Robust Trajectory Tracking Control of Space Manipulators in Extended Task Space

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Abstract: This study provides a new class of controllers for freeflying space manipulators subject to unknown undesirable disturbing forces exerted on the end-effector. Based on suitably defined taskspace non-singular terminal sliding manifold and the Lyapunov stability theory, we derive a class of estimated extended transposed Jacobian controllers which seem to be effective in counteracting the unstructured disturbing forces. The numerical computations which are carried out for a space manipulator consisting of a spacecraft propelled by eight thrusters and holonomic manipulator of three revolute kinematic pairs, illustrate the performance of the proposed controller.

Keywords: space manipulator, trajectory tracking, unstructured disturbance forces, robust finite-time task space control, Lyapunov stability

1. Introduction

Space manipulators are extensively applied in recent years in space operations such as an on-orbit high-tolerance assembly tasks, maintenance, inspection and precise capturing and transferring space debris [10]. Space manipulators are complex and highly non-linear dynamic systems composed of a spacecraft (platform or satellite) on which holonomic robotic manipulator is mounted. The spacecraft is actuated by thrusters or other propellers and the links of holonomic manipulator are usually driven by electric DC motors. There are two modes of operations of space manipulators: free-flying and free-floating. In the case of free-flying dynamic systems, a spacecraft's controller is active. On the other hand, in the free-floating operational mode, a platform controller is turned off. Due to high potential costs of a space mission, space manipulators have to be robust against actuator failures. This means that such dynamic systems are, by nature, over-determined (number of actuators is greater than the number of coordinates uniquely describing a configuration of the whole space robotic system). It is worth noting that the space operations requiring extremely high accuracy still make a great challenge for space manipulators due to various uncertainties of, e.g., parts or sub-parts to be assembled, end-effector tasks, etc., which are rigidly gripped and transferred by the end-effector along a desired trajectory mostly expressed in task (Cartesian) coordinates. In practice, for example, either a known or unknown sub-part (by its geometry and mass) introduces structural and/or parametric uncertainties in both kinematic and dynamic equations of the space manipulator. As a result, transferred sub-parts generate undesirable external disturbances (e.g., Coriolis and/or centrifugal forces) acting on the end-effector which may result in large tracking errors. When accomplishing the complicated assembly process, the space manipulator (or at least a holonomic manipulator) should attain possibly maximal manipulability measure [25]. Otherwise, a complex and time-consuming reconfiguration of the whole space manipulator has to be carried out to make it possible accomplishment of assembly operations by the end-effector. The control of space manipulators is usually decomposed into two stages. In the first stage, the space manipulator has to attain a close vicinity of a target using the thrusters (the rendezvous). In the second one, the target must be captured in a close range using only the holonomic manipulator actuators (the docking). Most of the works devoted to control process in Cartesian task coordinates concern an on-orbit docking maneuvers (e.g., with tumbling target) [17–19, 23] for which, the thrusters are turned off what implies a non-holonomic movement. In [17, 19, 23], the authors have applied a Jacobian transposed technique with full knowledge of kinematics and dynamics to steer space manipulator to a close target. An off-line control based on endogenous space approach and accurate knowledge of both kinematic nd dynamic equations has been offered in [18]. Let us note that control algorithms proposed in [17–19, 23] may lead, e.g., to configurations with small manipulability measure what prevents accomplishment of complicated assembly operations. In order to increase the mobility of in-orbit robotic systems, both thrusters of spacecraft and actuators of holonomic manipulator are assumed to be active (in works [11–13, 15, 16, 24]) in controlling the space manipulator during the trajectory tracking tasks. Unfortunately, a vast majority of control algorithms is expressed in the space of generalized (joint) coordinates [11–13, 24]. Due to the fact that a huge amount of trajectory tracking tasks is expressed in Cartesian (task) coordinates, the algorithms proposed in works [11–13, 24] are

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not suitable for accomplishment of such tasks. Only a very few works has been reported which tackle the problem of trajectory tracking control of fully actuated space manipulators in task coordinates. In [15, 16], a modified transpose Jacobian control algorithm has been presented, which employs stored data of the control command in the previous time step as a learning tool to yield improved performance. Nevertheless, the control laws from [15, 16] are only stable and require the full knowledge of kinematic equations.

In this work, a new class of controllers for space manipulator subject to undesirable forces exerted on the end-effector, is introduced. Due to unstructured nature of external disturbance forces, kinematics and dynamics of the mechanism is assumed herein to be uncertain. In order to tackle the trajectory tracking control problem subject to unstructured, a new non-singular terminal sliding manifold (TSM) is proposed. Based on the TSM introduced, we propose a new robust controller (incorporating a transposed extended estimated Jacobian matrix) which tackles unknown external forces and uncertainties of kinematic as well as dynamic equations. By fulfilment of a reasonable assumption regarding the Jacobian matrix, the proposed control scheme is shown to be finite-time stable. The remainder of the paper is organized as follows. Section 2 introduces kinematic and dynamic equations of the space manipulator including external forces acting on the end-effector. Section 3 sets up a class of robust controllers solving the trajectory tracking control problem in a finite-time. Section 4 presents computer example of the end-effector trajectory tracking by a space manipulator which is subject to external disturbance forces. Finally, some concluding remarks are drawn in Section 5.

2. Problem Formulation

The control scheme designated in the next section is applicable to holonomic mechanical systems comprising both non-redundant and redundant space manipulators considered here, which are described, in general, by the following dynamic equations, expressed in generalized coordinates [14, 19]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = Bv + d(q,\dot{q}), \tag{1}$$

where $q = (q_1,\dots,q_n)^{\mathrm{T}} \in \mathbb{R}^n$ denotes the space manipulator configuration including location and orientation of spacecraft with respect to a global coordinate system $OX_1X_2X_3$ and joint coordinates of the holonomic manipulator; $M(q)$ is the $n \times n$ positive definite inertia matrix; $C(q,\dot{q})\dot{q}$ denotes the n–dimensional vector representing centrifugal and Coriolis forces; $d(q,\dot{q})$ describes the n–dimensional external disturbance signal; $B(q)$ stands for the $n \times m$ non-singular control matrix; v is the m–dimensional vector of steering signals (forces generated by thrusters of spacecraft and torques provided by actuators of holonomic manipulator); n denotes the dimension of the space of generalized coordinates and m stands for the number of controls. By assumption, we have $m \geq n$ (safety and reliability of the space mission when a thruster is damaged). Without loss of generality, d is assumed to be upper estimated as follows

$$\|d\| \leq \alpha(t,q,\dot{q}), \tag{2}$$

where $\alpha(\cdot)$ is the time dependent, non-negative locally bounded Lebesgue measurable function. Location and orientation of the end-effector with respect to the global coordinate system $OX_1X_2X_3$ is described by the kinematic equation

$$p_e = f_e(q), \tag{3}$$

where $p_e \in \mathbb{R}^k$ denotes the coordinates of the end-effector; $f_e : \mathbb{R}^n \to \mathbb{R}^k$ represents k-dimensional mapping (in general, non-linear with respect to q) and k is the dimension of the task space. On account of the fact that space manipulator becomes mostly in practice a redundant mechanism with respect to a task to be accomplished, the following inequality holds true: $n \geq k$. Consequently, there exists a possibility to augment vector of the end-effector coordinates p_e by additional task coordinates p_a (specified by the user) of the following general form [20]:

$$p_a = f_a(q), \tag{4}$$

where $f_a : \mathbb{R}^n \to \mathbb{R}^{n-k}$ is at least twice continuously differentiable mapping with respect to q. From the practical point of view, redundant degrees of freedom of the mechanism may either satisfy additional task requirements (constraints) [20, 21] or optimize performance criteria reflecting the kinematic characteristics of the mobile manipulator [7]. Concatenating $f_e(q)$ with $f_a(q)$, one obtains generalized kinematic-differential mappings which relate q with augmented task coordinates $p = \begin{pmatrix} p_e \\ p_a \end{pmatrix}$

$$p = f(q), \quad \dot{p} = J(q)\dot{q}, \tag{5}$$

where $f = \begin{pmatrix} f_e \\ f_a \end{pmatrix}$ and $J = \dfrac{\partial f}{\partial q}$ is the $n \times n$ extended Jacobian matrix. The task accomplished by the space manipulator is to track both desired end-effector trajectory $p_d^e(t) \in \mathbb{R}^k$, $t \in [0,\infty)$ and auxiliary (user specified) trajectory $p_d^a(t) \in \mathbb{R}^{n-k}$. Introducing the task tracking error $e = f(q) - p_d(t)$, where $p_d = \begin{pmatrix} p_d^e \\ p_d^a \end{pmatrix}$, the finite time control problem may be formally expressed by means of the following equations:

$$\lim_{t \to T} e(t) = 0, \quad \lim_{t \to T} \dot{e}(t) = 0, \tag{6}$$

where $0 \leq T$ denotes a finite time of convergence of $f(q)$ to p_d and $e(t) = \dot{e}(t) = 0$ for $t \geq T$. The aim is to determine control v, for which, the corresponding trajectory $q = q(t)$ as being the solution of differential equations (1), accomplishes the space manipulator task (6).

3. Control of Space Manipulator in Extended Task Space

Before we propose the control law solving the kinematic task (6), some useful concepts are first introduced. Let $\hat{J} = \hat{J}(q)$ and $\hat{B} = \hat{B}(q)$ denote estimates of the uncertain generalized extended Jacobian matrix $J(q)$ and full rank control matrix $B(q)$ given by formulas (5), (1). In further considerations, $\hat{J}$ and full rank matrix $\hat{B}$ are assumed to fulfil the following inequalities:

$$0 < a \leq \lambda_{\min}\left(\hat{J}M^{-1}\hat{J}^{\mathrm{T}}\right), \tag{7}$$

and

$$0 \leq b_1 + b_2 < a, \tag{8}$$

where

$$0 \leq \left|\lambda_{\min}\left(\frac{\left(J - \hat{J}\right)M^{-1}\hat{J}^{\mathrm{T}} + \left(\left(J - \hat{J}\right)M^{-1}\hat{J}^{\mathrm{T}}\right)^{\mathrm{T}}}{2}\right)\right| \leq b_1;$$

$$0 \leq \left|\lambda_{\min}\left(\frac{JM^{-1}\left(B - \hat{B}\right)\hat{B}^{\#}\hat{J}^{\mathrm{T}} + \left(JM^{-1}\left(B - \hat{B}\right)\hat{B}^{\#}\hat{J}^{\mathrm{T}}\right)^{\mathrm{T}}}{2}\right)\right| \leq b_2;$$

$\hat{B}^{\#}$ denotes the Moore-Penrose pseudo-inverse matrix of $\hat{B}$. It is worth noting that inequality (8) can be in practice fulfilled by selection of measurement devices, which provide both kinematic and dynamic parameters of space manipulator with sufficient accuracy. Let us also observe that inequality (7) means non-singularity of extended estimated Jacobian matrix $\hat{J}(q)$. Nevertheless, relations (7)–(8) are only needed in the proof of the finite-time stability of the controller to be designated. The estimate of constant a from (7) can be determined based both on the numerical solution with respect to q of the following system of algebraic equations: $f(q) = p_d(t)$ and the knowledge of the components of the nominal kinematic and dynamic equations $J(q)$ and $M(q)$. Motivated partially by the computed torque methodology [8], we propose a new extended estimated transposed Jacobian control law of the following form:

$$v = \hat{B}^{\#}\hat{J}^{\mathrm{T}}u, \tag{9}$$

where $u \in \mathbb{R}^n$ is a new control to be determined. Applying (9) as a non-linear control law, inserting it to the right-hand side of (1) and determining $\ddot{q}$ gives

$$\ddot{q} = M^{-1}B\hat{B}^{\#}\hat{J}^{\mathrm{T}}u - M^{-1}C\dot{q} + M^{-1}d. \tag{10}$$

Let us also twice differentiate e with respect to time thus obtaining

$$\ddot{e} = JM^{-1}B\hat{B}^{\#}\hat{J}^{\mathrm{T}}u + \dot{J}\dot{q} - JM^{-1}C\dot{q} + JM^{-1}d - \ddot{p}_d. \tag{11}$$

In order to design a controller solving the space manipulator task (6), we introduce the following non-singular integral sliding vector variable $s \in \mathbb{R}^n$, defined in task coordinates as follows

$$s = \dot{e} - \dot{e}(0) + \int_0^t \left(\omega_0 e^{\alpha_1} + \omega_1 (\dot{e})^{\alpha_2}\right) d\tau, \tag{12}$$

where $\alpha_1 = \frac{a_1}{a_2}$; a_1, a_2 are positive odd numbers, $a_1 < a_2 < 2a_1$, $\alpha_2 = \frac{2\alpha_1}{1+\alpha_1}$; ω_0, w_1 stand for controller gains. The odd variables a_1 and a_2 ensure the finite-time convergence of the task errors $(e,\dot{e})$ to the origin $(e,\dot{e}) = (0,0)$. The time derivative of (12) results after simple calculation and taking into account (11) in the following expression:

$$\dot{s} = JM^{-1}B\hat{B}^{\#}\hat{J}^{\mathrm{T}}u + \mathcal{R}, \tag{13}$$

where $\mathcal{R} = \dot{J}\dot{q} - JM^{-1}C\dot{q} + JM^{-1}d - \ddot{p}_d + \omega_0 e^{\alpha_1} + \omega_1 (\dot{e})^{\alpha_2}$.

In further analysis, an upper estimate of $\|\mathcal{R}\|$ will be needed. It takes the form given below:

$$\|\mathcal{R}\| \le \mathcal{W}, \tag{14}$$

where $\mathcal{W} = w\left(\|\dot{q}\|^2 + \alpha\right) + \left\|\omega_0 e^{\alpha_1} + \omega_1 (\dot{e})^{\alpha_2} - \ddot{p}_d\right\|$, w denotes a construction parameter of the space manipulator. In what follows, we provide a useful lemma [8].

Lemma 1. If $s(t) = 0$ for $t \ge T \ge 0$ then task errors $(e,\dot{e})$ of (6) stably converge to the origin $(e,\dot{e}) = (0,0)$ in a finite time.

Based on (7)–(8), (12)–(14), we propose the following simple control law for the space manipulator solving the kinematic task (6):

$$u(t,q,s) = \begin{cases} -\frac{c}{a}\frac{s}{\|s\|}\left(\mathcal{W} + c_0\right) & \text{for} \quad s \neq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{15}$$

where c, c_0 denote controller gains to be specified further on. Applying the Lyapunov stability theory, we now derive the following result.

Theorem 1. If ω_0, ω_1, $c_0 > 0$ and $c = \frac{c'}{1 - \frac{b_1 + b_2}{a}}$ with $c' > 1$ then control scheme (9), (15) guarantees stable convergence in a finite time of the task tracking errors $(e,\dot{e})$ to the origin $(e,\dot{e}) = (0,0)$.

Proof. Consider a Lyapunov function candidate

$$V = \frac{1}{2}\langle s, s\rangle. \tag{16}$$

Computing the time derivative of (16) and replacing v by (9) results in the following expression:

$$\begin{aligned} \dot{V} = {} & \left\langle s, \hat{J}M^{-1}\hat{J}^{\mathrm{T}}u\right\rangle + \left\langle s, \left(J - \hat{J}\right)M^{-1}\hat{J}^{\mathrm{T}}u\right\rangle + \\ & \left\langle s, JM^{-1}\left(B - \hat{B}\right)\hat{B}^{\#}\hat{J}^{\mathrm{T}}u\right\rangle + \langle s, \mathcal{R}\rangle. \end{aligned} \tag{17}$$

Replacing u from (17) by (15) and then using (7)–(8), one obtains the inequality

$$\dot{V} \le \|s\|\left(\mathcal{W} + c_0\right)\left(-c + \frac{cb_1}{a} + \frac{cb_2}{a}\right) + \|s\|\mathcal{W}. \tag{18}$$

On account of (14) and using assumption regarding c_0 as well as $c = \frac{c'}{1 - \frac{b_1 + b_2}{a}}$ with $c' > 1$ from Theorem 1, we have

$$\dot{V} \le \|s\|\left(\mathcal{W} + c_0\right)\left(-c + \frac{cb_1}{a} + \frac{cb_2}{a} + 1\right) \le -(c' - 1)c_0\|s\|. \tag{19}$$

Based on (19) and (12), we conclude that TSM manifold $s = 0$ is stably attainable in a finite time. Finally, from Lemma 1, it follows that origin $(e,\dot{e}) = (0,0)$ can also be attained in a finite time $T \ge 0$. □

Two remarks may be made regarding the control law (9), (15) and Theorem 1.

– Remark 1. Let us observe that term $\frac{s}{\|s\|}$ in controller (15) will cause undesirable chattering effect in a small neighbourhood of $s = 0$. In order to eliminate the chattering, a known boundary layer technique of control law may now be utilized as follows

$$u(t,q,s,\varepsilon) = \begin{cases} -\frac{c}{a}\frac{s}{\|s\|}\left(\mathcal{W} + c_0\right) & \text{for} \quad \|s\| \ge \varepsilon \\ -\frac{c}{a}\frac{s}{\varepsilon}\left(\mathcal{W} + c_0\right) & \text{otherwise,} \end{cases} \tag{20}$$

where ε is a user-specified arbitrarily small positive real number (size of boundary layer). Let $e = e(t,\varepsilon)$ and $\dot{e} = \dot{e}(t,\varepsilon)$ be the solutions of control problem (1), (9), (12) and (20). Although boundary layer control is a well known technique, its desired property of uniform ultimate boundedness has been established for linear sliding variables s, e, $\dot{e}$ and diagonal actuation matrices of non-zero diagonal components (or of constant signs) in [22] as well as for dynamic systems fulfilling the so called matching conditions with known actuation matrices in work, e.g., [2], respectively. On the other hand, expression (12) is a non-linear differential equation with respect to $e = e(t,\varepsilon)$ and $\dot{e} = \dot{e}(t,\varepsilon)$. Moreover, B is uncertain non-diagonal actuation matrix. Consequently, the classic results regarding the ultimate uniform boundedness of $e = e(t,\varepsilon)$ and $\dot{e} = \dot{e}(t,\varepsilon)$ may not, in general, apply in our case. Hence, based on the results from work by [8], we may conclude that task errors $e = e(t,\varepsilon)$, $\dot{e} = \dot{e}(t,\varepsilon)$ converge uniformly with respect to time $t\,(t \geq T)$ to the origin $(e,\dot{e}) = (0,0)$ as $\varepsilon \to 0$, i.e. $e(t,\varepsilon)$, $\dot{e}(t,\varepsilon) \to 0$ as $\varepsilon \to 0$.

– **Remark 2.** Let us note that expressions (9), (15) present a transpose Jacobian controller. In such a context, the use of the transpose of the Jacobian matrix to robotic manipulators in [3, 4, 16] is a well-known technique.

However, works [3, 4, 16] present stability analysis for the set-point control problems. On the other hand, Theorem 1 provides stability analysis for the trajectory tracking of the space manipulator whose both kinematic and dynamic equations are uncertain as well as disturbances acting on the mechanism are unknown. In such a context, it is worth noting the fact that authors from works [5, 6] have also shown finite-time convergence of their controller using however the inverse of the Jacobian matrix.

4. Numerical Computations

In this section, we illustrate the performance of the proposed robust controller (9), (20) using the data of the planar space manipulator constructed in the Space Research Centre of the Polish Academy of Sciences [1] and schematically depicted in Fig. 1. This mechanism consists of a freeflying spacecraft

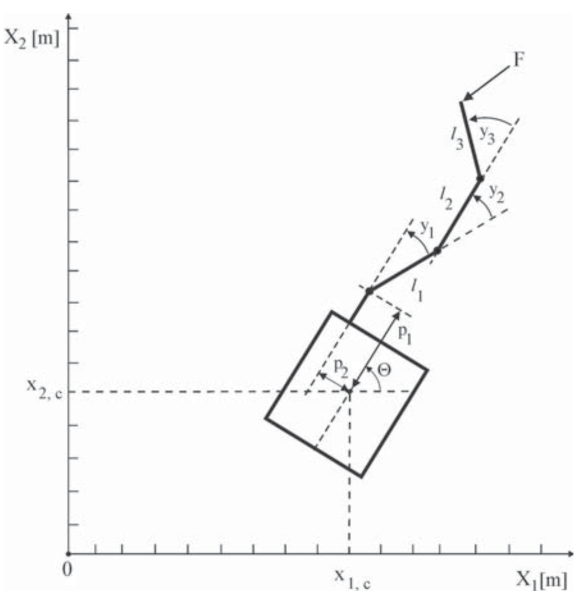

Fig. 1. A kinematic scheme of the space manipulator with force *F* acting on the end-effector
Rys. 1. Schemat kinematyczny manipulatora kosmicznego z siłą *F* działającą na koniec efektora

whose posture is described by two position variables $x_{1,c}$, $x_{2,c}$ and orientation angle θ with respect to the global coordinate system OX_1X_2. The planar holonomic manipulator (of three revolute kinematic pairs of the V-th order) whose configuration is described by the three joint angles y_1, y_2 and y_3, respectively is rigidly attached to the spacecraft at the point (p_1, p_2) – see Fig. 1. Consequently, vector of generalized coordinates q of the space manipulator from Fig. 1 equals $q = (x_{1,c}, x_{2,c}, \theta, y_1, y_2, y_3)^{\mathrm{T}}$ with $n = 6$.

In each of the four corners of the spacecraft, there are two thrusters that propel it. Moreover, the holonomic manipulator is actuated by three DC motors which generate three generalized torques in the joints of the kinematic pairs. Hence, the mechanism from Fig. 1 generates $m = 11$ steering signals. The space manipulator is assumed to operate in a three dimensional task space ($k = 3$). Hence, it becomes redundant mechanism with $n - k = 3$ redundant degrees of freedom. In the numerical computations, SI units are used. The nominal values of both kinematic and dynamic parameters of the space manipulator have been taken from work [1]. The nominal values of link lengths of the holonomic manipulator are equal to $l_1 = 0.449$ [m], $l_2 = 0.449$ [m] and $l_3 = 0.3103$ [m]. The coordinates (p_1, p_2) of the manipulator mounting point equal $(p_1, p_2) = (0.377$ [m], -0.001 [m]).

Distance w from the spacecraft geometry centre to each of corner is equal to $w = 0.2$ [m]. The dynamic parameters take the following nominal values: spacecraft mass $m_0 = 58.7$ [kg], masses of the first, second and third links of the holonomic manipulator equal $m_1 = 2.82$ [kg], $m_2 = 2.82$ [kg] and $m_3 = 4.64$ [kg], respectively; spacecraft inertia $I_0 = 2.42$ [kg·m²], links inertias are equal to $I_1 = 0.06$ [kg·m²], $I_2 = 0.06$ [kg·m²] and $I_3 = 0.05$ [kg·m²], respectively. The kinematic equations of the space manipulator from Fig. 1 take the following form:

$$p_e = f_e(q) = \begin{pmatrix} f_{e,1} \\ f_{e,2} \\ f_{e,3} \end{pmatrix} = \begin{pmatrix} x_{1,c} + p_1 c\theta - p_2 s\theta + l_1 c\theta 1 + l_2 c\theta 2 + l_3 c\theta 3 \\ x_{2,c} + p_1 s\theta + p_2 c\theta + l_1 s\theta 1 + l_2 s\theta 2 + l_3 s\theta 3 \\ \theta + y_1 + y_2 + y_3 \end{pmatrix}, \tag{21}$$

where $c\theta = \cos(\theta)$; $s\theta = \sin(\theta)$; $c\theta i = \cos\left(\theta + \sum_{j=1}^{i} y_j\right)$; $s\theta i = \sin\left(\theta + \sum_{j=1}^{i} y_j\right)$; $i = 1, \ldots, n$ and $n = 3$, respectively. Due to inequality $n - k = 3 > 0$, we can augment vector p_e by additional coordinates $p_a \in \mathbb{R}^3$ of the geometric centre of spacecraft and its orientation, as follows

$$p_a = f_a(q) = \begin{pmatrix} x_{1,c} \\ x_{2,c} \\ \theta \end{pmatrix}. \tag{22}$$

Disturbing signal d from (1) is assumed in the simulation to take the following form:

$$d(q) = \begin{bmatrix} \dfrac{\partial f_{e,1}(q)}{\partial q} \\ \dfrac{\partial f_{e,2}(q)}{\partial q} \end{bmatrix}^{\mathrm{T}} F, \tag{23}$$

where $F \in \mathbb{R}^2$ denotes external force vector (imitating the action of, e.g., a sub-part to be assembled) exerted on the end-effector. It takes the following form (external forces of a Brownian motion type):

$$F = \left(F_{x_1}, F_{x_2}\right)^{\mathrm{T}} = \left(0.3 + \eta_1(t),\, 0.5 + \eta_2(t)\right)^{\mathrm{T}}, \tag{24}$$

where $d\eta_i = \sqrt{t} X(t)\,dt$; $X(t) \sim N(0, 1)$; $i = 1, 2$; $t \in [0,25]$. The actuator matrix B in (1) equals

$$B(q) = Tr(\theta)\begin{bmatrix} B_{spacecraf} & \mathbb{O}_{3\times 3} \\ \mathbb{O}_{3\times 8} & \mathbb{I}_{3\times 3} \end{bmatrix}, \tag{25}$$

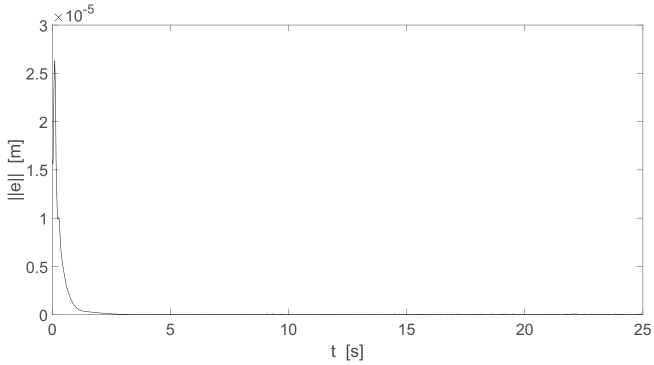

Fig. 2. The logarithm of the Euclidean norm of task errors *e* for controller (9), (20)

Rys. 2. Logarytm normy Euklidesowej błędow zadaniowych *e* dla sterownika (9), (20)

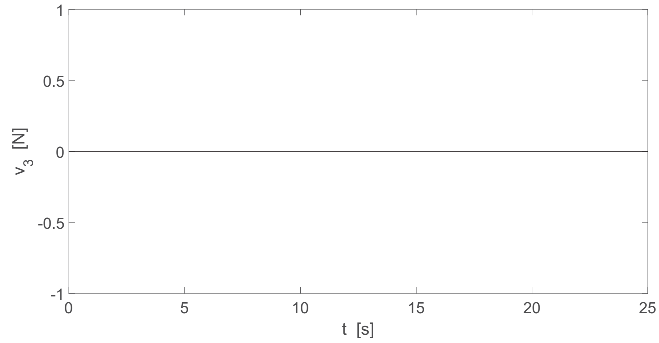

Fig. 5. Force v_3 of the third thruster for controller (9), (20)

Rys. 5. Siła v_3 trzeciego pędnika typu cold-gas dla sterownika (9), (20)

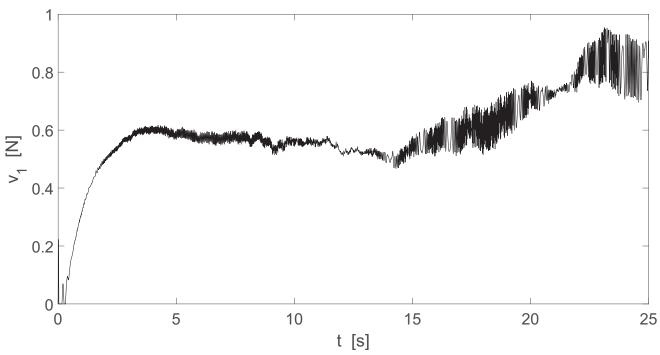

Fig. 3. Force v_1 of the first thruster for controller (9), (20)

Rys. 3. Siła v_1 pierwszego pędnika typu cold-gas dla sterownika (9), (20)

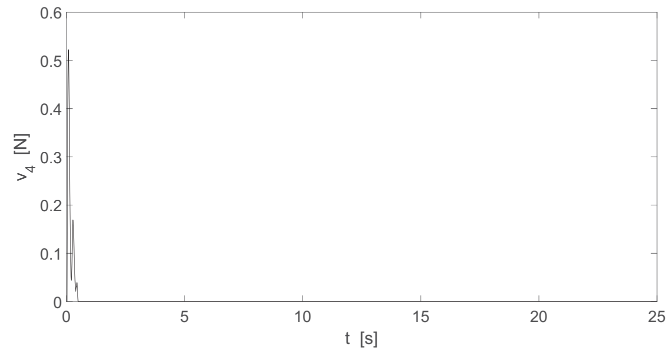

Fig. 6. Force v_4 of the fourth thruster for controller (9), (20)

Rys. 6. Siła v_4 czwartego pędnika typu cold-gas dla sterownika (9), (20)

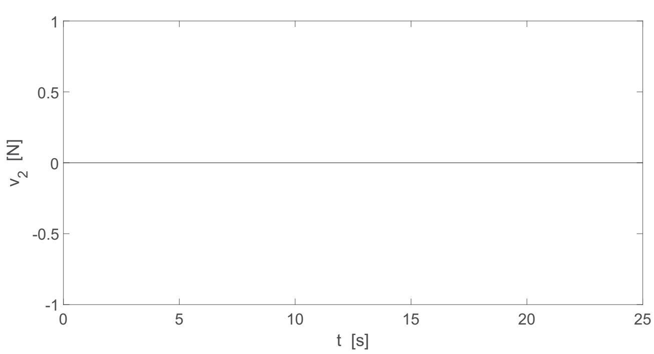

Fig. 4. Force v_2 of the second thruster for controller (9), (20)

Rys. 4. Siła v_2 drugiego pędnika typu cold-gas dla sterownika (9), (20)

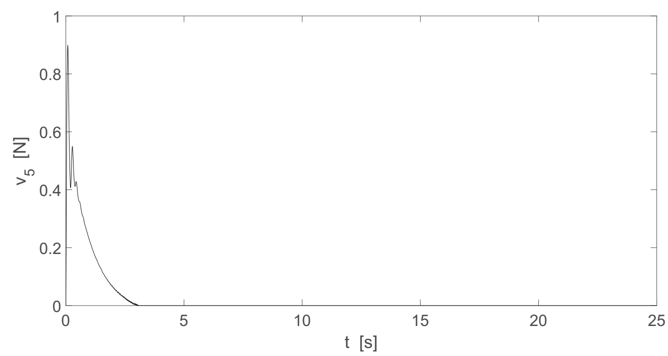

Fig. 7. Force v_5 of the fifth thruster for controller (9), (20)

Rys. 7. Siła v_5 piątego pędnika typu cold-gas dla sterownika (9), (20)

where

$$Tr(\theta) = \begin{bmatrix} Rot(\theta) & \mathbb{O}_{3\times3} \\ \mathbb{O}_{3\times8} & \mathbb{I}_{3\times3} \end{bmatrix};$$

$$Rot(\theta) = \begin{bmatrix} c(\theta) & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$B_{spacecraft} = \begin{bmatrix} -1 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 \\ \omega & -\omega & \omega & -\omega & \omega & -\omega & \omega & -\omega \end{bmatrix};$$

$\mathbb{I}_{3\times3}$ is the 3×3 identity matrix and $\mathbb{O}_{3\times3}$, $\mathbb{O}_{3\times8}$ denote 3×3 as well as 3×8 zero matrices, respectively. The estimates for controller (9), (20) are chosen as $a = 0.1$, $b_1 = 0.03$, $b_2 = 0.06$. In order to simplify numerical computations, rough conservative estimates of w_i, $i = 1, 2$ have been assumed. Hence, coefficients wi were chosen as follows $w_1 = 1$ and $w_2 = 0.02$, respectively. The initial configuration $q(0)$ and velocity $\dot{q}(0)$ are assumed to be equal to $q(0) = (0, 0, 0, -0.458, 2.19, -0.161)^{T}$ and $\dot{q}(0) = 0$, respectively. The task realized by controller (9), (20) is to track desired augmented trajectory $p_d(t)$ which takes the form

$$p_d(t) = \begin{pmatrix} 0.02\left(t + \exp(-t)\right) - 0.02 + 0.7077 \\ 0.5538 + 0.05\sin\left(\pi 0.02 t^2/6\right) \\ \pi/2 \\ 0.02\left(t + \exp(-t)\right) - 0.02 \\ 0 \\ 0 \end{pmatrix}. \qquad (26)$$

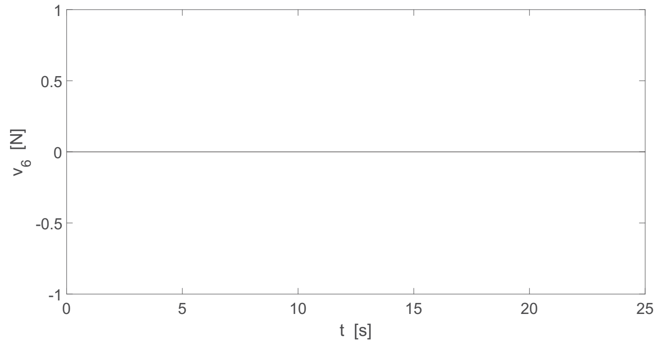

Fig. 8. Force v_6 of the sixth thruster for controller (9), (20)
Rys. 8. Siła v_6 szóstego pędnika typu cold-gas dla sterownika (9), (20)

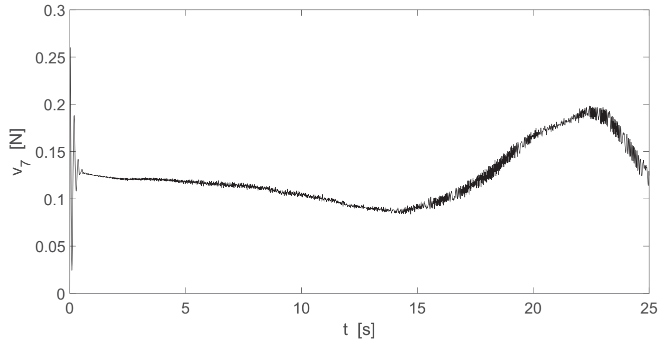

Fig. 9. Force v_7 of the seventh thruster for controller (9), (20)
Rys. 9. Siła v_7 siódmego pędnika typu cold-gas dla sterownika (9), (20)

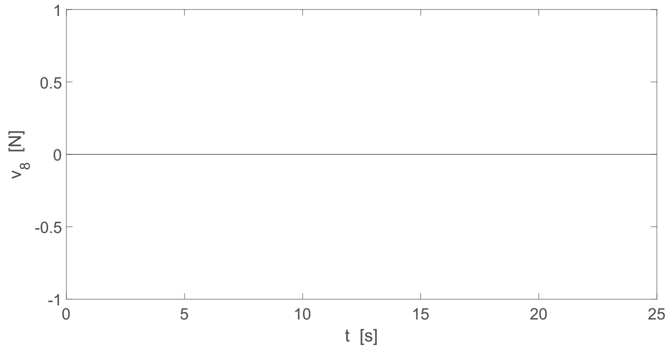

Fig. 10. Force v_8 of the eight thruster for controller (9), (20)
Rys. 10. Siła v_8 ósmego pędnika typu cold-gas dla sterownika (9), (20)

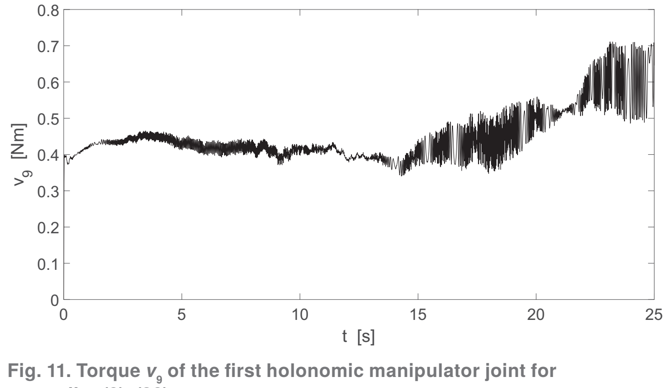

Fig. 11. Torque v_9 of the first holonomic manipulator joint for controller (9), (20)
Rys. 11. Moment napędowy v_9 pierwszego ogniwa manipulatora holonomicznego dla sterownika (9), (20)

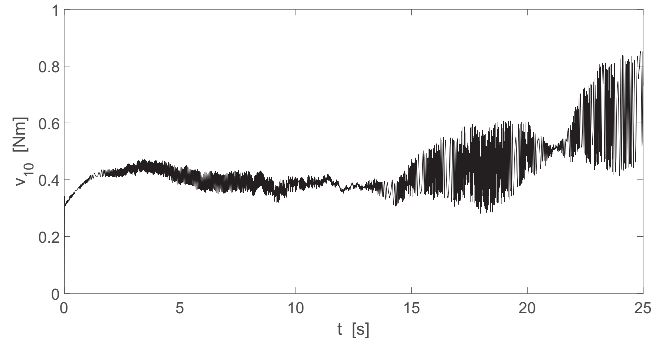

Fig. 12. Torque v_{10} of the second holonomic manipulator joint for controller (9), (20)
Rys. 12. Moment napędowy v_{10} drugiego ogniwa manipulatora holonomicznego dla sterownika (9), (20)

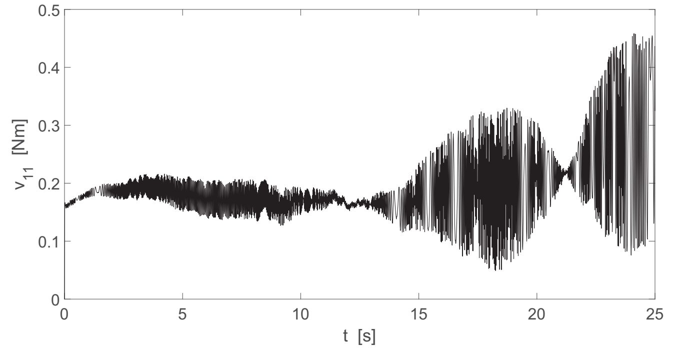

Fig. 13. Torque v_{11} of the third holonomic manipulator joint for controller (9), (20)
Rys. 13. Moment napędowy v_{11} trzeciego ogniwa manipulatora holonomicznego dla sterownika (9), (20)

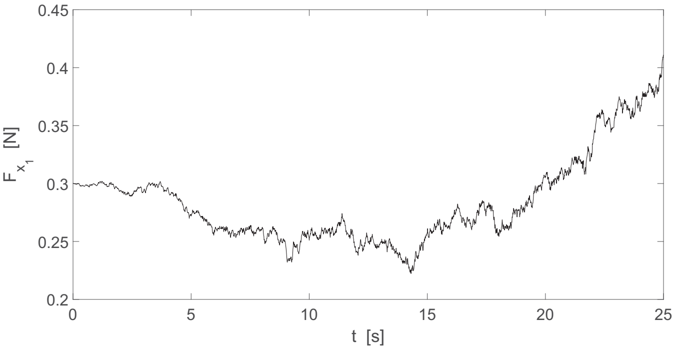

Fig. 14. Disturbing external force F_{x1} acting on the end-effector with controller (9), (20)
Rys. 14. Zewnętrzna siła zakłocająca F_{x1} działająca na koniec efektora dla sterownika (9), (20)

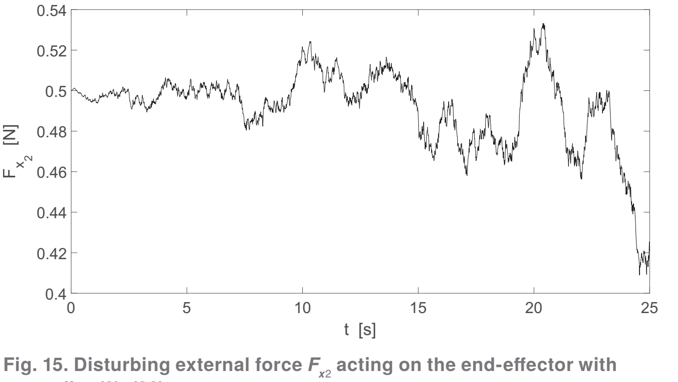

Fig. 15. Disturbing external force F_{x2} acting on the end-effector with controller (9), (20)
Rys. 15. Zewnętrzna siła zakłócająca F_{x2} działająca na koniec efektora dla sterownika (9), (20)

The estimates $\hat{J}$, $\hat{B}$ of the uncertain Jacobian matrix $J(q)$ and control matrix $B(q)$ are assumed in the computations to be equal to

$$\hat{J} = J + \Delta J, \qquad \hat{B} = B + \Delta B, \tag{27}$$

where each element $\Delta J_{i,j}$, $1 \le i, j \le 6$ 1 of matrix ΔJ and each element $\Delta B_{i,j}$, $1 \le i \le 6$, $1 \le j \le 11$ of ΔB were randomly generated according to normal distribution $0.1N(0, 1)$. In order to attain the convergence of task errors e at least less or equal to 10^{-6}, with simultaneous fulfilment of inequalities $0 \le v_i \le 1$ for $i = 1 : 8$ (forces generated by thrusters of the spacecraft), the following numerical values of gain coefficients are taken for controller (9), (20: $\omega_0 = 2$, $\omega_1 = 6$, $\alpha_1 = 3/5$, $\varepsilon = 0.01$, $c' = 2.4$, and $c_0 = 0.77$, respectively. On account of the physical properties of thrusters, we have equivalently modified computations of the thrusters forces v_i, $i = 1 : 8$ as follows [26]. If $v_i(t) < 0$ then $v_i(t) := 0$. Otherwise $v_i(t) := 2v_i(t)$, where symbol := means assigning a value to a variable. The results of numerical computations are depicted in Figs 2–15. As is seen from Fig. 2, our controller generates tracking errors e, which are practically for $t \ge 3$ equal to zero. Steering signals (eight forces generated by thrusters and three torques provided by DC motors of holonomic manipulator) are depicted in Figs 3–13. As is seen from Figs 3–13, control variables v_i, $i = 1 : 8$ fulfil inequalities $0 \le v_i \le 1$. Moreover, all the steering signals v_1, ..., v_{11} are absolutely continuous functions of time. Although, manipulator torques/forces (see Figs 3–13) present continuous mapping, the control signals shown in Figs 3–13 could not be feasible in real case due to the physical limitations of the actuators. In such a case a smoothing of torques/forces should be carried out. In order to eliminate additional phase delay related with recursive low-pass filters, one could apply Newton predictor enhanced Kalman filter (NPEKF) [9] which provides a wide bandwidth and significantly reduced phase lag. Finally, Figs 14–15 present one realization of external disturbing force $F = \left(F_{x_1}, F_{x_2}\right)^{\mathrm{T}}$ acting on the end-effector which tracks extended desired trajectory $p_d(t)$.

Let's know that the control algorithm (9), (15) is also able to converge to desired trajectory p_d starting from an arbitrary initial configuration. However, in order to maintain control constraints on usually imposed space craft thrusters, the control gains related to space craft should be sufficiently small. In practice, they should be chosen by trials and errors.

5. Conclusions

A new class of task space TSM controllers with finite-time stability when tracking a desired end-effector trajectory by the space manipulator has been proposed in this paper. On account of the fact that external forces acting on the mechanism are unknown, we have offered a sliding technique which seems to be effective in counteracting those undesirable forces. Moreover, our controller provides physically realizable steering signals. Another feature of the control law proposed is the elimination of the Jacobian matrix inverse (or pseudo-inverse) from the trajectory tracking. Instead, estimated extended Jacobian transpose matrix has been used. Applying the Lyapunov stability theory, control strategy (9), (15) is shown to be finite-time stable by fulfilment of practically reasonable assumptions. Numerical computations have shown that controller (9), (20) well performs under conditions of unknown external disturbances, uncertain kinematics and dynamics of the mechanism. Although our transposed estimated Jacobian controller needs some knowledge extracted from both the system kinematics and dynamics of the mechanism, the approach is able to handle uncertainties in kinematics and dynamics of the space manipulator as well as unknown external forces.

Acknowledgments

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Bibliography

1. Basmadji F.L., Chmaj G., Rybus T., Seweryn K., *Microgravity testbed for the development of space robot control systems and the demonstration of orbital maneuvers*, [In:] Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2019, Romaniuk R.S., Linczuk M., (Eds), Vol. 11176, International Society for Optics and Photonics. SPIE, 2019, 1158–1172, DOI: 10.1117/12.2537981.
2. Brogliato B., Neto A.T., *Practical stabilization of a class of nonlinear systems with partially known uncertainties*, "Automatica", Vol. 31, No. 1, 1995, 145–150, DOI: 10.1016/0005-1098(94)E0050-R.
3. Cheah C., *On duality of inverse Jacobian and transpose Jacobian in task-space regulation of robots*, [In:] Proceedings of the IEEE International Conference on Robotics and Automation, ICRA 2006, 2571–2576, DOI: 10.1109/ROBOT.2006.1642089.
4. Cheah C., Lee K., Kawamura S., Arimoto S., *Asymptotic stability of robot control with approximate Jacobian matrix and its application to visual servoing*, [In:] Proceedings of the 39th IEEE Conference on Decision and Control, Vol. 4, 2000, 3939–3944, DOI: 10.1109/CDC.2000.912329.
5. Defoort M., Floquet T., Kökösy A.-M., Perruquetti W., *A novel higher order sliding mode control scheme*, "Systems & Control Letters", Vol. 58, No. 2, 2009, 102–108, DOI: 10.1016/j.sysconle.2008.09.004.
6. Defoort M., Floquet T., Kökösy A.-M., Perruquetti W., *Higher order sliding modes in collaborative robotics*, [In:] Sliding Modes after the First Decade of the 21st Century: State of the Art, Lecture Notes in Control and Information Sciences, Fridman L., Moreno J., Iriarte R. (Eds), Vol. 412. Berlin, Heidelberg: Springer, 2012, 409–437, DOI: 10.1007/978-3-642-22164-4_15.
7. Galicki M., *Inverse kinematics solution to mobile manipulators*, "The International Journal of Robotics Research", Vol. 22, No. 12, 2003, 1041–1064, DOI: 10.1177/0278364903022012004.
8. Galicki M., *Constraint finite-time control of redundant manipulators*, "International Journal of Robust and Nonlinear Control", Vol. 27, No. 4, 2016, 639–660, DOI: 10.1002/rnc.3591.
9. Han J.D., He Y., Xu W., *Angular acceleration estimation and feedback control: An experimental investigation*, "Mechatronics", Vol. 17, No. 9, 2007, 524–532, DOI: 10.1016/j.mechatronics.2007.05.006.
10. Hu Q., Zhang J., *Maneuver and vibration control of flexible manipulators using variable-speed control moment gyros*, "Acta Astronautica", Vol. 113, 2015, 105–119, DOI: 10.1016/j.actaastro.2015.03.026.
11. Jia S., Jia Y., Xu S., Hu Q., *Maneuver and active vibration suppression of free-flying space robot*, "IEEE Transactions on Aerospace and Electronic Systems", Vol. 54, No. 3, 2018, 1115–1134, DOI: 10.1109/TAES.2017.2775780.

12. Jia S., Shan J., *Finite-time trajectory tracking control of space manipulator under actuator saturation*, "IEEE Transactions on Industrial Electronics", Vol. 67, No. 3, 2020, 2086–2096, DOI: 10.1109/TIE.2019.2902789.
13. Khaloozadeh H., Homaeinejad M. Reza, *Real-time regulated sliding mode controller design of multiple manipulator space free-flying robot*, "Journal of Mechanical Science and Technology", Vol. 24, 2010, 1337–1351, DOI: 10.1007/s12206-010-0403-7.
14. Moosavian S.A.A., *Dynamics and control of free-flying robots in space: A survey*, "IFAC Proceedings Volumes", Vol. 37, No. 8, 2004, 621–626, DOI: 10.1016/S1474-6670(17)32047-5.
15. Moosavian S.A.A., Papadopoulos E., *Free-flying robots in space: an overview of dynamics modeling, planning and control*, "Robotica", Vol. 25, No. 5, 2007, 537–547, DOI: 10.1017/S0263574707003438.
16. Moosavian S.A.A., Papadopoulos E., *Modified transpose Jacobian control of robotic systems*, "Automatica", Vol. 43, No. 7, 2007, 1226–1233, DOI: 10.1016/j.automatica.2006.12.029.
17. Nanos K., Papadopoulos E.G., *On the dynamics and control of free-floating space manipulator systems in the presence of angular momentum*, "Frontiers in Robotics and AI", Vol. 4, 2017, 26.1–26.19, DOI: 10.3389/frobt.2017.00026.
18. Ratajczak A., Ratajczak J., *Trajectory reproduction algorithm in application to an on-orbit docking maneuver with tumbling target*, [In:] 12th International Workshop on Robot Motion and Control, RoMoCo 2019, 172–177, DOI: 10.1109/RoMoCo.2019.8787367.
19. Rybus T., Seweryn K., Sasiadek J.Z., *Control system for free-floating space manipulator based on nonlinear model predictive control (NMPC)*, "Journal of Intelligent & Robotic Systems", Vol. 85, 2017, 491–509, DOI: 10.1007/s10846-016-0396-2.
20. Seraji H., *A unified approach to motion control of mobile manipulators*, "The International Journal of Robotics Research", Vol. 17, No. 2, 1998, 107–118, DOI: 10.1177/027836499801700201.
21. Seraji H., Colbaugh R., *Improved configuration control for redundant robots*, "Journal of Robotic Systems", Vol. 7, No. 6, 1990, 897–928, DOI: 10.1002/rob.4620070607.
22. Slotine J.-J.E., Li W., *Applied nonlinear control*. Englewood Cliffs, New Jersey: Prentice Hall, 1991.
23. Xu W., Hu Z., Yan L., Yuan H., Liang B., *Modeling and planning of a space robot for capturing tumbling target by approaching the dynamic closest point*, "Multibody System Dynamics", Vol. 47, 2019, 203–241, DOI: 10.1007/s11044-019-09683-3.
24. Yao Q., *Robust finite-time trajectory tracking control for a space manipulator with parametric uncertainties and external disturbances*, "Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering", Vol. 236, No. 2, 2022, 396–409, DOI: 10.1177/09544100211014754.
25. Yoshikawa T., *Manipulability of robotic mechanisms*, "The International Journal of Robotics Research", Vol. 4, No. 2, 1985, 3–9, DOI: 10.1177/027836498500400201.
26. Zappulla R., Virgili-Llop J., Zagaris C., Park H., Sharp A., Romano M., *Floating spacecraft symulator test bed for the experimental testing of autonomous guidance, navigation, and control of spacecraft proximity maneuvers and operations*, [In:] AIAA/AAS Astrodynamics Specialist Conference. Calhoun, 2016, 1–26, DOI: 10.2514/6.2016-5268.

Sterowanie odporne śledzeniem trajektorii manipulatora kosmicznego w rozszerzonej przestrzeni zadaniowej

Streszczenie: W pracy zaproponowano nową klasę sterowników dla manipulatorów kosmicznych przy uwzględnieniu nieznanych, niepożądanych sił zakłócających wywieranych na koniec efektora. W oparciu o odpowiednio zdefiniowane nieosobliwą, końcową rozmaitość ślizgową i teorię stabilności Lapunowa wyprowadzono klasę rozszerzonych estymowanych transponowanych sterowników Jakobianowych, ktore wydają się być efektywne w przeciwdziałaniu nieustrukturyzowanych sił zakłocających. Podejście zilustrowano również obliczeniami numerycznymi dla manipulatora kosmicznego składającego się z bazy napędzanej przez osiem pędników typu cold-gas i manipulatora holonomicznego o trzech parach kinematycznych obrotowych.

Słowa kluczowe: manipulator kosmiczny, śledzenie trajektorii, nieustrukturyzowane siły zakłocające, odporne skończone czasowo sterowanie w przestrzeni zadaniowej, stabilność Lapunowa

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