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## MAINTENANCE OPTIMIZATION FOR A PRODUCTION SYSTEM WITH INTERMEDIATE BUFFER AND REPLACEMENT PART ORDER CONSIDERED

### OPTIMALIZACJA KONSERWACJI SYSTEMU PRODUKCYJNEGO UWZGLĘDNIAJĄCA BUFOR POŚREDNI I ZAMÓWIENIA CZĘŚCI ZAMIENNYCH

*Existing research on maintenance is mostly devoted to maintenance planning without considering other related issues. However, optimizing maintenance separately may lead to unexpected system cost, due to the interaction between maintenance, buffer, and replacement parts. In this paper, a production system consisting of two serial machines and an intermediate buffer is studied. The upstream machine deteriorates with time, and the deterioration degrees are classified into different working conditions and represented by ascendant states. During the maintenance optimization for the upstream machine, the replacement part order and buffer inventory are both considered. Therefore, the system state is complex with the buffer level, machine working condition, and replacement parts taken into account together. One type of control-limit policy is applied based on system state, and then the system and decision process are modeled by discrete Markov method. Through policy-iteration algorithm, the control-limit policy is optimized for the minimal long-term expected cost rate. Numerical examples are delivered to illustrate the proposed method and for the parameter sensitivity analysis.*

**Keywords:** maintenance, replacement part, buffer, production system, Markov model.

*Prowadzone dotychczas badania nad konserwacją poświęcone są głównie harmonogramowi konserwacji, nie przywiązując uwagi do innych wiążących się z nią zagadnień. Jednakże, prowadzona niezależnie optymalizacja konserwacji może prowadzić do nieplanowanych kosztów z uwagi na powiązania między konserwacją, buforem i częściami zamiennymi. Niniejszy artykuł analizuje system produkcyjny składający się z dwóch urządzeń szeregowych oraz bufora pośredniego. Urządzenie na początku linii z czasem się zużywa, a stopień zużycia sklasyfikowano z uwagi na różne warunki pracy i przedstawiono za pomocą stanów wstępujących. W ramach optymalizacji konserwacji urządzenia na początku linii, rozważono zarówno zamówienia części zamiennych jak i zapasy bufora. Tak więc, na kompletny obraz stanu systemu składają się poziom bufora, warunki pracy urządzenia, oraz części zamienne. Jeden z rodzajów strategii poziomu kontroli oparty jest o stan systemu, następnie system i proces decyzyjny są modelowane przy wykorzystaniu ukrytych modeli Markowa. Strategia poziomu kontroli została zoptymalizowana dla minimalnego długofalowego i prognozowanego poziomu kosztu za pomocą algorytmu iteracji strategii. Przedstawiono również przykłady liczbowe aby zilustrować proponowaną metodę a także przeprowadzić analizę wrażliwości na zmiany parametrów.*

**Słowa kluczowe:** konserwacja, części zamienne, bufor, system produkcyjny, model Markowa.

#### 1. Introduction

In a production system, maintenance is very important for keeping machine availability and production line stability. Since the middle of last century, many categories of maintenance model have been studied [23]. However, maintenance activity is usually related to other issues in a production system, and the interaction between them may cause unexpected system cost, which makes the maintenance optimization complex [15].

Firstly, replacement part (or spare part) shortage will make the maintenance activity couldn't be implemented in time, and long-term storage of spare parts is also not suitable for the consideration of cost saving. Hence, some research was focused on maintenance optimization considering spare parts. Continuous review (s, S) policy for spare parts associated with certain maintenance strategy was studied in Zohrul Kabir and Al-Olayan [28], Vaughan [22], and Ling Wang

[17]. De Smidt-Destombes et al. took into account repair capacity, spare numbers, and maintenance frequency together, to investigate availability function of k-out-of-N systems, and achieved joint optimization [4–7]. Other examples for joint optimization of maintenance and spare parts can be seen in Brezavšček and Hudoklin [1], Ilgin and Tunali [11], Huang et al. [10], and Chien [3]. Secondly, buffer capacity is usually built to cope with unexpected interruptions due to preventive maintenance or failure. Therefore, some papers were devoted to joint optimization of maintenance and buffer size [2, 14, 19, 20, 24]). Some other studies were presented for one type of control-limit policy, which initiates the preventive maintenance based on the buffer level and the deterioration degree of machine, with the buffer size predetermined [8, 12, 13, 16, 18, 21].

However, there is no research presented to optimize the maintenance strategy with the replacement part (or spare part) order and intermediate buffer simultaneously considered. It can be understood

that in a production system with the replacement parts needed to be ordered, the buffer inventory is built not only for meeting the need of the downstream machine during the maintenance duration time, but also for that during the replacement part shortage time (or the waiting time for ordered replacement part), when the upstream machine is down. Therefore, the joint consideration of replacement part (or spare part) order and buffer inventory when optimizing maintenance is necessary. This study is aimed to fill this gap, and obtain a balance between maintenance cost, inventory cost, replacement part order cost, and production cost.

In this paper, a production system containing two serial machines is considered, and an intermediate buffer is built for coping with unexpected disruptions. The upstream machine deteriorates in time, and the increasing degrees of deterioration are classified into different states. The system studied is similar to that studied by Dimitrakos and Kyriakidis [8], but differs from Dimitrakos and Kyriakidis' paper in taking into account replacement part order. For two-unit series system, the condition-based maintenance optimization without a predetermined strategy structure has been studied by our group in an early paper [26]. Some other research on machine deterioration and maintenance optimization by our group can be seen in Zhou et al. [27] and Zhang et al. [25]. Then in a recent study, we have analyzed the intermediate buffer under an age-based maintenance policy [9]. In this paper, the replacement part order and buffer inventory are both considered during the condition-based maintenance optimization for the upstream machine. Two types of replacement part order, general order and urgent order, can be chosen according to the present system state. If the machine is found at a failure state, an urgent order is carried out immediately. Otherwise, a general order is carried out when the machine state equals to or is higher than the critical state corresponding to the current buffer level. As long as the ordered replacement parts arrive, the maintenance is initiated. The maintenance may be a preventive maintenance or a corrective maintenance, which depends on the present state of the upstream machine. Therefore, the purpose of this paper can be described as finding the conditions under which to place a general order for replacement parts, with the intent of doing preventive maintenance when the replacement parts arrive, and with the corrective maintenance and urgent order for replacement parts taken into account.

In this paper, the maintenance, replacement part order and buffer inventory are all considered, therefore, the system state contains three kinds of information, and the analysis of system action and incurred cost is very complex. For this problem, the system and decision processes are described by discrete Markov model, in which the replacement part state is divided into several states to represent different situations. Then based on policy-iteration algorithm, the minimal long-term expected cost rate is achieved, and the critical machine states corresponding to different buffer levels, i.e. the control parameters for judging whether or not carrying out a general replacement part order, are also determined.

The rest of this paper is organized as following. In section 2, the system is described. In section 3, the mathematic formulation is presented. The policy-iteration algorithm in this paper is introduced in section 4. Then in section 5, some numerical examples are delivered for illustrating the proposed method and analyzing parameter sensitivity. Finally the conclusion is given in section 6.

## 2. System descriptions

In the production system, two serial machines and an intermediate buffer are involved. The downstream machine  $A_2$  operates at a constant rate  $p_2$ . The upstream machine  $A_1$  operates at a rate  $p_1 + p_2$  if the buffer is not full. As long as the buffer is full, the operation rate of  $A_1$  is decreased to  $p_2$ . A practical example of the production system may be a work center consisting of an automated part feeder, an auto-

mated drilling machine, and an intermediate buffer, or be an assembly work shop consisting of a semi-finished good feeder, an assembly machine, and an intermediate buffer. In this study, the upstream machine  $A_1$  deteriorates with time, and the maintenance planning and replacement part order are considered for it (see fig. 1).

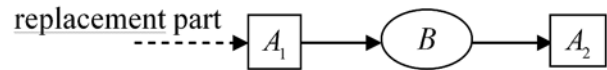


Fig. 1. The production system studied

### 2.1. Notation

$A_1$	the upstream machine
$A_2$	the downstream machine
$p_1$	the buffer accumulation speed
$p_2$	the buffer consumption speed or the operation rate of machine $A_2$
$B$	the buffer or the buffer size
$\lambda_1/\lambda_2$	the parameter of probability distribution function of preventive maintenance duration time/ corrective maintenance duration time
$c_p/c_f$	the cost of preventive maintenance / corrective maintenance during a unit time (say one day)
$\lambda_3/\lambda_4$	the parameter of probability distribution function of general replacement part order lead time / urgent replacement part order lead time
$c_{go}/c_{uo}$	the cost of a general replacement part order/ an urgent replacement part order
$h$	the inventory holding cost per unit per unit time (say one day)
$c_i$	the production cost of machine $A_1$ when it is at state $i$ and operates at a rate of $p_1 + p_2$ during a unit time (say one day)
$c_i'$	the production cost of machine $A_1$ when it is at state $i$ and operates at a rate of $p_2$ during a unit time (say one day)
$p_{ij}$	the transition probability of the machine state moving from state $i$ to state $j$ during a unit time (say one day)
$c$	the shortage cost during a unit time (say one day)
$W$	the state space of the system
$w$	the system state

### 2.2. The assumption and policy

In this study, the deterioration and maintenance of the upstream machine is considered, and one type of control-limit policy is implemented, which takes into account maintenance, replacement part order, and intermediate buffer. The purpose of this study is optimizing the control-limit policy to obtain the minimal long-term expected cost rate, which involves the buffer inventory holding cost, replacement part order cost, maintenance cost, production cost, and shortage cost.

At each time epoch, if machine  $A_1$  doesn't fail, and the machine state exceeds the critical value corresponding to the current buffer level, a general replacement part order is carried out. If machine  $A_1$  is found to encounter failure, an urgent replacement part order is carried out. As soon as the ordered replacement parts arrive, the maintenance is initiated. If machine  $A_1$  doesn't fail at the arrival time of the replacement part, a preventive maintenance is implemented. If machine  $A_1$  is found failed at the arrival time of the replacement part, a cor-

rective maintenance is required. Both the preventive maintenance and corrective maintenance can restore machine  $A_1$  to a new state. Additionally, it is supposed that the preventive maintenance and corrective maintenance of machine  $A_1$  are both non-preemptive (i.e. the maintenance can't be interrupted), and the duration time lengths of them follow geometrical distribution with the probability of success as  $\lambda_1$  and  $\lambda_2$  respectively. If the maintenance is not finished when the buffer is exhausted, a shortage cost is incurred. Conversely, if the buffer is not empty when the maintenance is finished, machine  $A_1$  is kept idle until the buffer is exhausted rather than being resumed to work immediately. The allowing of idle time of machine  $A_1$  is reasonable for the reduction of production cost [12]. According to the policy applied, it is seen that the interval between the time epochs when the maintenance is finished and the buffer is exhausted is a renewal cycle.

**2.3. System state space**

It is assumed that the system state is inspected at discrete equidistant time epochs  $t=0, 1, \dots$  (say every day), and a decision is made for machine  $A_1$ . The system state contains the information of machine state ' $i$ ', buffer level ' $b$ ', and replacement parts ' $s$ ', which is denoted by  $w = (i, b, s)$ . A decision is selecting an action ' $a$ ' from five possible actions  $\{0, 1, 2, 3, 4\}$ , depending on the current system state and the control parameters involved in the control-limit policy. The action  $a = 0$  represents doing nothing (leaving the machine  $A_1$  working, failure, or idle). The action  $a = 1$  and action  $a = 2$  represent carrying out a general replacement part order and carrying out an urgent replacement part order respectively. The action  $a = 3$  and action  $a = 4$  represent doing preventive maintenance and doing corrective maintenance respectively.

The states of machine  $A_1$  are classified into  $M+2$  states  $0, 1, \dots, M+1$ , corresponding to ascending deterioration degrees. State 0 represents a new machine, or the machine operates as new. State  $M+1$  represents a failure machine. The other states represent the intermediate working conditions. If the working condition of machine  $A_1$  is found to be at state  $i$  ( $0 \leq i \leq M+1$ ) at a time epoch, the state  $j$  ( $0 \leq j \leq M+1$ ) can be reached at the next time epoch with the transition probability  $p_{ij}$ , which only depends on the state  $i$  and state  $j$ . It is supposed that the state  $M+1$  can be reached from any state with non-zero probability during a unit time, i.e.  $p_{iM+1} > 0, 0 \leq i \leq M+1$ , and machine  $A_1$  can not improve on its own, i.e.  $p_{ij} = 0, j < i$ . The production cost rate of machine  $A_1$  at state  $i$  ( $0 \leq i \leq M$ ) is assumed to be  $c_i$  when it operates at rate  $p_1 + p_2$  and assumed to be  $c_i'$  when it operates at rate  $p_2$ .

The intermediate buffer size is predetermined to be  $B$ . If the buffer level is  $b$  ( $0 \leq b \leq B$ ) with the machine state found at  $i$  ( $0 \leq i \leq M$ ) at a time epoch, it will change to  $\min(B, b + p_1)$  at the next time epoch. If the buffer level is  $b$  ( $0 \leq b \leq B$ ) with the machine state found at  $M+1$  at a time epoch, it will change to  $\max(0, b - p_2)$  at the next time epoch.

It is supposed that two types of replacement part order policy, general order and urgent order, can be chosen for different lead time requirements. It is assumed that only one replacement part is needed for a preventive maintenance or a corrective maintenance, which can easily be released according to practical condition. In a renewal cycle, the replacement part order cost is  $c_{go}$  due to general order or  $c_{uo}$  due to urgent

order. The lead times of the two types of replacement part order both follow geometrical distribution, and the arrival rates of replacement part under general order and urgent order are  $\lambda_3$  and  $\lambda_4$  respectively. It is reasonable that the value of  $c_{uo}$  is larger than that of  $c_{go}$ , and the value of  $\lambda_4$  is larger than that of  $\lambda_3$ . The replacement part state is described by  $s \in \{0, go, uo, 1\}$ . State '0' represents the situation that the replacement part is not ordered, or the situation that the replacement part has been consumed by the maintenance and machine  $A_1$  is idle waiting for the buffer to be exhausted. State 'go' represents the situation that a general replacement part order is carried out, but the replacement part hasn't arrived. State 'uo' represents the situation that an urgent replacement part order is carried out, but the replacement part hasn't arrived. State 1 represents the situation that the replacement part has arrived, but the maintenance hasn't be finished.

Additionally, state 'PM' and state 'ID' are given to represent the situations that machine  $A_1$  is during a preventive maintenance and during an idle state respectively. Therefore, the state space of the system is as following:

$$W = \{0, \dots, M+1\} \times \{0, \dots, B\} \times \{0, go, uo, 1\} \cup \{(PM, b, 1) : 0 \leq b \leq B\} \cup \{(ID, b, 0) : 0 \leq b \leq B - p_2\} \quad (1)$$

Notation: because machine  $A_1$  could not improve by itself, some states involved in eq. (1) can't be reached from any other state. The actual system states that can be accessible under a control-limit policy are obtained by eliminating the inaccessible states from state space  $W$ , based on the policy and the system parameter setting.

**3. The mathematical formulation**

Based on the system descriptions in section 2, the mathematic formulation is presented for describing the cost incurred by feasible actions at different system states. When the current system state is found at  $w = (i, b, s)$  ( $w \in W$ ) and certain action is chosen, the expected cost from the current time epoch to the following time when the maintenance is finished and the buffer is exhausted (i.e. to the end of the current renewal cycle) is described by  $C(i, b, s)$ . Therefore, the mathematic formulation is as following:

$$C(i, b, 0) = \begin{cases} c_i + hb + \sum_{j=i}^{M+1} p_{ij} C(j, \min(b + p_1, B), 0), & a = 0 \\ c_i + hb + c_{go} + \lambda_3 \sum_{j=i}^{M+1} p_{ij} C(j, \min(b + p_1, B), 1) + (1 - \lambda_3) \sum_{j=i}^{M+1} p_{ij} C(j, \min(b + p_1, B), go), & a = 1 \end{cases} \quad (2)$$

$$C(i, B, 0) = \begin{cases} c_i' + hB + \sum_{j=i}^{M+1} p_{ij} C(j, B, 0), & a = 0 \\ c_i' + hB + c_{go} + \lambda_3 \sum_{j=i}^{M+1} p_{ij} C(j, B, 1) + (1 - \lambda_3) \sum_{j=i}^{M+1} p_{ij} C(j, B, go), & a = 1 \end{cases} \quad (3)$$

$$C(M+1, b, 0) \quad (0 \leq b \leq B) = hb + c_{uo} + c \max(0, p_2 - b) / p_2 + \lambda_4 C(M+1, \max(0, b - p_2), 1) + (1 - \lambda_4) C(M+1, \max(0, b - p_2), uo), \quad a = 2 \quad (4)$$

$$C(i, b, go) \quad (0 \leq i \leq M, 0 \leq b < B) = c_i + hb + \lambda_3 \sum_{j=i}^{M+1} p_{ij} C(j, \min(b + p_1, B), 1) + (1 - \lambda_3) \sum_{j=i}^{M+1} p_{ij} C(j, \min(b + p_1, B), go), \quad a = 0$$

$$\begin{aligned}
 C(i, B, go) \quad (0 \leq i \leq M) &= c_i' + hb + \lambda_3 \sum_{j=i}^{M+1} p_{ij} C(j, B, 1) + (1 - \lambda_3) \sum_{j=i}^{M+1} p_{ij} C(j, B, go), \quad a = 0 \\
 C(M + 1, b, go) \quad (0 \leq b \leq B) &= hb + c \max(0, p_2 - b) / p_2 + \lambda_3 C(M + 1, \max(0, b - p_2), 1) \\
 &\quad + (1 - \lambda_3) C(M + 1, \max(0, b - p_2), go), \quad a = 0 \\
 C(M + 1, b, uo) \quad (0 \leq b \leq B) &= hb + c \max(0, p_2 - b) / p_2 + \lambda_4 C(M + 1, \max(0, b - p_2), 1) \\
 &\quad + (1 - \lambda_4) C(M + 1, \max(0, b - p_2), uo), \quad a = 0 \\
 C(i, b, 1) \quad (0 \leq i \leq M, 0 \leq b \leq B) &= C(PM, b, 1), \quad a = 3 \\
 C(M + 1, b, 1) \quad (p_2 < b \leq B) &= c_f + hb + \lambda_2 C(ID, b - p_2, 0) + (1 - \lambda_2) C(M + 1, b - p_2, 1), \quad a = 4 \\
 C(M + 1, b, 1) \quad (0 \leq b \leq p_2) &= c_p + hb + c(p_2 - b) / p_2 + \lambda_2 C(0, 0, 0) + (1 - \lambda_2) C(M + 1, 0, 1), \quad a = 4 \\
 C(PM, b, 1) \quad (p_2 < b \leq B) &= c_p + hb + \lambda_1 C(ID, b - p_2, 0) + (1 - \lambda_1) C(PM, b - p_2, 1), \quad a = 3 \\
 C(PM, b, 1) \quad (0 \leq b \leq p_2) &= c_p + hb + c(p_2 - b) / p_2 + \lambda_1 C(0, 0, 0) + (1 - \lambda_1) C(PM, 0, 1), \quad a = 3 \\
 C(ID, b, 0) \quad (p_2 < b \leq B) &= hb + C(ID, b - p_2, 0), \quad a = 0 \\
 C(ID, b, 0) \quad (0 \leq b \leq p_2) &= hb + c(p_2 - b) / p_2 + C(0, 0, 0), \quad a = 0
 \end{aligned}
 \tag{5}$$

In eq. (2) and eq. (3), the upper expression on the right of the equal sign represents that the action 0 (do nothing) is adopted, and the lower expression represents that the action 1 (carrying out a general replacement part order) is adopted, when the current system state is found at state  $(i, b, 0)$  ( $0 \leq i \leq M, 0 \leq b < B$ ) and state  $(i, B, 0)$  ( $0 \leq i \leq M$ ) respectively. In eq. (4), the expression on the right of the equal sign represents that the action 2 (carrying out an urgent replacement part order) is adopted when machine  $A_1$  fails and no replacement part is ordered. In eq. (5), the preventive maintenance is initiated, when the ordered replacement part has arrived and the machine state is not at  $M+1$ . In eq. (6), the action 3 (carrying out the preventive maintenance) is adopted when the machine state is found during preventive maintenance ( $i = PM, s = 1$ ). In eq. (7), the action 3 is adopted, and the shortage cost from the current time epoch to the next is calculated by  $c(p_2 - b) / p_2$ , with the buffer level  $b$  is equal to or smaller than the operation rate of machine  $A_2$ . Other equations can be explained similar to eq. (2) ~ eq. (7).

#### 4. The policy-iteration algorithm

Based on section 2, the control-limit policy applied is described by  $B+1$  control parameters, respectively corresponding to different buffer levels, i.e. 0, 1, ...,  $B$ . For clarity, the  $B+1$  control parameters are denoted by  $cp_0, cp_1, \dots, cp_B$ . During each renewal cycle, when the machine state  $i$  ( $i \leq M$ ) is found to exceed the control parameter corresponding to current buffer level, a general replacement part order is carried out. For example, the interpretation of  $cp_2=5$  is that if the buffer inventory between the machines is equal to 2, a general replacement part order should be carried out only if the current machine state exceeds 5. If the control parameter is set as  $M+1$  for buffer level  $b$ , it

means that the replacement part order will not be carried out when the buffer level is  $b$ , unless machine  $A_1$  encounters failure.

In this paper, the policy-iteration algorithm is applied to obtain the minimal long-term expected cost rate and determine the related  $B+1$  control parameters. The main method of this algorithm is successively generating a new policy with smaller long-term expected cost rate than the current one, until the two neighboring policies is the same, or their long-term expected cost rates are of equal values.

Therefore, the steps of policy-iteration algorithm for the optimal control-limit policy in this paper are as following:

##### Step 1

Set the system parameters, including the machine parameters ( $M, p_1, p_2, p_{ij}, c_i, c_i'$ ), maintenance parameters ( $\lambda_1, \lambda_2, c_p, c_f$ ), buffer parameter ( $B, h$ ), shortage parameter ( $c$ ), and replacement part parameters ( $\lambda_3, \lambda_4, c_{go}, c_{uo}$ ). Set the initial policy as  $(cp_0, cp_1, \dots, cp_B) = (M + 1, M + 1, \dots, M + 1)$ .

##### Step 2

Eliminate the inaccessible system states from state space  $W$  in eq. (1), based on the system description and parameter setting. The actual system state space or accessible system state space is denoted as  $W'$ .

##### Step 3

Based on the current policy, determine which action is chosen when the system is found at each accessible state. Then solve the following equations, and obtain the value of  $g$  and values of  $v(i, b, s)$  for all states in  $W'$ .  $g$  is the long-term average cost rate for the current policy:

$$\begin{aligned}
 v(i, b, 0) \\
 (0 \leq i \leq M, 0 \leq b < B) &= \\
 \left\{ \begin{aligned}
 v_0(i, b, 0) &= c_i + hb - g + \sum_{j=i}^{M+1} p_{ij} v(j, \min(b + p_1, B), 0), \quad a = 0 \\
 v_1(i, b, 0) &= c_i + hb + c_{go} - g + \lambda_3 \sum_{j=i}^{M+1} p_{ij} v(j, \min(b + p_1, B), 1) + (1 - \lambda_3) \sum_{j=i}^{M+1} p_{ij} v(j, \min(b + p_1, B), go), \quad a = 1
 \end{aligned} \right. \tag{8}
 \end{aligned}$$

$$\begin{aligned}
 v(i, B, 0) \\
 (0 \leq i \leq M) &= \left\{ \begin{aligned}
 v_0(i, B, 0) &= c_i' + hb - g + \sum_{j=i}^{M+1} p_{ij} v(j, B, 0), \quad a = 0 \\
 v_1(i, B, 0) &= c_i' + hb + c_{go} - g + \lambda_3 \sum_{j=i}^{M+1} p_{ij} v(j, B, 1) + (1 - \lambda_3) \sum_{j=i}^{M+1} p_{ij} v(j, B, go), \quad a = 1
 \end{aligned} \right. \tag{9}
 \end{aligned}$$

$$\begin{aligned}
 v(M+1, b, 0) \quad (0 \leq b \leq B) &= hb + c_{uo} - g + c \max(0, p_2 - b) / p_2 + \lambda_4 v(M+1, \max(0, b - p_2), 1) \\
 &\quad + (1 - \lambda_4) v(M+1, \max(0, b - p_2), uo), \quad a = 2 \\
 v(i, b, go) \quad (0 \leq i \leq M, 0 \leq b < B) &= c_i + hb - g + \lambda_3 \sum_{j=i}^{M+1} p_{ij} V(j, \min(b + p_1, B), 1) \\
 &\quad + (1 - \lambda_3) \sum_{j=i}^{M+1} p_{ij} V(j, \min(b + p_1, B), go), \quad a = 0 \\
 v(i, B, go) \quad (0 \leq i \leq M) &= c_i + hb - g + \lambda_3 \sum_{j=i}^{M+1} p_{ij} V(j, B, 1) + (1 - \lambda_3) \sum_{j=i}^{M+1} p_{ij} V(j, B, go), \quad a = 0 \\
 v(M+1, b, go) \quad (0 \leq b \leq B) &= hb + c \max(0, p_2 - b) / p_2 - g + \lambda_3 v(M+1, \max(0, b - p_2), 1) \\
 &\quad + (1 - \lambda_3) v(M+1, \max(0, b - p_2), go), \quad a = 0 \\
 v(M+1, b, uo) \quad (0 \leq b \leq B) &= hb + c \max(0, p_2 - b) / p_2 - g + \lambda_4 v(M+1, \max(0, b - p_2), 1) \\
 &\quad + (1 - \lambda_4) v(M+1, \max(0, b - p_2), uo), \quad a = 0 \\
 v(i, b, 1) \quad (0 \leq i \leq M, 0 \leq b \leq B) &= v(PM, b, 1), \quad a = 3 \\
 v(M+1, b, 1) \quad (p_2 < b \leq B) &= c_f + hb - g + \lambda_2 v(ID, b - p_2, 0) + (1 - \lambda_2) v(M+1, b - p_2, 1), \quad a = 4 \\
 v(M+1, b, 1) \quad (0 \leq b \leq p_2) &= c_p + hb - g + c(p_2 - b) / p_2 + \lambda_2 v(0, 0, 0) + (1 - \lambda_2) v(M+1, 0, 1), \quad a = 4 \\
 v(PM, b, 1) \quad (p_2 < b \leq B) &= c_p + hb - g + \lambda_1 v(ID, b - p_2, 0) + (1 - \lambda_1) v(PM, b - p_2, 1), \quad a = 3 \\
 v(PM, b, 1) \quad (0 \leq b \leq p_2) &= c_p + hb - g + c(p_2 - b) / p_2 + \lambda_1 v(0, 0, 0) + (1 - \lambda_1) v(PM, 0, 1), \quad a = 3 \\
 v(ID, b, 0) \quad (p_2 < b \leq B) &= hb - g + v(ID, b - p_2, 0), \quad a = 0 \\
 v(ID, b, 0) \quad (0 \leq b \leq p_2) &= hb - g + c(p_2 - b) / p_2 + v(0, 0, 0), \quad a = 0 \\
 v(0, 0, 0) &= 0
 \end{aligned}
 \tag{10}$$

The adding of Eq. (10) makes the number of the equations and that of the unknowns equal. The system state (0, 0, 0) is arbitrary selected.

*Step 4*

For each  $b$  ( $0 \leq b \leq B$ ), calculate the values of  $v_0(M, b, 0)$  and  $v_1(M, b, 0)$  according to eq. (8) or eq. (9). If  $v_0(M, b, 0) < v_1(M, b, 0)$ , change the value of  $cp_b$  to  $M+1$ . Otherwise, calculate the values of  $v_0(M-1, b, 0)$  and  $v_1(M-1, b, 0)$ , and compare them. If  $v_0(M-1, b, 0) < v_1(M-1, b, 0)$ , change the value of  $cp_b$  to  $M$ . Otherwise, calculate the values of  $v_0(M-2, b, 0)$  and  $v_1(M-2, b, 0)$ , and compare them. Do the calculation and comparison as above until finding  $v_0(M-x, b, 0) < v_1(M-x, b, 0)$  ( $0 \leq x \leq M$ ) and changing the value of  $cp_b$  to  $M-x+1$ .

If the inequality  $v_0(M-x, b, 0) \geq v_1(M-x, b, 0)$  is obtained for all  $x$  ( $0 \leq x \leq M$ ), change the value of  $cp_b$  to 0. After the above process is repeated for all  $b$  ( $0 \leq b \leq B$ ), a new policy is generated.

*Step 5*

Compare the new policy and the last policy. If the two policies are the same or the long-term expected cost rates of them are of equal values, stop the iteration. Then the optimal policy is the last two policies, and the minimal long-term expected cost rate is the value of  $g$  under them. Otherwise, treat the new policy as the current one and return to step 3 to repeat the iteration.

**5. Numerical examples**

In this paper, the maintenance planning, buffer inventory, and replacement part order are taken into account for optimizing the long-term expected cost rate and control-limit policy. In order to investigate the influence of system parameters on the optimization result,

numerical examples are delivered under various situations with different maintenance parameters, replacement part order parameters, buffer inventory parameters, and shortage parameters.

**5.1. Sensitivity analysis for maintenance parameter**

The system parameters related to maintenance are  $\lambda_1, \lambda_2, c_p,$  and  $c_f$ . The change of them will cause preference variation of the optimal policy. In this section, the minimal long-term expected cost rate and the optimal policy are investigated in different numerical examples with increasing values of  $c_p$  and  $c_f$  respectively.

*Numerical example 1:* changing the corrective maintenance cost rate  $c_f$

The system parameters are set as following.  $M=5, B=4, \lambda_1 = 0.7, \lambda_2 = 0.5, \lambda_3 = 0.6, \lambda_4 = 0.9, c_p = 4, c_g = 5, c_u = 8, h = 0.7, c = 10, c_i = 0.6(i+1)$  ( $0 \leq i \leq M$ ),  $c_i' = 0.2(i+1)$  ( $0 \leq i \leq M$ ),  $p_1 = 1, p_2 = 1$ . The transition probability from state  $i$  to state  $j$  of machine  $A_1$  is described by the element at the crossing of row  $i+1$  and column  $j+1$  in matrix  $P$ :

$$P = \begin{bmatrix} 0.2 & 0.35 & 0.23 & 0.15 & 0.06 & 0.01 \\ 0 & 0.2 & 0.36 & 0.3 & 0.12 & 0.02 \\ 0 & 0 & 0.18 & 0.5 & 0.26 & 0.06 \\ 0 & 0 & 0 & 0.14 & 0.66 & 0.2 \\ 0 & 0 & 0 & 0 & 0.1 & 0.9 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The corrective maintenance cost rate  $c_f$  is changed from 4 to 8 in the increment of 0.5. For each  $c_f$ , the optimal control-limit policy is obtained, and the long-term expected cost rate  $g$  is minimized, by using the method in section 4. The result is shown in table 1.

*Numerical example 2:* changing the preventive maintenance cost rate  $c_p$

The system parameters are set as numerical example 1, except that the value of  $c_f$  is set as 6, and the value of  $c_p$  is changed from 2 to 6 in the increment of 0.5 (it is reasonable to assume that the preventive maintenance cost rate is not higher than the corrective maintenance cost rate). For each  $c_p$ , the optimal control-limit policy is obtained, and the long-term expected cost rate  $g$  is minimized, by using the method in section 4. The result is shown in table 2.

Table 1. the optimization result of changing the corrective maintenance cost rate  $c_f$

$c_f$	$cp_0$	$cp_1$	$cp_2$	$cp_3$	$cp_4$	min $g$
4.0	5	2	1	1	0	4.3612
4.5	5	2	1	0	0	4.4273
5.0	5	2	1	0	0	4.4933
5.5	5	2	1	0	0	4.5593
6.0	5	2	1	0	0	4.6253
6.5	5	2	1	0	0	4.6914
7.0	5	2	1	0	0	4.7574
7.5	5	2	1	0	0	4.8234
8.0	5	1	1	0	0	4.8840

Table 2. the optimization result of changing the preventive maintenance cost rate  $c_p$

$c_p$	$cp_0$	$cp_1$	$cp_2$	$cp_3$	$cp_4$	min $g$
2.0	5	1	0	0	0	4.3011
2.5	5	1	0	0	0	4.3893
3.0	5	1	0	0	0	4.4775
3.5	5	2	1	0	0	4.5564
4.0	5	2	1	0	0	4.6253
4.5	5	2	1	0	0	4.6943
5.0	5	2	1	0	0	4.7632
5.5	5	3	2	1	1	4.8265
6.0	5	3	2	1	1	4.8783

In each optimal policy inside of table 1 and table 2, the control parameters corresponding to higher buffer levels are smaller than or equal to those corresponding to lower buffer levels. It is for the reason that the replacement part order should be carried out when the buffer level is too high or the value of machine state is too large, to prevent too much inventory holding cost or the failure of machine. However, for the same buffer inventory level, the optimal control parameter is gradually decreased when increasing  $c_f$  (see table 1), and gradually increased when increasing  $c_p$  (see table 2). It can be explained that when the value of  $c_f$  becomes larger, the cost incurred by corrective maintenance also becomes larger. Then the replacement part is expected to be ordered earlier for larger possibility to perform preventive maintenance. Similarly, when the value of  $c_p$  becomes larger, the cost incurred by preventive maintenance also becomes larger. Then the replacement part is not expected to be ordered too early for frequent preventive maintenance.

The minimal long-term average cost is found becoming larger when increasing  $c_f$  or  $c_p$ . It can be explained that the long-term expected cost rate is equal to the ratio of the expected total cost in a renewal cycle and the expected time length of the renewal cycle, and

the maintenance cost is an important section of the total cost. Additionally, it is seen that the increments between the adjacent minimal long-term expected cost rates are almost to be same values (about 0.066) in table 1. And some increments in table 2 also seem to be stable. In order to investigate the character of the increment of minimal long-term expected cost rate when the value of  $c_f$  or  $c_p$  is increased by a determined quantity and to be much larger, numerical example 3 and 4 are delivered.

*Numerical example 3:* The corrective maintenance cost rate  $c_f$  is changed from 4 to 20 in the increment of 0.5. Other parameters are set as numerical example 1. For each  $c_f$ , the optimal control-limit policy is obtained, and the increments of the adjacent minimal long-term expected cost rates are calculated. The result is depicted in figure 2.

*Numerical example 4:* The preventive maintenance cost rate  $c_p$  is changed from 2 to 18 in the increment of 0.5. The corrective maintenance cost rate  $c_f$  is set as 18. Other parameters are set as numerical example 1. For each  $c_p$ , the optimal control-limit policy is obtained, and the increments of the adjacent minimal long-term expected cost rates are calculated. The result is depicted in figure 3.

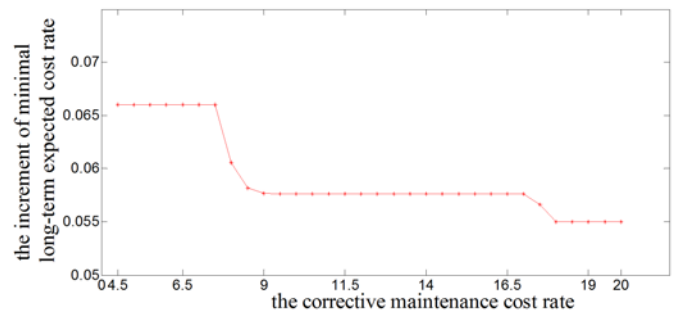


Fig. 2. The increment of minimal long-term expected cost rate when increasing  $c_f$

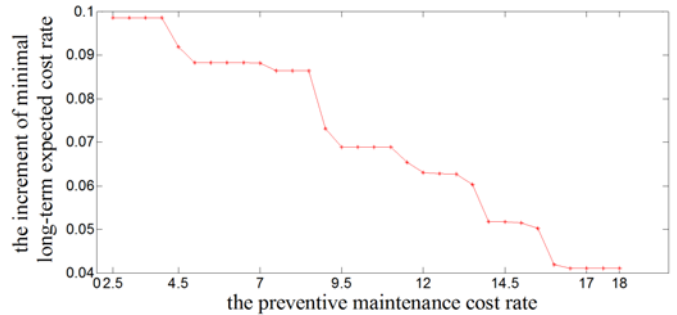


Fig. 3. The increment of minimal long-term expected cost rate when increasing  $c_p$

In Figure 2 and Figure 3, the increment of adjacent minimal long-term expected cost rates appears to be stable in some certain ranges when the value of  $c_f$  or  $c_p$  is increased by a determined quantity. However, the increment does not always keep unchanged. When  $c_f$  or  $c_p$  is much larger, the increment gradually becomes smaller.

## 5.2. Sensitivity analysis for replacement part order parameter

In this section, the minimal long-term expected cost rate and the optimal policy are investigated in different numerical examples with increasing values of  $c_u$  and  $c_g$  respectively.

*Numerical example 5:* changing the urgent replacement part order cost  $c_u$

The system parameters are set as numerical example 1, except that the value of  $c_f$  is set as 7, and the value of  $c_u$  is changed from 6 to 14 in the increment of 1 (it is reasonable that the urgent replacement part order cost is larger than the general replacement part order cost). For each value of  $c_u$ , the optimal control-limit policy is obtained, and the long-term expected cost rate  $g$  is minimized, by using the method in section 4. The result is shown in table 3.

*Numerical example 6:* changing the general replacement part order cost  $c_g$

The system parameters are set as numerical example 5, except that the value of  $c_u$  is set as 11, and the value of  $c_g$  is changed from 2 to 10 in the increment of 1. For each value of  $c_g$ , the optimal control-limit policy is obtained, and the long-term expected cost rate  $g$  is minimized, by using the method in section 4. The result is shown in Table 4.

Table 3. the optimization result of changing the urgent replacement part order cost  $c_u$

$c_u$	$cp_0$	$cp_1$	$cp_2$	$cp_3$	$cp_4$	min $g$
6	5	2	1	0	0	4.7511
7	5	2	1	0	0	4.7542
8	5	2	1	0	0	4.7574
9	5	2	1	0	0	4.7605
10	5	2	1	0	0	4.7637
11	5	2	1	0	0	4.7668
12	5	2	1	0	0	4.7700
13	5	2	1	0	0	4.7731
14	5	2	1	0	0	4.7763

Table 4. the optimization result of changing the general replacement part order cost  $c_g$

$c_g$	$cp_0$	$cp_1$	$cp_2$	$cp_3$	$cp_4$	min $g$
2	5	1	0	0	0	4.2388
3	5	1	0	0	0	4.4178
4	5	1	0	0	0	4.5968
5	5	2	1	0	0	4.7668
6	5	2	1	0	0	4.9262
7	5	2	1	0	0	5.0856
8	5	2	1	0	0	5.2450
9	5	2	2	1	0	5.4017
10	5	2	2	1	0	5.5553

In each optimal policy inside of table 3 and table 4, the control parameters corresponding to higher buffer levels are smaller than or equal to those corresponding to lower buffer levels, which is similar to the situations of table 1 and table 2 for the same reason. As for the same buffer inventory level, the corresponding optimal control parameter is not changed when increasing  $c_u$  (see table 3), but gradually gets larger when increasing  $c_g$  (see table 4). Additionally, the increment between adjacent minimal long-term expected cost rates with increasing  $c_g$  is obviously larger than that with increasing  $c_u$ . It means that the optimization result is much more sensitive to  $c_g$  than to  $c_u$ . It can be explained that the economical way of restoring machine  $A_1$  as new is performing general replacement part order and

preventive maintenance, and then much more general replacement part orders should be carried out in a long term under an optimal policy. Note that although the corrective maintenance is not preferred in an optimal policy, the optimization result is still sensitive to  $c_f$  (see table 1). It is because that the machine may encounter failure when waiting for a general replacement part order, and the success rate of corrective maintenance is not high.

In order to investigate the character of the increment of minimal long-term expected cost rate when the value of  $c_u$  or  $c_g$  is increased by a determined quantity and to be much larger, numerical example 7 and 8 are delivered.

*Numerical example 7:* The system parameters are set as numerical example 1, except that the value of  $c_f$  is set as 7, and the value of  $c_u$  is changed from 6 to 64 in the increment of 2. For each value of  $c_u$ , the optimal control-limit policy is obtained, and the increments of the adjacent minimal long-term expected cost rates are calculated. The result is depicted in Figure 4.

*Numerical example 8:* the value of  $c_f$  is set as 7, and the urgent replacement part order cost is set as 32. The value of  $c_g$  is changed from 2 to 31 in the increment of 1. Other parameters are set as numerical example 1. For each value of  $c_g$ , the optimal control-limit policy is obtained, and the increments of the adjacent minimal long-term expected cost rates are calculated. The result is depicted in Figure 5.

Both in figure 4 and figure 5, the increments of minimal long-term

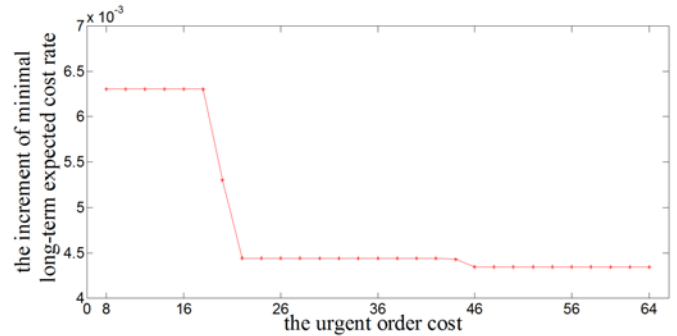


Fig. 4. The increment of minimal long-term expected cost rate when increasing  $c_u$

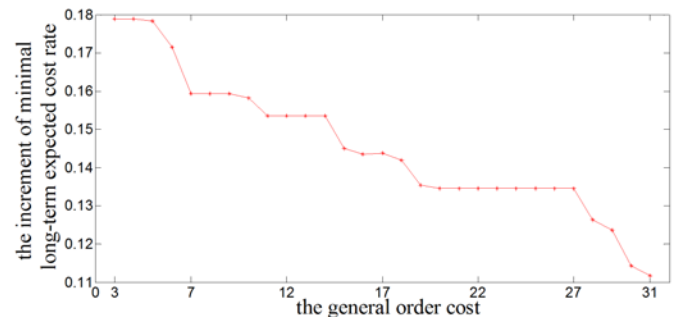


Fig. 5. The increment of minimal long-term expected cost rate when increasing  $c_g$

expected cost rate are decreased, although in some ranges it remains unchanged. It is seen that the increment with increasing  $c_u$  is very small (lower than 0.0065, about 0.14% of the minimal long-term expected cost rate). Therefore, the minimal long-term expected cost rate can be considered as stable when the value of  $c_u$  is increased.

**5.3. Sensitivity analysis for buffer inventory parameter**

In this section, the minimal long-term expected cost rate and the optimal policy are investigated in numerical examples with increasing value of  $h$ .

*Numerical example 9:* changing the buffer inventory parameter  $h$

The system parameters are set as numerical example 1, except that the value of  $c_f$  is set as 7, and the value of  $h$  is changed from 0.1 to 1.7 in the increment of 0.2. For each value of  $h$ , the optimal control-limit policy is obtained, and the long-term expected cost rate  $g$  is minimized, by using the method in section 4. The result is shown in table 5.

Table 5. The optimization result of changing the inventory holding cost rate  $h$

$h$	$cp_0$	$cp_1$	$cp_2$	$cp_3$	$cp_4$	min $g$
0.1	5	2	2	1	1	3.8357
0.3	5	2	2	1	1	4.1494
0.5	5	2	1	0	0	4.4568
0.7	5	2	1	0	0	4.7574
0.9	5	1	0	0	0	5.0401
1.1	5	1	0	0	0	5.3110
1.3	5	0	0	0	0	5.5787
1.5	5	0	0	0	0	5.8305
1.7	5	0	0	0	0	6.0824

In each optimal policy inside of table 5, it is seen that the control parameters corresponding to higher buffer levels are smaller than or equal to those corresponding to lower buffer levels, which is similar to the situations of increasing  $c_f$ ,  $c_p$ ,  $c_u$ , and  $c_g$  for the same reason. For the same buffer inventory level, the control parameter is gradually decreased when  $h$  is increased. It can be explained that when the buffer inventory holding cost rate is increased, it is preferred to perform the replacement part order and the maintenance earlier under an optimal policy, to prevent too much buffer inventory holding cost.

It is found that the minimal long-term expected cost rate becomes larger with increasing  $h$ . In order to investigate the character of the increment of minimal long-term expected cost rate when the value of  $h$  is increased by a determined quantity and to be much larger, numerical example 10 is delivered.

*Numerical example 10:* The system parameters are set as numerical example 1, except that the value of  $c_f$  is set as 7, and the value of  $h$  is changed from 0.1 to 8.8 in the increment of 0.3. For each value of  $h$ , the optimal control-limit policy is obtained, and the increments of the adjacent minimal long-term expected cost rates are calculated. The result is depicted in figure 6.

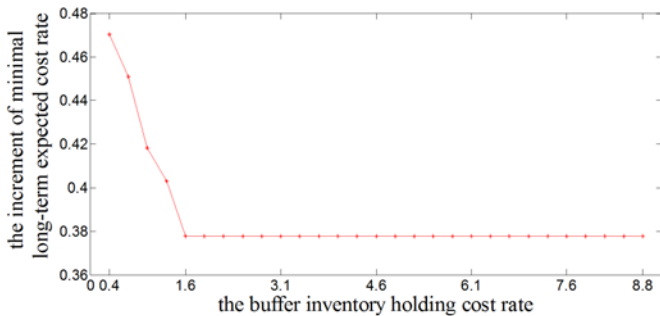


Fig. 6. The increment of minimal long-term expected cost rate when increasing  $h$

In Figure 6, the increment of minimal long-term expected cost rate is continually decreased until the value of  $h$  is increased to 1.6. It means that when the value of  $h$  is equal to or larger than 1.6, under the parameter setting in the numerical example, the minimal long-term expected cost rate is proportional to  $h$ .

**5.4. Sensitivity analysis for shortage parameter**

In this section, the minimal long-term expected cost rate and the optimal policy are investigated in numerical examples with increasing value of  $c$ .

*Numerical example 11:* changing the shortage cost rate  $c$

The system parameters are set as numerical example 1, except that the value of  $c_f$  is set as 7, and the value of  $c$  is changed from 3 to 27 in the increment of 3. For each value of  $c$ , the optimal control-limit policy is obtained, and the long-term expected cost rate  $g$  is minimized, by using the method in section 3. The result is shown in table 6.

Table 6. The optimization result of changing the shortage cost rate  $c$

$c$	$cp_0$	$cp_1$	$cp_2$	$cp_3$	$cp_4$	min $g$
3	5	1	0	0	0	4.4215
6	5	1	1	0	0	4.5701
9	5	2	1	0	0	4.7106
12	5	2	1	0	0	4.8509
15	5	2	1	0	0	4.9912
18	5	2	1	0	0	5.1315
21	5	2	1	0	0	5.2718
24	5	2	1	0	0	5.4121
27	5	2	1	0	0	5.5523

In table 6, it is seen that in each optimal policy the control parameters corresponding to higher buffer levels are smaller than or equal to those corresponding to lower buffer levels, which is similar to the situations with increasing  $c_f$ ,  $c_p$ ,  $c_u$ ,  $c_g$ , and  $h$  for the same reason. For certain lower buffer inventory level ( $b=0, 1, \text{ or } 2$ ), the corresponding control parameter is gradually increased when  $c$  is increased. It can be explained that when the shortage cost rate gets larger, it is more expected in an optimal policy to build higher buffer inventory level preparing for maintenance duration and preventing shortage. However, for certain higher buffer inventory level ( $b=3 \text{ or } 4$ ), the corresponding control parameter is not increased when  $c$  is increased. It is because that for higher buffer inventory levels, the replacement part order is preferred to be carried out earlier to prevent the shortage caused by machine failure.

The minimal long-term expected cost rate is found to become larger with increasing  $c$ . In order to investigate the character of the increment of minimal long-term expected cost rate when the value of  $c$  is increased by a determined quantity and to be much larger, numerical example 12 is delivered.

*Numerical example 12:* The system parameters are set as numerical example 1, except that the value of  $c_f$  is set as 7, and the value of  $c$  is changed from 3 to 61 in the increment of 2. For each value of  $c$ , the optimal control-limit policy is obtained, and the increments of the adjacent minimal long-term expected cost rates are calculated. The result is depicted in figure 7.

In figure 7, it is seen that when the value of  $c$  is smaller than 7, the increment is continually decreased with increasing  $c$ . It is also seen that in a wide range (the value of  $c$  is about 7 ~49), the increment of minimal long-term expected cost rate keeps unchanged (equaling to 0.0935). It means that the minimal long-term expected cost rate



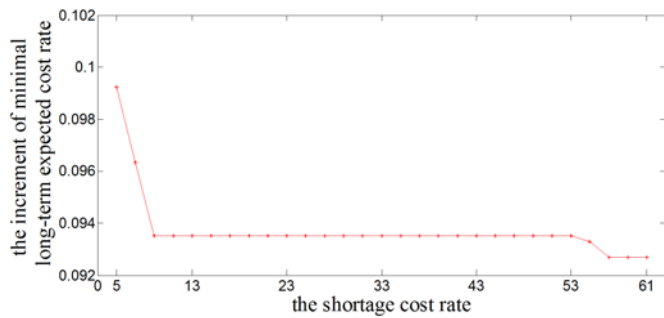


Fig. 7. The increment of minimal long-term expected cost rate when increasing  $c$

is proportional to  $c$  in the range and under the parameter setting of numerical example 12.

## 6. Conclusion

In this paper, a production system consisting of two serial machines and an intermediate buffer is studied. The deterioration of the upstream machine is considered. One type of control-limit policy is applied, which takes into account the maintenance, replacement part order, and buffer level. The system and decision process are modeled by discrete Markov method, and through a policy-iteration algorithm, the long-term expected cost rate and control policy are optimized.

Numerical examples are delivered for parameter sensitive analysis. The result shows that in all cases the optimal control parameters corresponding to higher buffer levels are smaller than or equal to those corresponding to lower buffer levels. It is also shown that the change of urgent replacement part order cost doesn't have obvious effect on

the optimal result. However, the increasing of any other parameter (the maintenance cost rate, general replacement part order cost, buffer inventory holding cost rate, or shortage cost rate) makes the minimal long-term expected cost rate become larger, and under some situations, the minimal long-term expected cost rate is proportional to the parameter. For the same buffer inventory level, the optimal control parameter gradually gets larger with increasing preventive maintenance cost rate or general replacement part order cost, and gets smaller with increasing corrective maintenance cost rate or buffer inventory holding cost rate. When the shortage cost rate is increased, the optimal control parameters for lower buffer levels gradually become larger; however, the optimal control parameters for higher buffer levels keep stable.

Additionally, the maintenance time duration and replacement part lead time are both assumed to follow geometric distribution in this paper. If they follow some known continuous distribution, the states of replacement part order and maintenance can be respectively divided into several states, and then the continuous Markov model is transformed into discrete Markov model which can be analyzed by using the method proposed in this paper. However, the system state will be changed to be much larger in the situation. An effect method for solving this problem is needed to be studied in future.

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