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Heuristic control of nonlinear power systems: Application to the infinite bus problem

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In this paper, we apply the heuristic method for determination of control functions for controllability analysis of nonlinear power systems. The problem of control of quasi-linear systems under proper assumptions on the nonlinear term is considered in the general statement. Making use of the Green's function solution of nonlinear systems, the exact and approximate controllability conditions are expressed in terms of unknown controls in an explicit form. The way of resolving controls determination is discussed. As a particular application, a one-machine infinite-bus system is considered described by a coupled system of three first order ordinary differential equations. Two heuristic forms of admissible controls are considered providing approximate controllability within the same amount of time having different intensities. Results of numerical simulations are presented and discussed.

Key words: power systems, nonlinear control, heuristic method, harmonic control, piecewise constant control

1. Introduction

Due to continuously increasing demands on efficiency and reliability, power systems become more complicated and focused infrastructures [1]. With the development of large-scale interconnected power systems, the efficiency of electric generation and transmission has been increased, causing a substantial increase in operation complexity of power systems [2]. Hence, the application of advanced control systems is necessary to ensure a proper operation of power systems [3]. Generally, power systems are considered as nonlinear systems, so that nonlinear methods must be used to achieve a better performance [4].

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For a long time, linear controls has been used to design controllers. In spite of the fact that controllers are validated through simulations, practical robustness cannot be guaranteed with the aid of linear power systems model. Hence, it is necessary to use nonlinear characteristics and nonlinear methods to gain better power [5]. Different control strategies have been employed to design high performance power systems, such as automatic generation control [6], model predictive control [7], robust H^∞ output feedback control [8], optimal control [9], adaptive control, variable structure control, etc. One of the most commonly considered model power systems are the so-called one-machine infinite-bus (OMIB) systems introduced by De Mello and Concordia in [10]. Many studies have been devoted to analysis and control of OMIB systems such as [5, 11–14] (see also related references therein).

In spite the fact that explicit solution of nonlinear power systems would allow to reduce the computational complexity of simulations which is very important especially from qualitative analysis point of view, rigorous analysis of these systems, in most cases, is impossible due to the lack of exact methods of integration of governing nonlinear system of differential equations. This paper addresses some aspects of explicit analysis of some quasi-linear power systems. Under quite general assumptions, making use of nonlinear Green's function technique [15-17], the general solution of the governing system is represented in an explicit form in terms of all system parameters and control function. Then, exact and approximate controllability conditions are derived by satisfying given terminal conditions. As an application, controllability of a most commonly used model of OMIB is studied. Some heuristic controls are proposed, the efficacy of which are confirmed by a numerical simulation.

2. Heuristic method in controllability analysis of nonlinear systems

For some control systems, especially for those whose state is described by nonlinear state constraints, the rigorous determination of controls guaranteeing a required performance is difficult and even impossible. More often, numerical methods of approximation are used. Nevertheless, because of high computational costs, numerical methods may require a burdensome machinery time especially if the analysis is required to carry out in multiple ranges of system parameters. Thus, the development of a simple method for control determination is a challenging problem for many applied areas.

One of the efficient rigorous method for determination of controls governing systems with nonlinear state constraints, is the so-called heuristic approach developed in [17, 18]. In simple words, the heuristic approach, based on some specific considerations, allows to construct parametric families of controls guaranteeing a required performance. As a rule, some of the parameters are determined by a direct substitution of the control into the state constraints, the other parameters are chosen to minimize some cost functional. The mentioned considerations usually correspond to the peculiarities of the control system under study. For instance, if the behavior of a system is expected to be periodic, it is necessary to choose periodic controls.

In order to illustrate the heuristic method, consider a control system whose state is described by the vector function $\boldsymbol{s} : \mathcal{U} \times \mathbb{R}^+ \to \mathbb{R}^n$, where $\mathcal{U}$ is the set of admissible controls. Let the state function satisfies the system of abstract differential equations

$$\frac{\partial^k \boldsymbol{s}}{\partial t^k} + N\left(\frac{\partial^{k-1}\boldsymbol{s}}{\partial t^{k-n}}, \ldots, \frac{\partial \boldsymbol{s}}{\partial t}, \boldsymbol{s}\right) = \boldsymbol{f}(\boldsymbol{u}, t), \quad t > 0, \tag{1}$$

where $k \in \mathbb{N}$ is finite, and appropriate *linear* homogeneous Cauchy conditions

$$\left.\frac{\partial^n \boldsymbol{s}}{\partial t^n}\right|_{t=0} = 0, \qquad n = 0, 1, \ldots, k-1. \tag{2}$$

Partial derivatives are used above in order to indicate that $\boldsymbol{s}$ depends on two parameters: $\boldsymbol{u}$ and t. Here, N is a nonlinear vector, $\boldsymbol{f}$ is the source function, satisfying existence and uniqueness conditions for the solution to (1), (2).

Then, under additional assumption that N is a generalized homogeneous function, i.e.,

$$N(\theta \cdot \boldsymbol{s}_{k-1}, \ldots, \theta \cdot \boldsymbol{s}_1, \theta \cdot \boldsymbol{s}) = \theta \cdot N(\boldsymbol{s}_{k-1}, \ldots, \boldsymbol{s}_1, \boldsymbol{s}),$$

where θ is the Heaviside function, it is shown in [15, 16] that the general solution of (1), (2) can be represented as follows:

$$\boldsymbol{s}(\boldsymbol{u}, t) = \int_0^t \mathbf{G}(t-\tau)\boldsymbol{f}(\boldsymbol{u}, \tau) g(t-\tau)\, \mathrm{d}\tau. \tag{3}$$

Here, $\mathbf{G}$ is the nonlinear Green's matrix of (1), (2), g is a scalar function chosen in numerical purposes. See [16] for error bound, numerical determination of g, particular forms of N leading to validity of (3) and special representation of $\mathbf{G}$ through the solution of homogeneous part of (1).

Thus, (3) provides an explicit relation between the state and control functions which can be used to derive controllability conditions for the considered system. For the general procedure, refer to [17]. Here, we will describe the procedure shortly for two specific types of controllability.

Let the aim of the control be providing the required condition

$$\left.\frac{\partial^n \boldsymbol{s}}{\partial t^n}\right|_{t=T} = \boldsymbol{s}_n, \qquad n = 0, 1, \ldots, k-1, \tag{4}$$

at a prescribed amount of time T, where $\boldsymbol{s}_n$ are given constant vectors. Then, if by a choice of $u \in \mathcal{U}$, (4) is fulfilled exactly, (1), (2) is called *exactly controllable*. If by a choice of $u \in \mathcal{U}$, (4) is fulfilled approximately with an admissible accuracy, (1), (2) is called *approximately controllable*.

Making use of (3), for the exact fulfillment of (4), i.e., exact controllability of (1), (2), it is necessary and sufficient that for at least one $\boldsymbol{u} \in \mathcal{U}$,

$$\left[\frac{\partial^n}{\partial t^n}\int_0^t \mathbf{G}(t-\tau)\boldsymbol{f}(\boldsymbol{u},\tau)g(t-\tau)\,\mathrm{d}\tau\right]\Bigg|_{t=T} = \boldsymbol{s}_n, \qquad n = 0,1,\ldots,k-1. \tag{5}$$

The approximate controllability of (1), (2) is checked by evaluating the system

$$\left|\left[\frac{\partial^n}{\partial t^n}\int_0^t \mathbf{G}(t-\tau)\boldsymbol{f}(\boldsymbol{u},\tau)g(t-\tau)\,\mathrm{d}\tau\right]\Bigg|_{t=T} - \boldsymbol{s}_n\right| \leqslant \varepsilon_n, \tag{6}$$

for $n = 0,1,\ldots,k-1$.

There exist several methods for analysis of (5) or (6). Since these are systems of, in general, nonlinear constraints on $\boldsymbol{u}$, evaluation of (5) or (6) can be computationally very costly. A very intuitive and less costly method of derivation of $\boldsymbol{u}$ has been recently developed in [18] consisting in the following. First, parametric families of controls $\boldsymbol{u} = \boldsymbol{u}\,(t;\alpha_1,\ldots,\alpha_M)$, $M \in \mathbb{N}$ is constructed based on some heuristic considerations corresponding, e.g., to physical treatment of the control system. Then, the constructed $\boldsymbol{u}$ is substituted into (5) or (6) and expressions for $\alpha_1,\ldots,\alpha_M$ satisfying the constraints are reduced. In the range of values of $\alpha_1,\ldots,\alpha_M$ for which $\boldsymbol{u} \in \mathcal{U}$, the system is exact controllable. This method is referred to as heuristic method. Application of the heuristic method can be found in related articles [19–23].

3. Nonlinear heuristic control over single-machine infinite-bus

In this section, we consider a one-machine infinite-bus system described by the following coupled system of first order quasi-linear ordinary differential equations (for the derivation of the state equations refer to [24]):

$$\begin{aligned}
\dot{\delta} &= \omega - \omega_s\,,\\
\dot{\omega} &= \frac{P_m}{H}\omega_0 + \frac{D}{H}(\omega-\omega_0) - \frac{E_q' V_s}{x_{d\Sigma}' H}\sin(\delta)\,\omega,\\
\dot{E}_q' &= -\frac{1}{T_d'}E_q' + \frac{1}{T_d'}\frac{x_d - x_d'}{x_{d\Sigma}}V_s\cos(\delta) + \frac{x_{d\Sigma}'}{T_d' x_{d\Sigma}}u.
\end{aligned} \tag{7}$$

Here, δ is the power angle, ω is the corresponding angular velocity, ω_s is the synchronous angular velocity, P_m is the mechanical power, which is assumed to be constant, H describes the inertia, D describes the damping, E'_q is the electric potential, V_s is the infinite bus voltage, $x_{d\Sigma}$, $x'_{d\Sigma}$, x_d, and x'_d are the generator's reactances, T'_d is the time constant of the field winding when the stator circuit is closed, u is the controlled voltage.

For the sake of simplicity, assume that the initial configuration is given by

$$\delta(0) = 0, \qquad \omega(0) = 0, \qquad E'_q(0) = 0. \tag{8}$$

Let the aim of the control is the determination of a function u chosen from the set of admissible controls

$$\mathcal{U} = \left\{u \in L^2[0, T], \ \ |u| \leqslant \epsilon, \ \ \mathrm{supp}(u) \subseteq [0, T]\right\},$$

such that for a given T, the following configuration is implemented:

$$\delta(T) = \delta_T, \qquad \omega(T) = \omega_T, \qquad E'_q(T) = E'_{qT}, \tag{9}$$

where δ_T, ω_T, E'_{qT} are given constant.

The Green's matrix of coupled system (7) is not found explicitly, and we involve the standard finite difference method to construct it. The numerical Green's matrix is substituted into (5) or (6) to study the exact or approximate controllability of (7), respectively. Now, let us involve some heuristic solutions, constructed in [17, 18]. First, we try the periodic control

$$u(t) = u_0 \left[\theta(t) - \theta(t - T)\right] \sin\left(\alpha t + \beta\right), \tag{10}$$

where u_0, α and β are free parameters that will be chosen to achieve the given terminal data. Evidently, in this case, it is possible to find the range of values of those parameters, for which $u \in \mathcal{U}$. Indeed, straightforward integration results in

$$\|[\theta(t) - \theta(t - T)] \sin\left(\alpha t + \beta\right)\|^2_{L^2[0,T]} = \frac{T}{2} + \frac{\sin(2\beta) - \sin\left(2\alpha T + 2\beta\right)}{4\alpha},$$

implying that $u \in L^2[0, T]$ as soon as

$$0 < \frac{T}{2} + \frac{\sin(2\beta) - \sin\left(2\alpha T + 2\beta\right)}{4\alpha} < \infty.$$

Apparently, when $\alpha^2 + \beta^2 > 0$, which is a quite natural assumption, because otherwise $u \equiv 0$, the norm in finite. Thus, the admissible values of these parameters are defined through

$$\left\{\alpha^2 + \beta^2 > 0\right\} \cup \left\{\frac{T}{2} + \frac{\sin(2\beta) - \sin\left(2\alpha T + 2\beta\right)}{4\alpha} > 0\right\}.$$

On the other hand, in order to ensure that $|u| \leqslant \epsilon$, it is sufficient that $|u_0| \leqslant \epsilon$.

Thus, evaluation of (5) or (6) becomes a simple problem of nonlinear programming with respect to u_0, α and β constrained by the above restrictions. The solution of the corresponding problem has been carried out numerically using MATLAB software and the result of simulation is presented below.

Numerical analysis of (6) with $\varepsilon = 10^{-4}$ under restrictions on α and β above, shows that for small values of α (u_0 and β being fixed), control influences the stability of the system and it may become unstable for $T > 50s$. Increasing α changes the situation making the system approximately controllable in finite time. Indeed, as Figures 1 and 2, when $\alpha \leqslant 1$, the solution of system (7), (8) controlled by harmonic control (10), demonstrate an aperiodic, increasing in time behavior leading to instability. Thus, let $\alpha \gg 1$.

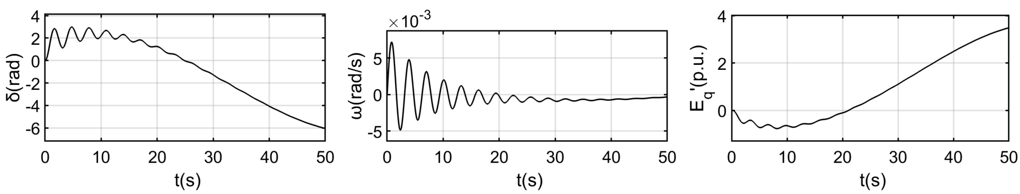

Figure 1: Controlled δ, ω and E'_q: harmonic regime (10) with $u_0 = 1$, $\alpha = 0.01$, $\beta = 0$

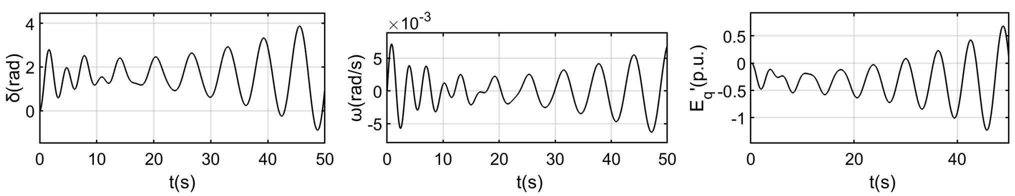

Figure 2: Controlled δ, ω and E'_q: harmonic regime (10) with $u_0 = 1$, $\alpha = 1$, $\beta = 0$

Consider a particular case, when (see (9))

$$\delta_T = 2.8, \qquad \omega_T = 0, \qquad E'_{qT} = 0.8.$$

As Figure 3 shows, for large values of α, all the three controlled quantities have a quickly damping behavior. This means that once an equilibrium is achieved at $T \approx 40$, this state will remain achieved for $T > 40$ as well. See the relevant work [20] for controllability of nonlinear systems in infinite time. In this particular case, $u_0 = 1$, $\alpha = 10$ and $\beta = 0$, provide (6) with $\varepsilon = 10^{-4}$.

Second, we involve the piecewise control regime [18]

$$u(t) = \sum_{k=1}^{K} u_k \theta\left(t - t_k\right), \qquad 0 \leqslant t_1 < \cdots < t_K \leqslant T, \tag{11}$$

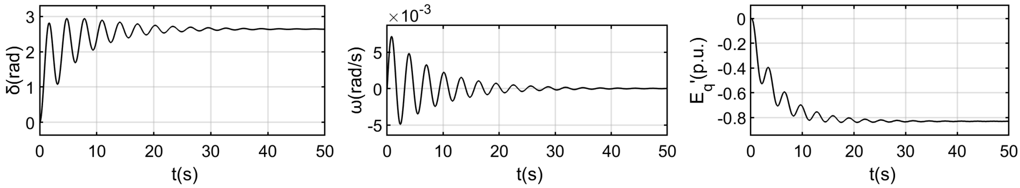

Figure 3: Controlled δ, ω and E'_q: harmonic regime (10) with $u_0 = 1$, $\alpha = 10$, $\beta = 0$

where K, u_k and t_k are free parameters to ensure appropriate type of controllability. Consider a particular case with

$$\delta_T = 1, \qquad \omega_T = 0, \qquad E'_{qT} = -0.25.$$

Evaluation of (6) shows that $K = 3$, $u_1 = 0.5$, $t_1 = 5$, $u_2 = 0.25$, $t_2 = 15$ and $u_3 = 0.1$, $t_3 = 25$ provides the required state in $T \approx 35$ (i.e., faster than the previous case). The total intensity of controls is $u_1 + u_2 + u_3 = 0.85$, while in the previous case the intensity of controls is 1 (see Figure 4). Note that in this case $u \in L^2[0, T]$ for all finite values of the parameters and $|u| \leqslant \epsilon$ is satisfied with $\epsilon = 0.83$.

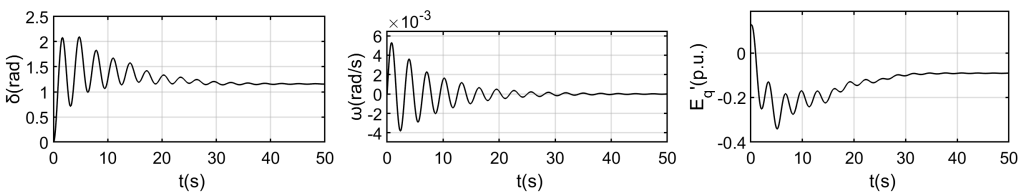

Figure 4: Controlled δ, ω and E'_q: piecewise constant regime (11) with $u_1 = 0.5$, $t_1 = 5$, $u_2 = 0.25$, $t_2 = 15$ and $u_3 = 0.1$, $t_3 = 25$

4. Conclusions

A general approach for determination of control functions based on heuristic considerations providing exact or approximate controllability of systems described by coupled systems of nonlinear equations is developed in this paper. The general solution of the nonlinear system is written explicitly by means of the nonlinear Green's function providing and explicit dependence of the solution and control function. Substituting the Green's function solution into the required terminal conditions, constraints for the system exact and exact and approximate controllability are reduced for the control function. The method is demonstrated on the example of a nonlinear power system corresponding to a one-machine infinite-bus system. Two particular forms of the control function are considered:

harmonic control described by sin function and piecewise constant function described by the Heaviside step function. On the basis of the heuristic solutions, numerical simulations are easily repeated for different values of the system parameters.

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