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NUMERICAL MODELING OF TRANSIENT ACOUSTIC FIELD USING FINITE ELEMENT METHOD

Numerical solution of acoustic wave equation in the time domain is performed. The appropriate mathematical model is described by the partial differential wave equation supplemented with appropriate boundary conditions. The goal is to obtain the time evolution of the distribution of acoustic pressure, which can serve as the basis for the solution of various subsequent problems. The paper discusses the results of the numerical analysis of a semi-circular acoustic diffuser in a free field, realized by a fully adaptive higher-order finite element method implemented in our own codes Agros2D and Hermes. The results are compared with the data obtained by the commercial code Comsol Multiphysics.

1. INTRODUCTION

There exist numerous acoustic problems where knowledge of only steady state or harmonic acoustic field is insufficient. In such cases it is necessary to solve more complicated transient phenomena, which is typical for room acoustics. Of great importance are, for example, reflected sound waves produced by an acoustic diffuser.

The diffuser [1] is an acoustic element that uniformly disperses sound regardless of the angle of incidence. It is used to adjust the sound level distribution in the concert halls, theatres or areas that require perfect acoustics. Due to the fact that the design and measurement of diffusers is very complicated and expensive, numerical modelling could be an effective alternative of solving such tasks in this field.

2. MATHEMATICAL MODEL

The continuous mathematical model of the problem is given by the corresponding non-stationary partial differential equation supplemented with appropriate boundary conditions. The wave equation will be derived for ideal fluids from the Newton's law of motion, continuity equation and equation of state for the adiabatic processes [2].

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Newton's law of motion is used in the form of a highly simplified Navier-Stokes equation for compressible fluids. The equation will be used in the form of

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\mathrm{grad}\, p\,, \tag{1}$$

where the symbol ρ stands for the specific mass, $\boldsymbol{v}$ represents the velocity (we consider the velocity $\boldsymbol{v}$ rather small), t is time and p denotes the acoustic pressure.

The other mentioned equations are the continuity equation

$$\mathrm{div}\left(\rho \boldsymbol{v}\right) + \frac{\partial \rho}{\partial t} = 0\,, \tag{2}$$

and state equation for adiabatic processes

$$p \cdot \rho^{-\kappa} = C\,, \tag{3}$$

where κ is the Poisson ratio (for air $\kappa = 1.4$) and C is a general constant.

The derivation starts from the application of operator divergence to (1) and continues with substitution into (2). Then we apply double time differentiation to (3), combine both obtained terms and put $c^2 = p\kappa / \rho$. In this way we obtain the partial differential equation that describes the acoustic waves in the time domain

$$-\mathrm{div}\left(\frac{1}{\rho}\mathrm{grad}\, p\right) - \frac{1}{\rho c^2} \cdot \frac{\partial^2 p}{\partial t^2} = 0\,, \tag{4}$$

where c is the speed of sound in gas at the standard Earth sea-level conditions.

3. ILLUSTRATIVE EXAMPLE

Consider a semi-circular diffuser in the free space in front of the planar signal source. Fig. 1-left part shows the principal arrangement with the geometrical dimensions of. The problem is considered planar. The basic dimensions of the impedance-matched area (representing the free space) are $5\,\mathrm{m}$ and $8\,\mathrm{m}$, respectively. The source of signal has a square platform with dimensions of 0.2 m and the signal spreads from one side. It is located at the distance of 5 m form the reflective element whose radius is $0.3\,\mathrm{m}$. There are also points (probes $0-6$) around the diffuser (marked by small black dots) along the radius of $2\,\mathrm{m}$, where the values of acoustic pressure are investigated.

3.1. Boundary conditions and environment properties

The solution of partial differential equation (4) requires correct boundary conditions. These are defined by either the values or normal derivatives of the acoustic pressure p along particular edges of the area of solution.

The boundary conditions for reflective surfaces, axis of symmetry and impedance-matched boundary are specified by the Neumann boundary condition

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial n_0} = D_{\mathrm{n}}\,, \tag{5}$$

where the values of function D_{n} are defined according to Fig. 1-right part..

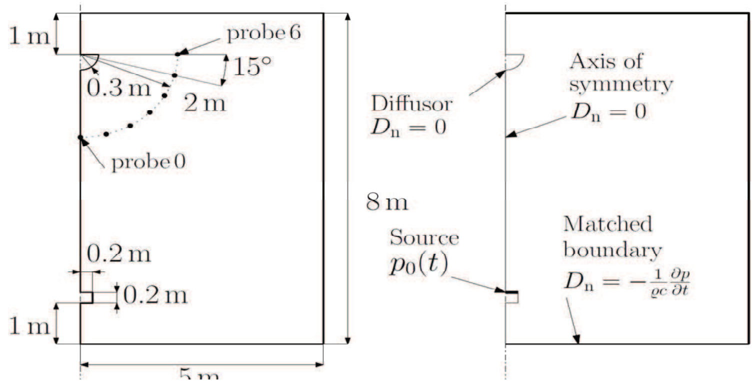

Fig. 1. Left: geometrical arrangement of the system, right: boundary conditions

The surface source of signal is defined by Dirichlet boundary condition

$$p = p_0(t), \tag{6}$$

where $p_0(t)$ is a function of the Gauss monocycle pulse

$$p_0(t) = A\cdot \mathrm{e}^{-\pi^2\cdot f_0^2\cdot\left(t-1/f_0\right)^2}\,. \tag{7}$$

Here, the symbol $A = 100\,\mathrm{m}^3\,\mathrm{s}^{-1}$ is the amplitude of the pulse and $f_0 = 1\,\mathrm{kHz}$ denotes the pulse bandwidth. The illustrative example is solved in the air with material parameters: the specific mass density $\rho = 1.2\,\mathrm{kg\ m}^{-3}$ and speed of sound $c = 343\,\mathrm{m\ s}^{-1}$ (for the ambient temperature $T_0 = 20$ °C).

4. NUMERICAL SOLUTION

The solution of (4) was performed using the higher-order finite element method (*hp*-FEM). This method is implemented in the C++ library Hermes [3] developed by hpfem.org group and it is used in Agros2D application [4] developed by the group at the University of West Bohemia in Pilsen. Agros2D allows solving partial differential equations and exhibits a lot of unique features suitable for numerical modeling (interactive geometry creation, support of curvilinear elements and hanging nodes, scripting support in Python language, particle tracing and much more).

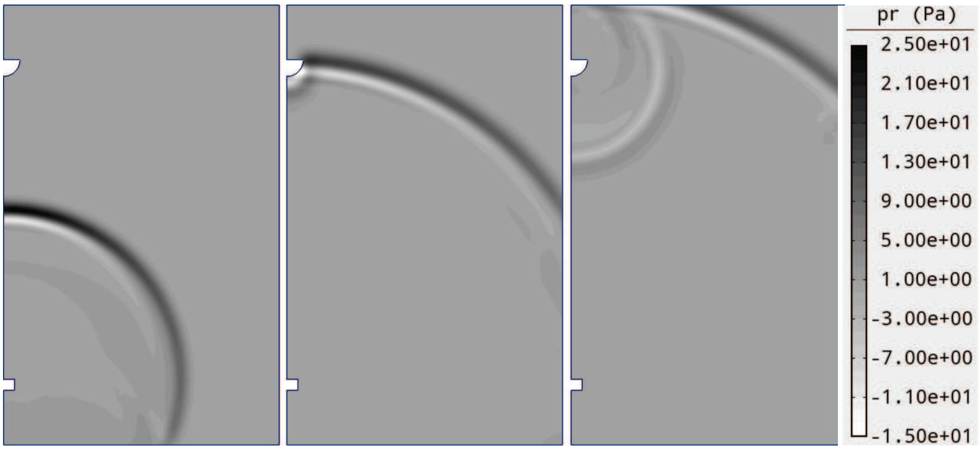

Fig. 2. Distribution of acoustic pressure in time steps:
left - 10×10^{-3} s, middle- 18×10^{-3} s and right- 22×10^{-3} s

The elements covering the area are of the order of 3 and their maximum surface was set to $0.03\,\mathrm{m}^2$. The transient was calculated for time ranging from $0\,\mathrm{s}$ to $0.025\,\mathrm{s}$ with the length of the fixed time step $\Delta t = 3.125\times10^{-5}$ s.

The result of the numerical solution in the time domain is the time evolution of the distribution of acoustic pressure p in the whole area of solution in the above time range. This distribution at three selected time instants is shown in Fig. 2. The time-dependent values of the acoustic pressure at the position of probes 0, 3 and 6 are depicted in Fig. 3.

The same example was also solved by the commercial software Comsol Multiphysics 4.2 [5] using the module called TRANSIENT PRESSURE ACOUSTICS. The goal was to verify the results obtained by the Agros2D application.

Due to different algorithms for meshing the definition area, the resultant mesh differed in this case substantially. Nevertheless, the dimensions of the largest elements of the mesh were the same as in case of the Agros2D code. The time step in Comsol was used adaptively, with the maximum length of the time step 8.79×10^{-5} s.

The comparison of results at the probe 0 placed on the axis of symmetry is carried out in Fig. 4. The curves calculated by Agros2D and Comsol are very similar. The small differences are mainly caused by the accuracy of the numerical solution. Due to the shape of the Gauss monocycle pulse, the results from Agros2D could be improved by using of adaptive time stepping.

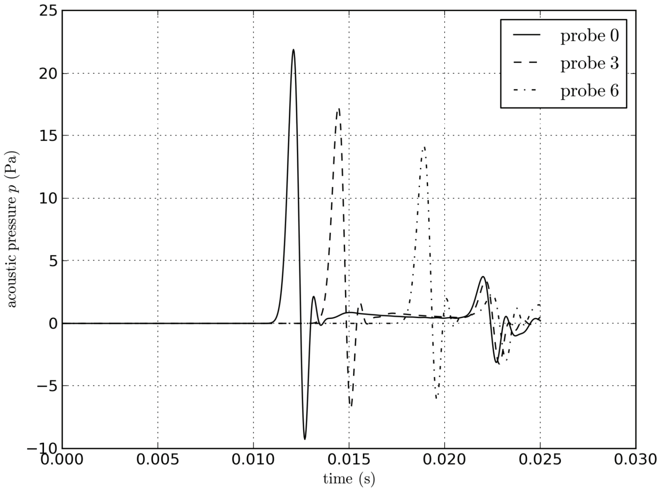

Fig. 3. Time dependence of acoustic pressure at place of selected probes

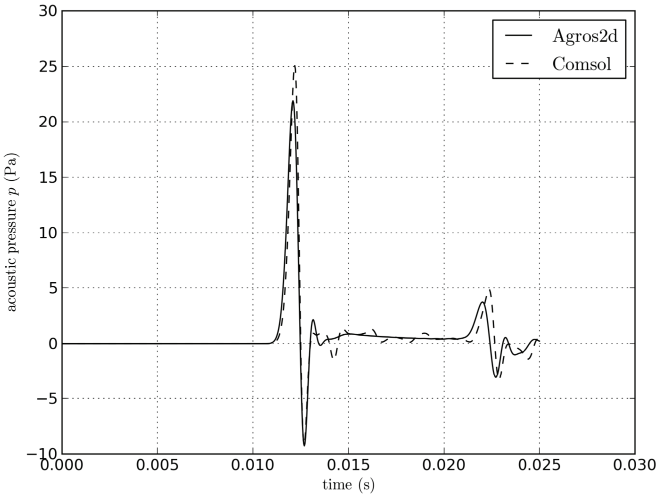

Fig. 4. Comparison of time dependence of acoustic pressure at probe 0 obtained by both used codes

5. CONCLUSION

Modelling of acoustic transient filed was performed by using the appropriate acoustic module of Agros2D application. It has been tested on several examples and in all cases the results were consistent with the results obtained using the commercial software Comsol Multiphysics. This paper describes the results from the semi-circular acoustic diffuser reflecting a signal in the form of the Gauss monocycle pulse.

The future work in this field will be focused on the correlation of signals for obtaining the required time window for the next signal processing (discrete Fourier transform and calculation of diffusity and scattering coefficient) and the verification of results by experimental measurements.

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