_o c_o u_o^2 e^{-2\alpha x}}{2} \tag{18}$$

the power lost (absorbed) in a layer with thickness of dx_1 amount to:

$$\Delta W = 2\alpha W(x) dx_1 \tag{19}$$

Then using above denotation, we arrive at:

$$u_2 = \frac{\Delta W}{4\pi\mu c_o r_1} \approx \frac{\Delta W}{4\pi\mu c_o a} \tag{20}$$

Above relation describes value of streaming velocity in the dependence on wave parameter, such as its amplitude and the environment parameters represented by viscosity, speed of sound and density of undisturbed medium.

3. SET-UP FOR EXPERIMENTAL INVESTIGATIONS

The testing set up designed for investigation of the field's distribution of a finite amplitude elastic wave radiated by a focusing source was used as the initial step leading to more extensive experimental investigations of acoustic streaming.

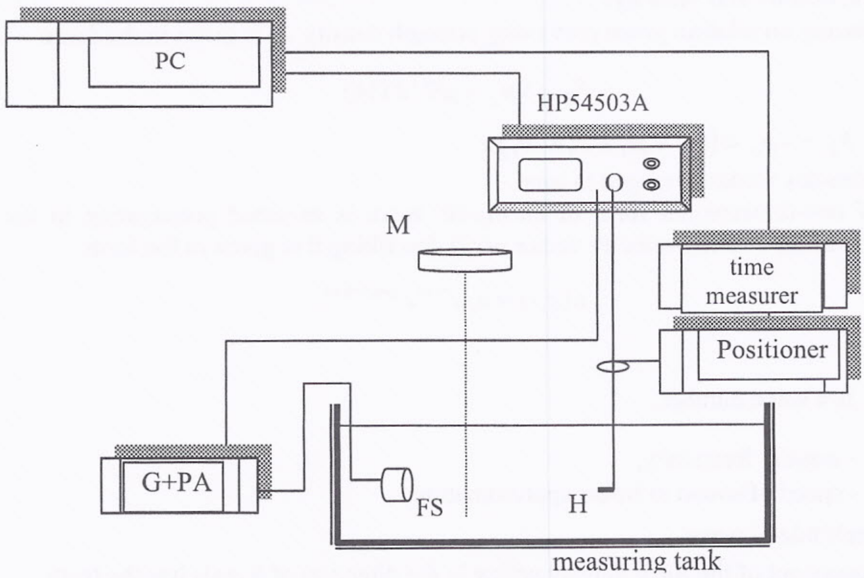


Fig. 1. Scheme of the measuring set up

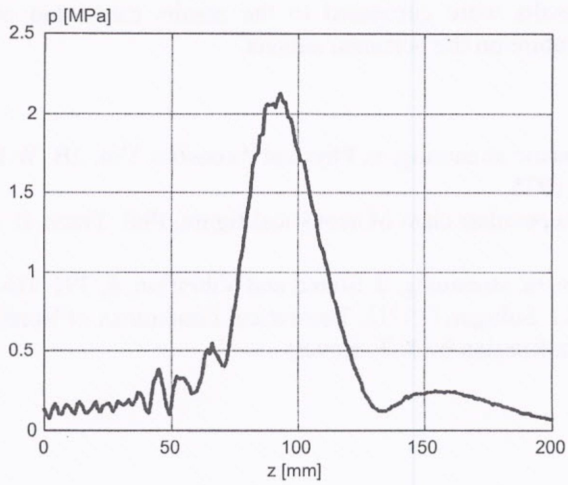
Measurement was made in water environment of parameters as follows:

$\rho \approx 1*10^3$ kg m^{-3}

t ≈ 20 °C

$c_0 \approx 1480\ m\ s^{-1}$
$\alpha_0 \approx 0.056\ m^{-1}$ (f = 1.5 MHz)
$\mu \approx 1*10^{-3}$ Pa s
a = 1 mm
dx = 0.1 mm

During measurement, the focal area was determined experimentally by means of finding the focal point in the pressure distribution along the beam axis and next in the plane perpendicular to the bean axis placed in the focal distance. Examples of such obtained



pressure distributions are shown in Fig. Fig. 2 and 3.

Fig. 2. Pressure distribution along the beam axis of focusing source
f=1.5 MHz, d=100 mm, p_o=127 kPa

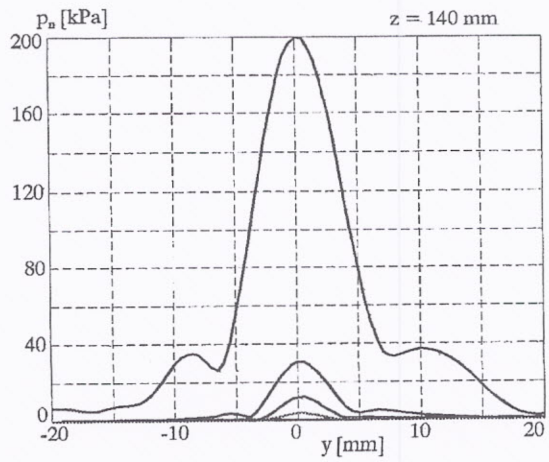


Fig. 3. Transverse pressure distribution of the first four harmonic components in the focal plane

Values of the velocity of the acoustic streaming determined experimentally in the focal area changed from 0.59 cm/s when acoustic power was about 10 W to 1.37 cm/s when acoustical power was about 23 W. Experimental data were in reasonable agreement with ones predicted theoretically.

CONCLUSIONS

Takings into account cognitive aspects and great importance of the knowledge in the sphere of velocity of acoustic streaming the introductory experimental investigations have been made. The results were compared to the results calculated on the basis of theory presented in the literature on the pertinent subject.

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