Webster's Delay Formula Revision by $M_{+\Delta}/G_{+\Delta}/1$ Queuing Model Usage

Leszek Mindur

The International University of Logistics and Transport in Wroclaw, Poland

Grzegorz Sierpiński

Silesian University of Technology, Poland

Webster's delay model was published for the first time almost 60 years ago (exactly in 1958). Next year will be a round anniversary. It became an occasion to verify the model. The method described in the paper discusses thesis of general shifted distribution of service times. Proposes delay model allows to use variance of service time as one of the parameters of a traffic flow at signalized intersection.

Keywords: Delay Model, Signalized Intersection, Variance of Service Time.

1. INTRODUCTION

At a constant growth rate of a number of vehicles entering the city, an effect of a strong congestion is observed still more frequently. Such a situation is typical for a morning and afternoon rush hours but in the strict city centre area the congestion may sustain for more than ten hours during a day. The congestion effect means among others time delays extensive. (e. g. [22]). Congestion has a significant impact on the environment. In addition to the increase in time losses, noise and emissions of harmful substances to the environment are increased [18]. Correct estimation of time delays in transport network can support decision making to improve transport system and transport management [4], [5], [16]. These activities are important to proper economy development [19].

A traffic signal usage belongs to the methods of traffic control at intersections. It considerably facilitates decreasing a number of collision points. First of all traffic signal effectiveness depends on a proper selection of an individual signal length. Time delays are the factors on the basis of which it is possible to measure the effectiveness of traffic signals and traffic state at intersections. The level of service (LOS) is estimated on the basis of time delays in most of current capacity methods (for example HCM2010, Polish method (2004)) [6], [10], [11], [17], [24], [25].

There were many delay models in history (more e. g. [2], [3], [7], [8], [9] [12], [13], [30]). Webster's model was the first model widely applied to estimate an average time delay at signalized intersection. Afterwards the model and algorithm of a cycle length optimization were being used many times in capacity methods (for example Swedish method, Canadian method and British method).

In the paper one kind of traffic flow models was used. They give a wide range of possibilities for testing complex road traffic states and processes. They also aim at a proper representation of a transport system dynamics. Queuing theory is characterized by a wide range of applications. Queuing models are used in many aspects of life, such: traffic flow, scheduling, facility design and employee management [14], [20], [30]. Implementation of a queuing theory for description of a signalized intersection simplifies creating a mathematical model of a real system. The paper presents a proposed delay model.

Symbols used in this paper are shown in Table 1.

Table 1. Symbols definitions.

t_s – average service time for a usual model [s]
Δ – minimal time distance between vehicles [s]
λ' – arrival rate for a compressed model [veh/s]
λ – arrival rate [veh/s]
ρ – flow ratio [-]
σ^2 – variance of service time [s]
$\overline{L}$ – average number of the vehicles in the system [veh]
$\overline{L_q}$ – average number of the vehicles in the queue [veh]
$\overline{W}$ – average waiting time in the system [s/veh]
$\overline{W_q}$ – average waiting time in the queue [s/veh]
μ – service rate [veh/s]
d – average delay per vehicle (Webster's delay model) [s]
d_M – average delay per vehicle (proposed delay model) [s]
T_c – length of cycle [s]
G_e – effective green signal duration [s]
σ_μ^2 – variance of service time [s]

2. WEBSTER'S MODEL

Webster's delay model was based on four general assumptions, however the most significant were these two:

- the model estimates delays at fixed time signals;
- vehicles arrive at an intersection at random.

Webster's formula to estimate average delay per vehicle consists of three parts. A deterministic Clayton's model is the first element and average waiting time in a queue for queuing system M/D/1 is the second one. A modification estimated on the basis of simulation is the last part. Graphic interpretations of Webster's formula are shown in Figure 1.

The formula is presented in the following form [27], [28]:

$$d = \frac{T_c \cdot \left(1 - \frac{G_e}{T_c}\right)^2}{2 \cdot \left(1 - \frac{G_e}{T_c} \cdot \rho\right)} + \frac{\rho^2}{2 \cdot \lambda \cdot (1-\rho)} - 0{,}65 \cdot \left(\frac{T_c}{\lambda^2}\right)^{\frac{1}{3}} \cdot \rho^{\left(2 + 5 \cdot \frac{G_e}{T_c}\right)} \qquad (1)$$

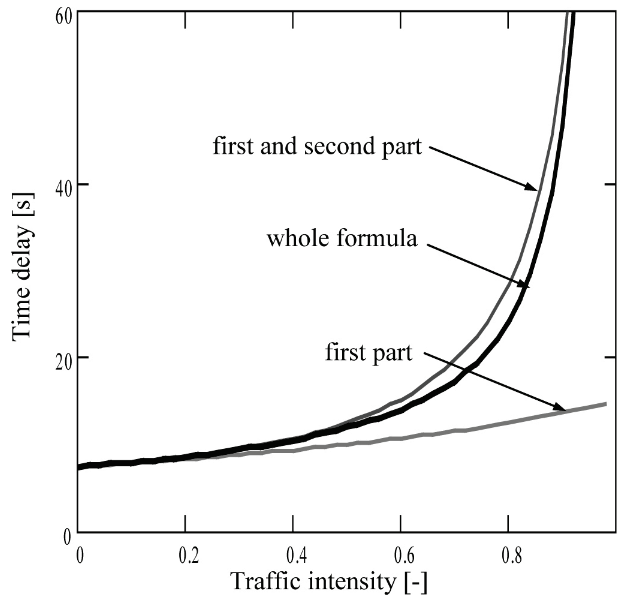

Fig. 1. Webster's model - graphic interpretation.

A. *Deterministic part*

The first part of the Webster's formula belongs to Clayton and makes it possible to estimate time delay for uniform state of traffic flow. The deterministic model (firstly described by Clayton) is estimated according to several assumptions:

- uniform arrivals at the arrival rate during the cycle;
- uniform departures at the saturation flow rate.

The maximal value of a queue length is given at the beginning of a green phase (figure 2). Vehicles do not wait until a queue clears out.

The formula describing an average waiting time in the deterministic model is the following:

$$\overline{W} = \frac{T_c \cdot \left(1 - \frac{G_e}{T_c}\right)^2}{2 \cdot \left(1 - \frac{G_e}{T_c} \cdot \rho\right)} \qquad (2)$$

B. *Random part*

The second part of the Webster's formula (eq. 1) additionally includes a stochastic nature of arrivals. It is a random delay. Webster took assumption about Poisson distribution of arrivals to inlet. He used average waiting time in a queue from M/D/1 queuing model to estimate that delay time (eq. 3).

$$\overline{W_q} = \frac{\lambda}{2\cdot\mu\cdot(\mu-\lambda)} = \frac{\rho}{2\cdot\mu\cdot(1-\rho)} = \frac{\rho^2}{2\cdot\lambda\cdot(1-\rho)} \quad (3)$$

- the average waiting time from M+Δ/G+Δ/1 queuing model with the usage of compressed queuing processes theory (described by Woch, [29]).

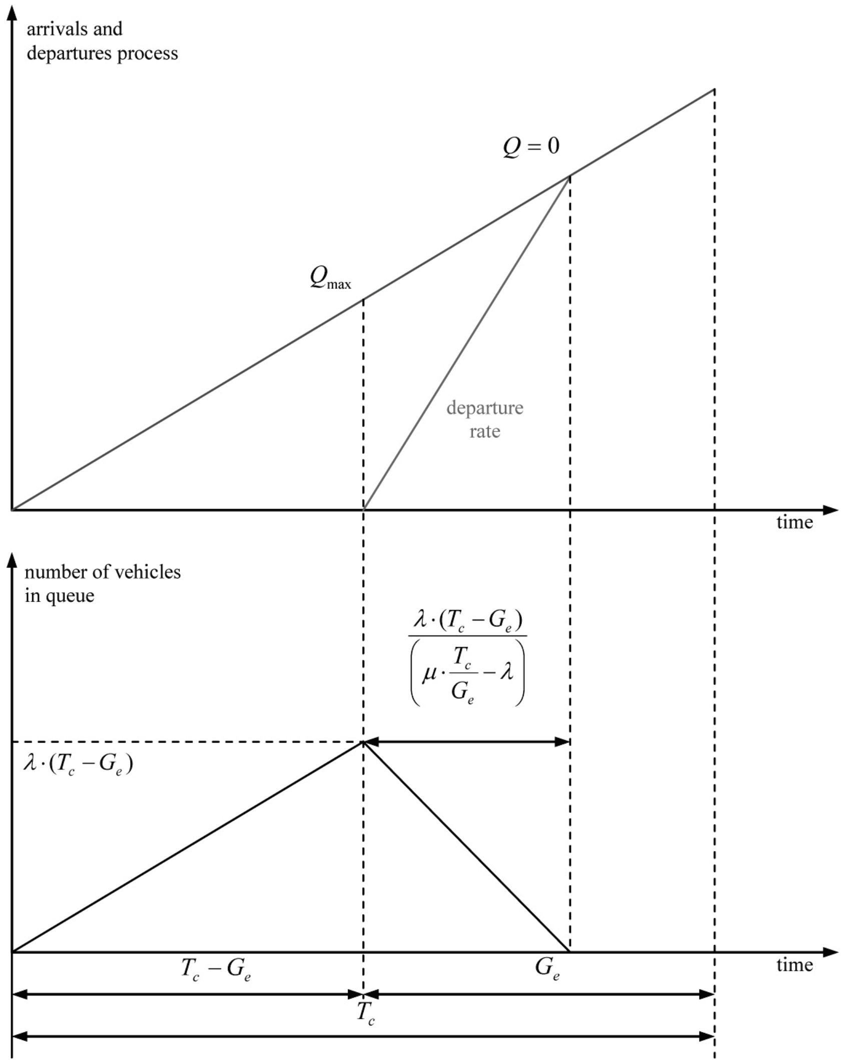

Fig. 2. Clayton's model.

3. PROPOSAL OF MODIFICATION

Constant service time was one of the Webster's simplifications, while in fact service times of succeeding vehicles are not identical. Therefore, there is a necessity to take this variance into consideration. The variance of service times is taken into account by average waiting time in a queue for queuing original system $M_{+\Delta}/G_{+\Delta}/1$ used with compressed queuing processes as a part of the proposed time delay model.

Proposed model, like most of delay models, contains of two elements: a constant part and a random part. The model includes:

- the deterministic model (described by Clayton);

A. *$M_{+\Delta}/G_{+\Delta}/1$ model*

A service time at signalized intersection has different distributions. It means that one distribution of a service time in model cannot be accepted. That is why the paper proposed another approach. $M_{+\Delta}/G_{+\Delta}/1$ queuing system has a single server (only one service channel). The interarrival times have the exponential shifted distribution. $G_{+\Delta}$ means a general shifted distribution of the service times. An average service time (ts) and a variance of the service time (σ^2) are used to describe service in this system. Compressed queuing processes are based on two assumptions [29]:

- average service time for a compressed model ts' equals:

$$t_s' = t_s - \Delta \tag{4}$$

- reverse of arrival rate for a usual model 1/λ equals:

$$\frac{1}{\lambda} = \frac{1}{\lambda'} + \Delta \text{ ; it means that } \lambda' = \frac{\lambda}{1 - \lambda \cdot \Delta} \tag{5}$$

Graphic interpretation of compressed process has been shown in figure 3.

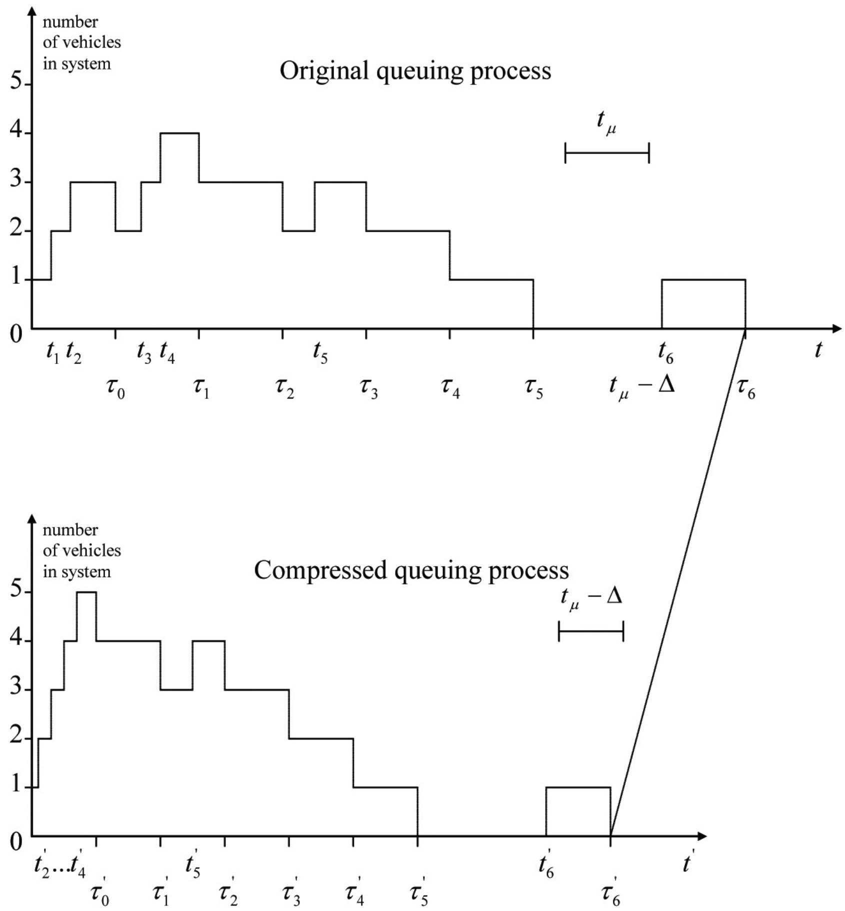

Fig. 3. Examples of original and compressed queuing processes (Woch, [29]).

In case of described waiting time in general M/G/1 model, the imbedded Markov chain can be used [14], [21]. In this assumption a behaviour of a system is observed in the discrete moments, when the successive number of vehicles leaves the system.

Pollaczek-Khintchin formula can be obtained in a following way (an average number of the vehicles in a system):

$$\overline{L} = \rho + \frac{\rho^2 + \lambda^2 \cdot \sigma^2}{2 \cdot (1 - \rho)} \tag{6}$$

By using this formula (6) and Little's equations (7), average waiting time in a queue at intersection inlet can be obtained.

$$\overline{L} = \lambda \cdot \overline{W}; \qquad \overline{L} = \overline{L_q} + \frac{\lambda}{\mu}; \tag{7}$$

$$\overline{L_q} = \lambda \cdot \overline{W_q}; \qquad \overline{W} = \overline{W_q} + \frac{1}{\mu}.$$

A required equation takes now the following form:

$$\overline{W_q} = \frac{\lambda \cdot (\mu^{-2} + \sigma^2)}{2 \cdot (1 - \rho)} = \frac{\rho^2 \cdot (1 + \sigma^2 \cdot \mu^2)}{2 \cdot \lambda \cdot (1 - \rho)} \tag{8}$$

Now, by using compressed queuing processes assumptions (4) and (5), the final formula describing average waiting time in a queue for original $M_{+\Delta}/G_{+\Delta}/1$ can be estimated [23]:

$$\overline{W_q} = \frac{\lambda \cdot \sigma^2 + \lambda \cdot \left(\frac{1}{\mu} - \Delta \right)^2}{2 \cdot (1-\rho)} \cdot (1 - \mu \cdot \Delta) \qquad (9)$$

B. ***Modified delay model***

The proposed time delay model is a sum of an average waiting time from the historical Clayton's model (2) and an average waiting time from the $M_{+\Delta}/G_{+\Delta}/1$ queuing model which uses the theory of compressed queuing processes (9). This approach allows to assume general shifted distribution of service time at intersection, which is more realistic to uniform service time. The last one is possible in case of straight direction of traffic flow.

A proposed delay model takes the following form [23]:

$$d_M = \frac{T_c \cdot \left(1 - \frac{G_e}{T_c}\right)^2}{2 \cdot \left(1 - \frac{G_e}{T_c} \cdot \rho\right)} + \frac{\lambda \cdot \sigma_\mu{}^2 + \lambda \cdot \left(\frac{1}{\mu} - \Delta \right)^2}{2 \cdot (1-\rho)} \cdot (1 - \mu \cdot \Delta) \qquad (10)$$

The variance of service times is a measure of dispersion in a process of vehicle service. Time delay was estimated for three different values of variance of service times: $\sigma_\mu^2 = 0$, in case of constant service times; $\sigma_\mu^2 = 4$ and $\sigma_\mu^2 = 10$ for empiric cases. It is shown in figure 4. This relation shows strongly dependence on time delay on variance of service times, which help to estimate time delay more accurately.

The relationship between time delay and values of green splits is shown in figure 5.

Minimal time distance between vehicles is a second new value involved in the proposed model. The relationship between time delay and flow intensity for three different values of minimal time distance between vehicles is shown in figure 6.

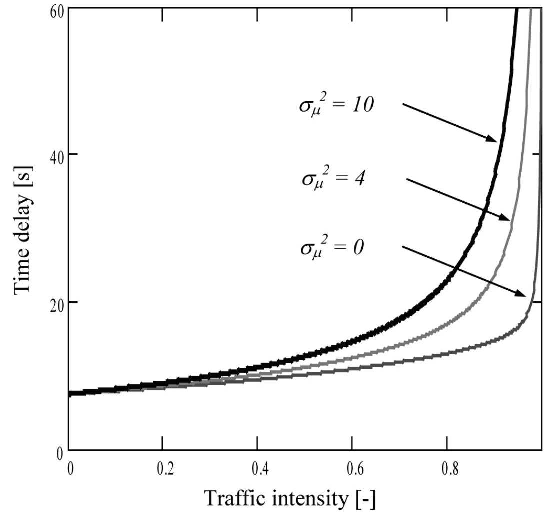

Fig. 4. The time delay for exemplary values of variance of service times σ_μ^2.

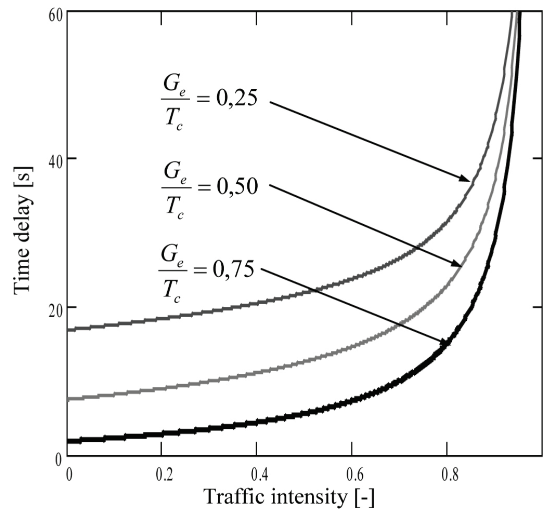

Fig. 5. The time delay for exemplary values of green splits.

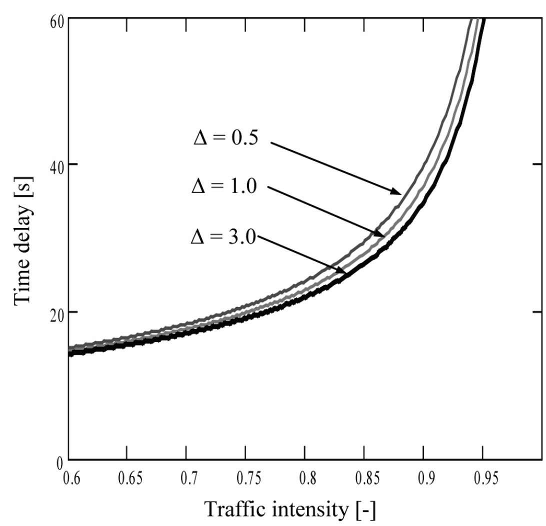

Fig. 6. The time delay for exemplary values of minimal time distance between services.

C. *Verification*

Both the proposed model and Webster's one have been verified. Two methods of a survey have been applied for this purpose. The first method (field test) covers measurements carried out at signalized intersections in Katowice (Poland). During these researches video technique was used in order to record all stages of vehicle transition - from entering to leaving an intersection. The second method covers simulations in VISSIM (Fig. 7) (user can put various intensity of arrivals in this simulation program). [26]

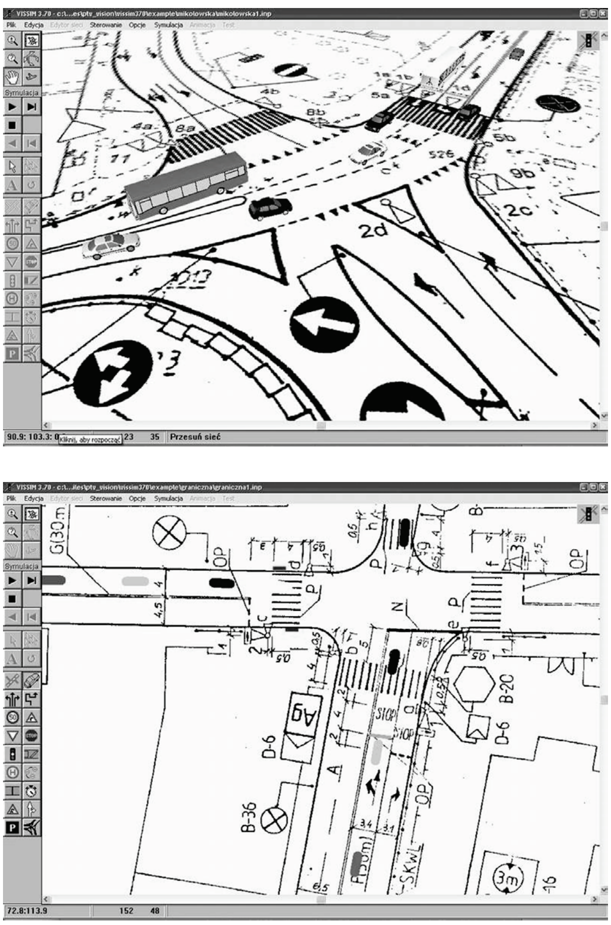

Fig. 7. Visualization of selected VISSIM's simulations of intersections in Katowice (Poland).

The verification has proved that:

- in case of independent movements with a small green split (green signal below 25 percent of length of cycle), time delay estimated on the basis of proposed model is almost equal with value from simulation (relative error below 10 percent);
- in case of independent movements with a big green split (green signal above 50 percent of length of cycle), relative error is high however it is still below 12 percent (the difference is maximum 1.5 second);
- in case of collision with pedestrians and two relations on a single line relative error is below 10 percent - only to 0.6 flow ratio;
- correlation coefficient in all cases is high and above 0.96.
- Estimated delay from proposed model has been also compared with values from Webster's model and Polish method [25]. The analysis of results shows that:
- in case of real measurements at signalized intersections maximal relative error has been observed in Webster's model (34 percent);
- average relative error was the lowest in proposed model – 2.4 percent (average relative error in Polish method – 3.7 percent, average relative error in Webster's model – 5.1 percent);
- time delay calculated in proposed model gives better estimation than Webster's model (in all cases relative error was lower than in Webster's model);
- proposed delay model gives better estimation of time delay than Polish method, but only to flow ratio 0.1-0.9 range (queuing models have no usage above this range).

4. CONCLUSIONS

The realized measurements of time delays and simulations of traffic flow of selected intersections in Katowice (Poland) proved that Webster's model, which is 60 years old, still gives good estimation of time delays. That is why in some countries this formula is still in use for isolated intersection.

Webster's model and the proposed in the paper delay model are steady-state models. It means that they give results in case of volume smaller than the saturation flow. Nevertheless, both models could be used to estimate delay time at signalized intersection with a small traffic flow. What is more, the proposed model of using general shifted distribution of service time gives better estimation than Webster's model.

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Date submitted: 2017-08-08
Date accepted for publishing: 2017-09-17

Leszek Mindur
The International University of Logistics and Transport in Wroclaw, Poland
lmindur@vp.pl

Grzegorz Sierpiński
Silesian University of Technology, Poland
grzegorz.sierpinski@polsl.pl