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Thermal dynamics of a building

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Abstract

The work presents the thermal dynamics of the building. The mathematical model covers the issues of heat exchange and temperature control in the room due to changing environmental conditions. The presented work describes the operation of the control system due to thermal losses and thermal inertia of the building. Mathematical model introduces a new method of thermal loss analysis in the conditions of variable thermal dynamics of the building.

Keywords: Transient model; Thermal loss; Thermal inertia; Building

Nomenclature

A – heat exchange surface, flow cross-section
c – specific heat
G – transfer function (transmittance)
k – overall heat transfer coefficient
K_d – derivative gain

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K_i – integrating gain
K_p – proportional gain
m – mass
s – Laplace operator
t – time
T – temperature
w – flow velocity
x – discretized distance

Greek symbols

ρ – density

1 Introduction

Heat losses in building energy analysis in steady-state conditions have significant impact on building's energy consumption. It is important where and how high they are, especially in the case of thermal imaging measurement, whose aim is to determine heat losses of a building. Thermal dynamics analysis of a building can give a guideline which rooms should be more isolated and which can be less. As a result, it is possible to reduce investment costs while maintaining the functionality of individual rooms without incurring higher exploitation and investment costs. This paper presents methodology for theoretical thermal dynamics analysis of a building in view of temperature changes in rooms over time.

2 Theoretical description and mathematical model

According to literature, dynamics analysis of buildings is carried out with the use of thermoelectric analogy, which is described quite well in the following works [1–3]. The 'novelty' described in these paper is a theoretical research on thermal dynamics of buildings due to changes in thermostat settings regulated by PID (proportial, integral, derivative) rules of the continuous regulator, which have not yet been published (for example in the known literature [4–6]). In [4], the model is proposed to simplify the building thermal behavior. The building internal mass is represented by a thermal network of lumped thermal mass and the parameters are identified using operation data. Genetic algorithm (GA) estimators are developed in [4] to identify these parameters. The paper [5] presents a simplified physical model for estimating the average air temperature in multi-zone heating systems. The model is used in an inferential control scheme to improve the control of boilers in heating systems. The paper [6] describes the model-based building systems control idea. The article [6] illustrates this idea using a simple computational scenario pertaining to the control of the thermal environment in buildings, and report on a prototypical implementation of a model-based lighting control strategy – the paper describe control system based on thermostat without using the continuous PID regulator.

Papers [7,8] describe basics connected with thermal management and energy balance measurements of buildings, as well as rules for preparing the energy balance of buildings. The papers also describe an initial form of the applied equations. Mathematical description of heat transfer between heated and unheated rooms is not difficult because the heat transfer between rooms depends on the materials used, building partitions, thermal parameters of mutual neighborhood and separation walls. It is the process of formulating the equations describing processes of heat exchange that is tedious for simple typical configuration. An example of one of those is shown in Fig. 1. Geometry of rooms is shown in Fig. 2.

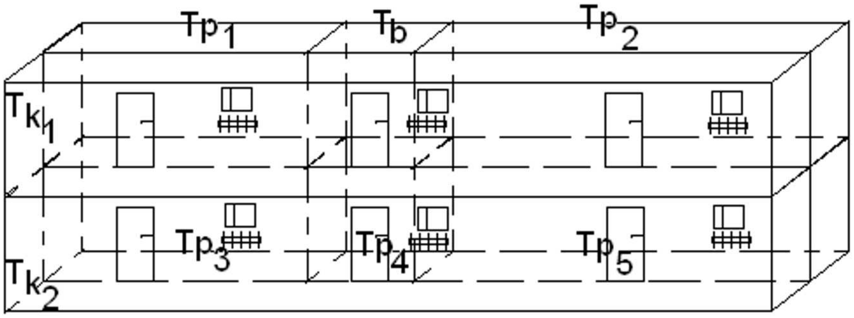

Figure 1: Structure of the modelled rooms.

Heat gains and losses in the reference room T_b are related to heat transfer through wall partitions, which may result from: rooms insulation, operating electrical equipment, number of people in the room, ventilation system, losses to the ground and environment, etc. The temperature in the room depends on heat gains and losses in the room. When heat losses are higher than heat gains, there is a constant decrease in temperature. When heat losses are lower than heat gains, temperature in the room increases. A balance between the gains and losses guarantees maintaining a certain temperature. This balance is maintained only for steady-states, for which heat-providing systems are designed. In working conditions we have to deal with dynamic changes in heat gains and losses, which in turn generates diverse and often patchy heat transfer distribution between rooms. As a result, precise determination of heat transfer between partitions indicating volume of the heated room can be difficult to determine. In this section there are examples of calculations of daily temperature changes over time for the reference room, T_b. The room is adjacent to heated rooms, T_{p1}, T_{p2}, T_{p3}, T_{p4}, T_{p5}, and unheated corridors, T_{k1} and T_{k2}, which are in contact with ambience and, depending on geometric distribution, with the ground.

The calculations allow us to determine temperature changes, which result from imbalance between the heat demand (losses) and heat supplied (gains).

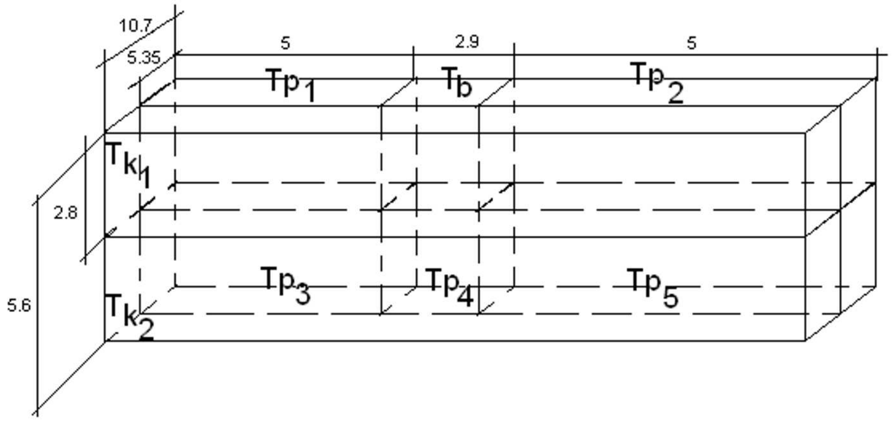

Figure 2: Geometry of the modelled rooms.

Mathematical model of the room b is shown below. In the model the following assumptions and simplifications were adopted:

1. The model does not examine the effect of material sizes, insulation thickness on the obtained temperature courses in the rooms.
2. The model is used to predict the temperature change over time due to change in thermostat setting.
3. The proposed model and the results obtained are purely hypothetical.
4. The change of the air temperature outside the building is modeled as a random signal.
5. The model describes daily temperature changes in winter.
6. Heat gains from solar radiation and electrical equipment are omitted.
7. Air humidity changes and heat gains and losses of the ventilation system are omitted.

8. Sinusoidal changes of expected temperature inside the room associated with the regulator are assumed.
9. Ambient temperature is described by a signal that is rising and then falling in a linear manner.
10. Partitions of the heated walls have the same heat transfer coefficients both in walls bordering with the ambience and in walls bordering with unheated rooms.
11. Warm water supply at the inlet to the heating radiator is carried out at a temperature of 50 °C.

The equations present:

- the temperature change over time in the analyzed room b:

$$\begin{aligned}\frac{dT_b}{dt} m_b c_b &= A_{b,p1} k_{b,p1} \left(T_{p1} - T_b\right) + A_{b,p2} k_{b,p2} \left(T_{p2} - T_b\right) \\ &+ A_{b,p4} k_{b,p4} \left(T_{p4} - T_b\right) + A_{b,roof} k_{b,roof} \left(T_{ambient} - T_b\right) \\ &+ A_{b,windowwall} k_{b,windowwall} \left(T_{ambient} - T_b\right) + A_{b,k1} k_{b,k1} \left(T_{k1} - T_b\right) \\ &+ A_{b,heat\,exchanger} k_{b,heat\,exchanger} \left(T_{heat\,exchanger} - T_b\right) \,, \qquad (1)\end{aligned}$$

- T_{p1} temperature change in the room p_1:

$$\begin{aligned}\frac{dT_{p1}}{dt} m_{p1} c_{p1} &= A_{b,p1} k_{b,p1} \left(T_b - T_{p1}\right) + A_{k1,p1} k_{k1,p1} \left(T_{k1} - T_{p1}\right) \\ &+ A_{p1,p3} k_{p1,p3} \left(T_{p3} - T_{p1}\right) \\ &+ A_{p1,windowwall} k_{p1,windowwall} \left(T_{ambient} - T_{p1}\right) \\ &+ A_{p1,roof} k_{p1,roof} \left(T_{ambient} - T_{p1}\right) \\ &+ A_{b,heat\,exchanger} k_{b,heat\,exchanger} \left(T_{heat\,exchanger} - T_{p1}\right) \,, \qquad (2)\end{aligned}$$

- T_{p2} temperature change in the room p_2:

$$\begin{aligned}\frac{dT_{p2}}{dt} m_{p2} c_{p2} &= A_{b,p2} k_{b,p2} \left(T_b - T_{p2}\right) + A_{k1,p2} k_{k1,p2} \left(T_{k1} - T_{p2}\right) \\ &+ A_{p5,p2} k_{p5,p2} \left(T_{p5} - T_{p2}\right) \\ &+ A_{p2,windowwall} k_{p2,windowwall} \left(T_{ambient} - T_{p2}\right) \\ &+ A_{p1,roof} k_{p1,roof} \left(T_{ambient} - T_{p2}\right) \\ &+ A_{p2,heatexchanger} k_{p2,heatexchanger} \left(T_{heatexchanger} - T_{p2}\right) \,, \qquad (3)\end{aligned}$$

- T_{p3} temperature change in the room p$_3$:

$$\begin{aligned}
\frac{dT_{p3}}{dt} m_{p3} c_{p3} &= A_{k2,p3} k_{k2,p3} \left(T_{k2} - T_{p3}\right) + A_{p4,p3} k_{p4,p3} \left(T_{p4} - T_{p3}\right) \\
&+ A_{p3,p1} k_{p3,p1} \left(T_{p1} - T_{p3}\right) \\
&+ A_{p3,windowwall} k_{p3,windowwall} \left(T_{ambient} - T_{p3}\right) \\
&+ A_{p3,ground} k_{p3,ground} \left(T_{ground} - T_{p3}\right) \\
&+ A_{p3,heat\,exchanger} k_{p3,heat\,exchanger} \left(T_{heat\,exchanger} - T_{p3}\right) \,,
\end{aligned} \tag{4}$$

- T_{p4} temperature change in the room p$_4$:

$$\begin{aligned}
\frac{dT_{p4}}{dt} m_{p4} c_{p4} &= A_{b,p4} k_{b,p4} \left(T_b - T_{p4}\right) + A_{k2,p4} k_{k2,p4} \left(T_{k2} - T_{p4}\right) \\
&+ A_{p3,p4} k_{p3,p4} \left(T_{p3} - T_{p4}\right) + A_{p5,p4} k_{p5,p4} \left(T_{p5} - T_{p4}\right) \\
&+ A_{p4,windowwall} k_{p4,windowwall} \left(T_{ambient} - T_{p4}\right) \\
&+ A_{p4,ground} k_{p4,ground} \left(T_{ground} - T_{p4}\right) \\
&+ A_{4,heat\,exchanger} k_{4,heat\,exchanger} \left(T_{heat\,exchanger} - T_{p4}\right) \,,
\end{aligned} \tag{5}$$

- T_{p5} temperature change in the room p$_5$:

$$\begin{aligned}
\frac{dT_{p5}}{dt} m_{p5} c_{p5} &= A_{k2,p5} k_{k2,p5} \left(T_{k2} - T_{p5}\right) + A_{p4,p5} k_{p2,p5} \left(T_{p4} - T_{p5}\right) \\
&+ A_{p2,p5} k_{p2,p5} \left(T_{p2} - T_{p5}\right) \\
&+ A_{p5,windowwall} k_{p5,windowwall} \left(T_{ambient} - T_{p5}\right) \\
&+ A_{p5,ground} k_{p5,ground} \left(T_{ground} - T_{p5}\right) \\
&+ A_{p5,heat\,exchanger} k_{p5,heat\,exchanger} \left(T_{heat\,exchanger} - T_{p5}\right) \,,
\end{aligned} \tag{6}$$

- T_{k1} temperature change in the corridor k$_1$:

$$\begin{aligned}
\frac{dT_{k1}}{dt} m_{k1} c_{k1} &= A_{b,k1} k_{b,k1} \left(T_b - T_{k1}\right) + A_{k1,p1} k_{k1,p1} \left(T_{p1} - T_{k1}\right) \\
&+ A_{k2,k1} k_{k2,k1} \left(T_{k2} - T_{k1}\right) + A_{k1,p2} k_{k1,p2} \left(T_{p2} - T_{k1}\right) \\
&+ A_{k1,windowwall} k_{k1,windowwall} \left(T_{ambient} - T_{k1}\right) \\
&+ A_{k1,roof} k_{k1,roof} \left(T_{ambient} - T_{k1}\right) \\
&+ A_{k1,heat\,exchanger} k_{k1,heat\,exchanger} \left(T_{heat\,exchanger} - T_{k1}\right) \,,
\end{aligned} \tag{7}$$

- T_{k2} temperature change in the corridor k$_2$:

$$\begin{aligned}\frac{dT_{k2}}{dt}m_{k2}c_{k2} &= A_{k2,k1}k_{k2,k1}\left(T_{k1}-T_{k2}\right)+A_{k2,p4}k_{k2,p4}\left(T_{p4}-T_{k2}\right)\\ &+ A_{k2,p3}k_{k2,p3}\left(T_{p3}-T_{k2}\right)+A_{k2,p5}k_{k2,p5}\left(T_{p5}-T_{k2}\right)\\ &+ A_{k2,windowwall}k_{k2,windowwall}\left(T_{ambient}-T_{k2}\right)\\ &+ A_{k2,ground}k_{k2,ground}\left(T_{ground}-T_{k2}\right)\\ &+ A_{k2,heat\,exchanger}k_{k2,heat\,exchanger}\left(T_{heat\,exchanger}-T_{k2}\right)\ . \end{aligned}\tag{8}$$

The equation describing the heating medium in the analyzed room b has the form

$$\rho A c_p\frac{dT_{heat\,exchanger}}{dt}=\rho w A c_p\frac{T_{in}-T_{heat\,exchanger}}{\frac{\triangle x}{2}}-\rho w A c_p\frac{T_{heat\,exchanger}-T_{out}}{\frac{\triangle x}{2}}$$
$$-A_{b,heat\,exchanger}k_{b,heat\,exchanger}\left(T_{heat\,exchanger}-T_b\right)\ . \tag{9}$$

The regulator of the flow through the heat centre is described by the following dependence:

$$G_{PID}\left(s\right)=K_p+\frac{K_i}{s}+K_d s\,. \tag{10}$$

The following time constants of the regulator were subjected to optimization by using of the methodology presented in [9,10]:

- proportional gain K_p,
- integral gain K_i,
- derivative gain K_d.

The equations presented above may be modified by costs related to covering the heat demand. As a result, we obtain a model system of continuous calculation of costs for multi-room buildings. The analyzed heat transfer system may be extended from the daily mode to the whole heating period, which results in prolonged time of calculations.

3 Calculations of the modelled room temperature

This chapter describes calculations made in accordance with the presented model by using the commercial Matlab-Simulink software [11]. Figure 3 shows the thermostat setting that changes according to the assumptions made for the purpose of calculations. Figure 4 shows the adopted ambient temperature changes. Figure 5 shows results of the calculations for rooms adjacent to the tested room. Figure 6 summarizes the temperature changes on the radiator in the tested room and ambient temperature changes.

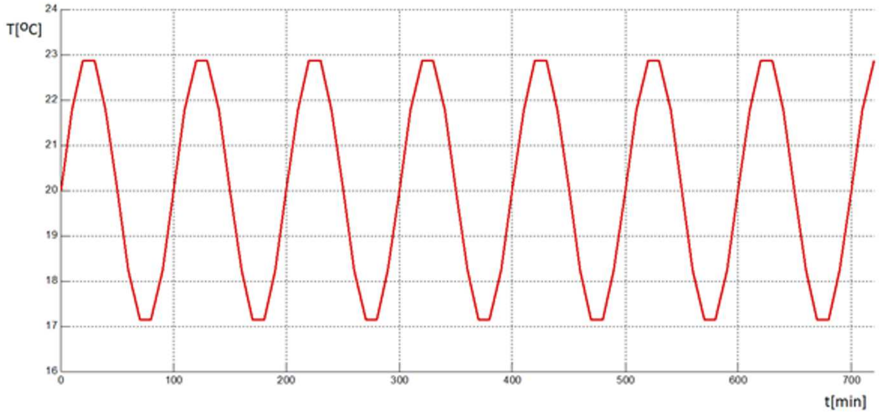

Figure 3: The temperature on the outer surface of the radiator in the heated room resulting from the thermostat setting.

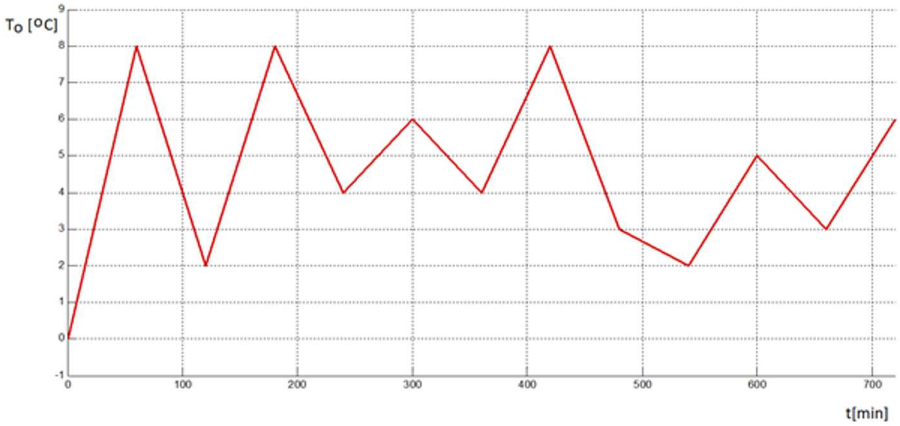

Figure 4: Ambient temperature.

The calculations show that the unheated rooms (e.g. underheated corridors adjacent to heated rooms and ambience) significantly contribute to heat losses of the heated rooms. It is so because the separation walls are not isolated and often the doors or windows in the stairwells are not airtight. As heating of the corridors and ventilation losses associated with them can worsen the energy balance of the building and, as a result, increase exploitation costs of the heating system, they should be isolated properly. This is especially noticeable in Fig. 5, where the temperature increase in the corridors bordering with heated rooms is quite

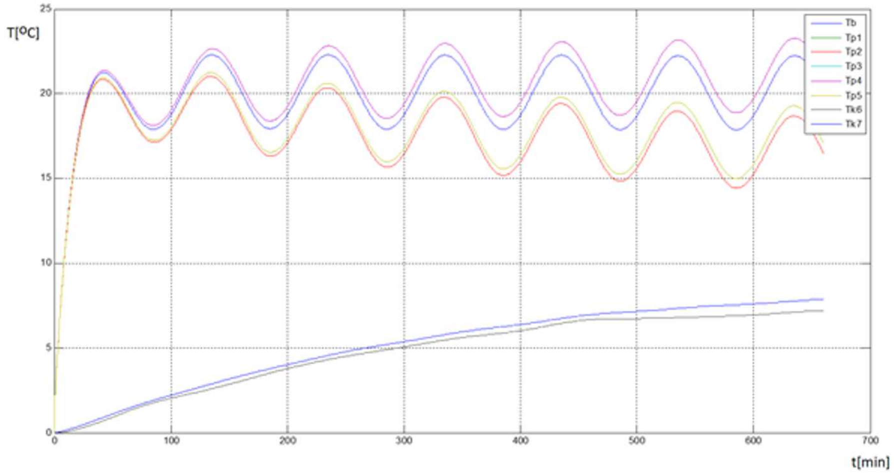

Figure 5: Temperature changes in the tested room, T_b, and adjacent rooms, T_{p1}–T_{k7}.

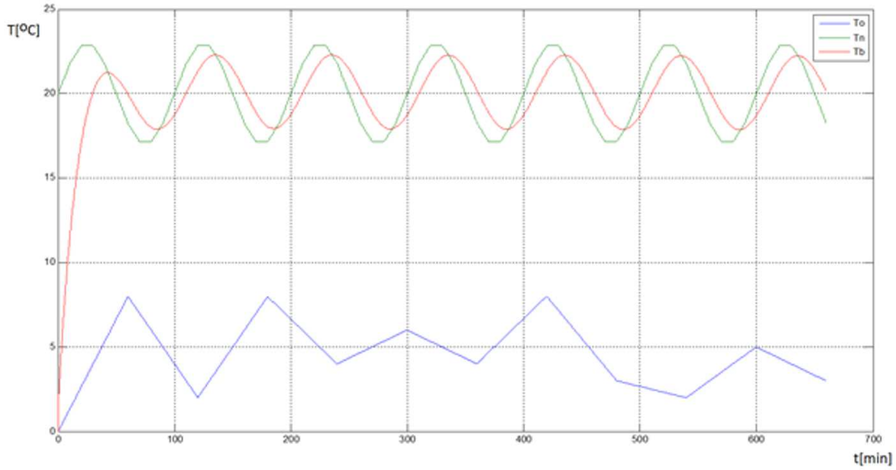

Figure 6: Inertia of b room heating system with reference to the thermostat setting T_n and ambient temperature T_o.

slow. It results from the influence of ambient temperature and heat transfer coefficients for non-insulated walls. Ambient temperature oscillations around its average $T_o = 5$°C are smoothed by heat transfer through the walls. As a result, these oscillations do not affect much the temperature in the rooms.

4 Summary

On the basis of the performed theoretical calculations, it can be concluded that thermal inertia of the unheated rooms adjacent to the directly heated rooms is connected primarily with the heat transfer coefficient value as well as thickness and mass of the partition walls. The influence of unheated rooms on heat losses of the heated rooms is so significant that it is legitimate to isolate them from each other. Heat losses resulting from temperature in the corridors may also have a significant impact on heat losses of the whole building. This gives a guideline that also caring about the airtightness and good isolation between partitions inside a building (between unheated corridors and heated rooms) improves energy savings of the building.

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