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## COMPARISON OF HIGHER HARMONIC CONTENTS IN SALIENT POLE SYNCHRONOUS GENERATOR WITH RADIAL INCISIONS ON THE SOLID POLE SURFACE

**Abstract:** The paper presents a comparison of higher harmonics in the induced voltages in the stator windings of three salient pole synchronous generators with different rotor construction. The comparison is done for generators with solid rotor without a skew, with solid rotor with radial incisions and with a rotor which consists of laminated electrotechnical steel with a skew. The calculations of the magnetic field distribution in the air gap are made in FEMM program. Comparison of the induced voltage waveforms in the stator windings is based on experimental investigations.

**Keywords:** salient pole synchronous generator, solid rotor, higher harmonics

### 1. Introduction

The occurrence of higher harmonics in the voltage induced in the stator windings of a salient pole synchronous generator is the subject of many considerations [1-15]. From those articles it follows that generating higher harmonics in induced stator voltages is dependent on:

- a type of stator winding (frequently the single-layer and the double-layer ones are used) [1, 6-8],
- a shape of the stator and rotor magnetic circuit [4, 6, 14],
- an inner eccentricity of a stator and a rotor [3, 10-12, 14],
- a control system of the field current or field voltage [2, 6, 7, 9],
- a saturation of magnetic circuit [4, 13, 14],
- a presence of the damping cage [15].

The shape of a salient pole synchronous generator pole shoe has a decisive influence on magnetic flux density distribution in the air gap and on the waveform of induced stator voltages [4, 6, 14] especially when the bars of the damping cage are placed inside (a pole shoe) [7, 15].

In the case of cooperation of a synchronous generator with power network, the presence of a damping cage allows to shorten many transient processes [16, 17]. In the case of the generator operating in autonomous regime, the presence of a damping cage is debatable. The role of the damping cage can be taken by solid iron. Elements of the synchronous generators pole shoes are made either as a solid or with electrotechnical sheets with a thickness up to 2.5 mm [18].

An influence of a length of an air gap on the eddy currents (and power loss) in the solid elements of a salient pole synchronous generators is more visible in low-power generators (because of a small initial length of the air gap). One of the methods of reducing eddy currents is constructing pole shoes with electrotechnical sheets [18], or as will be shown in this paper, by making radial incisions across the width of a pole shoe. The depth of an incision depends on the type of material (its magnetic permeability and electrical conductivity) and the frequency of magnetic field and generated eddy currents (under the influence of the field).

Eddy current penetration depth is determined as [18, 19]:

$$\lambda = \frac{5 \cdot 10^5}{\sqrt{f\gamma\mu_r}} \quad (1)$$

Where:  $\lambda$  – depth of eddy currents penetration,  $f$  – frequency,  $\gamma$  – electrical conductivity,  $\mu_r$  – relative magnetic permeability.

This paper presents the influence of the pole shoe radial incision on the shape of the normal magnetic flux density distribution in the air gap and on the waveform of voltages induced in the stator windings.

In the examined 5.5 kVA salient pole synchronous generator, a length of the air gap between stator and solid pole shoe is expressed:

$$\delta = \frac{\delta_0}{\cos \alpha} \quad (2)$$

Where:  $\delta_0$  – initial length of the air gap in longitudinal axis,  $\alpha$  – electrical angle of the pole shoe which is calculated in relation to the longitudinal axis.

Magnetic flux density distributions presented in this article are determined in the air gap of the salient pole synchronous generator rated data:  $S_N = 5.5$  kVA,  $I_N = 7.9$  A,  $U_N = 400$  V (Y),  $\cos\varphi_N = 0.8$ ,  $n_N = 3000$  rpm,  $Q_s = 24$  (number of stator slots),  $p_b = 1$  (number of pole pairs), with the rotor skew  $\alpha_q = 15^\circ$  (one stator tooth pitch) and without a rotor skew. Geometry of the field model is created in FEMM program [6, 7, 20]. Figure 1 shows the relative influence of a pole shoe radial incision on the effective air gap in relation to inner stator radius  $r_s$ .

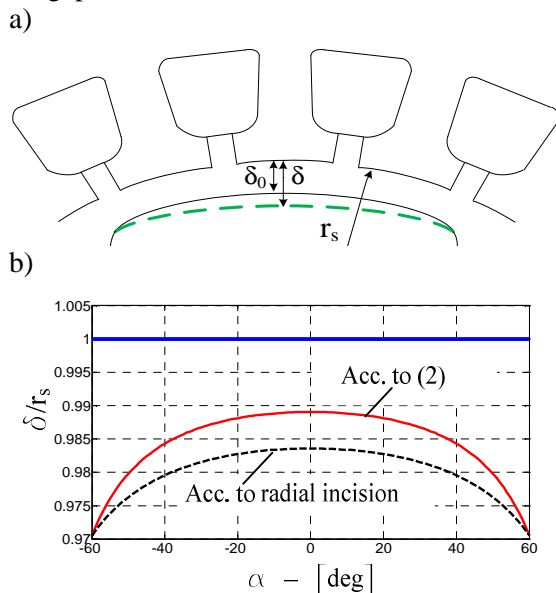


Fig. 1. Influence of the radial incision on the air gap length a) a piece of the magnetic circuit, b) length of the air gap vs.  $\alpha$

Figure 1 shows that the introduction of the radial incisions on the pole shoe surface results in even loss of the steel (in the pole shoe). However, the introduction of an even air gap between the stator and the pole shoe as compared to the actual curvature (of the pole shoe) have the effect of flattening the resultant magnetic flux density in the air gap (a similar effect as in case of the pole shoe saturation). The advantage of such radial incisions of the solid rotor is that it can be done on a simple lathe.

## 2. Distributions of magnetic flux density

Figure 2 shows the distribution of the magnetic flux lines of the 5.5 kVA salient pole synchronous generator with the current linkage of field winding in the no load steady state.

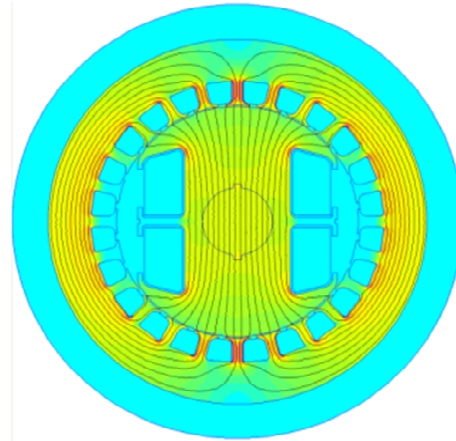


Fig. 2. Magnetic flux distribution lines in the 5.5-kVA salient pole synchronous generator in the no load steady state

Figure 3 shows a comparison of distribution of the normal component of magnetic flux density in the air gap for non-linear magnetic circuit of the 5.5 kVA synchronous generator with and without the radial incisions for the rotor with and without the skew. The depth of the radial incisions on the circumference of the rotor is equal to  $2\delta_0$ .

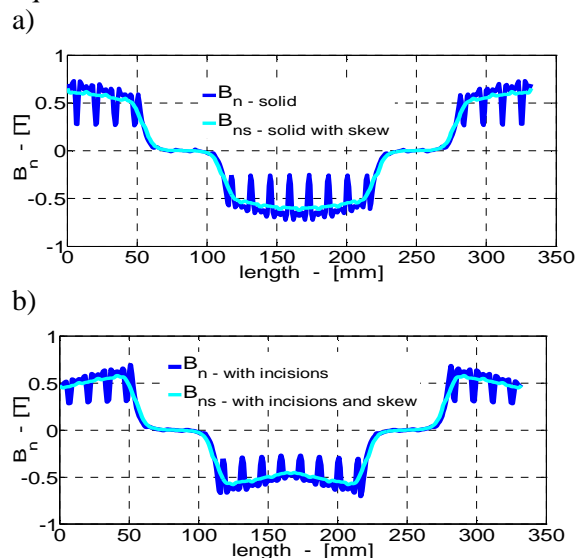


Fig. 3. Comparison of magnetic flux density distributions for the model with the a) solid rotor b) solid rotor with the radial incision

Shown in Figure 1 the simplified air gap length increase from  $\delta_0$  to  $\delta$  in the zone above pole shoe is reduced by the influence of the zone of the stator teeth and slots (Fig. 3). Permeance change on the stator teeth and slots is caused by dips in the distribution of the normal component of the magnetic flux density (Fig. 3). Vis-

ible dips are determined by the quotient of minimum to maximum magnetic flux density [18, 19]

$$B_{\min} / B_{\max} = 2u / (1 + u^2) \quad (3)$$

Where:  $u = b_s / 2\delta_0 + \sqrt{1 + (b_s / 2\delta_0)^2}$ ,  $b_s$  – length of stator slot opening.

The problem of permeance changes on the stator tooth – slot opening was firstly solved by F. W. Carter in 1901. In the design of electrical machines, magnetic flux density dips (due to slot opening) take into account the so-called Carter factor that increases the initial length of the air gap  $\delta$  [19]. Figure 4 shows the participation of the higher harmonics in the magnetic flux density distributions (shown in Fig. 3) due to Fourier analysis.

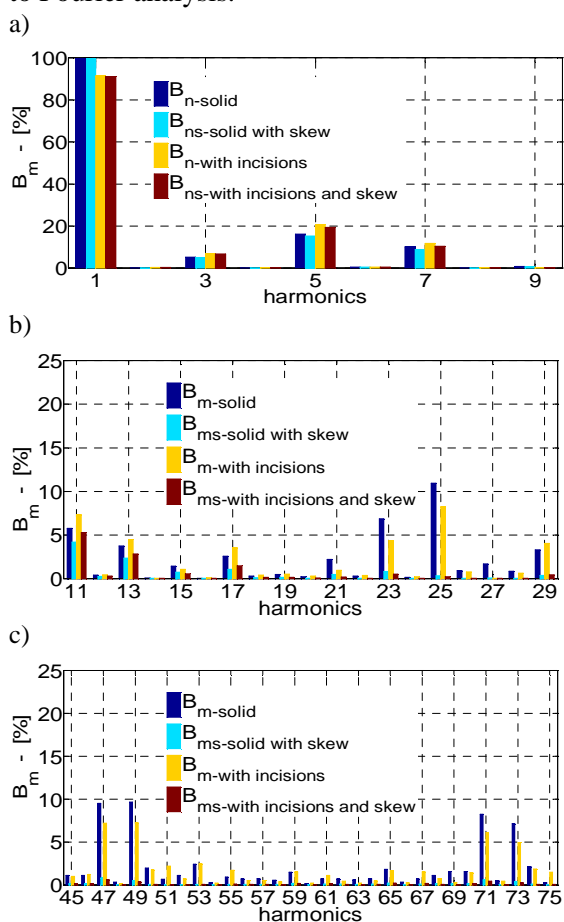


Fig. 4. Comparison of harmonic contents in the magnetic flux density distributions a) from 1st to 9th order, b) from 11th to 29th, c) from 45th to 75th

From Figure 4 it can be seen that the  $2\delta_0$  radial incision causes:

- 10% reduction of the fundamental component of the magnetic flux density (Fig. 4a),

- a few percent increase of the odd harmonics up to the 17th order,
- a few percent reduction of the amplitude of  $kQ_s \pm 1$  higher order harmonics ( $k$  – integer number).

The resultant increase in harmonics up to the 17th order (Fig. 4) is depend on the number of incisions (which is the percentage loss of steel along the length of the pole shoe and can vary from  $B_{nv}$  to  $B_{nv(\text{with incisions})}$  (without a skew) and from  $B_{nsv}$  to  $B_{nsv(\text{with incisions and skew})}$  (with a skew). The effect of flattening the shape of the pole shoe can be reduced by replacing the radial incision by incisions defined by relation (2). But such incision must be made with numerically controlled machine tools. Presented magnetic flux distributions (Fig. 2) have an impact on the content of higher harmonics in the induced stator voltages. However, the participation of individual  $v$ -harmonics are depend not only on magnetic flux distribution (Fig. 3) but also on the values of the winding coefficient (mostly for single-layer and double-layer winding), on the winding distribution, slot opening and used rotor skew [8].

### 3. Experimental investigation

Experimental investigation of the 5.5 kVA salient pole synchronous generator with the three rotor structures (solid, solid with radial incisions and made of electrotechnical sheets) are carried out in the measurement set shown in Figure 5.



Fig. 5. View of the measurement set for investigation of the 5.5 kVA salient pole synchronous generator

Figure 6 shows two pole shoes of the salient pole rotors. The first one with a solid structure, with the length of the air gap  $\delta$  determined by equation (2). The second one with solid structure with radial incisions of pole shoe surface (made on the lathe with 1.2 mm knife width).

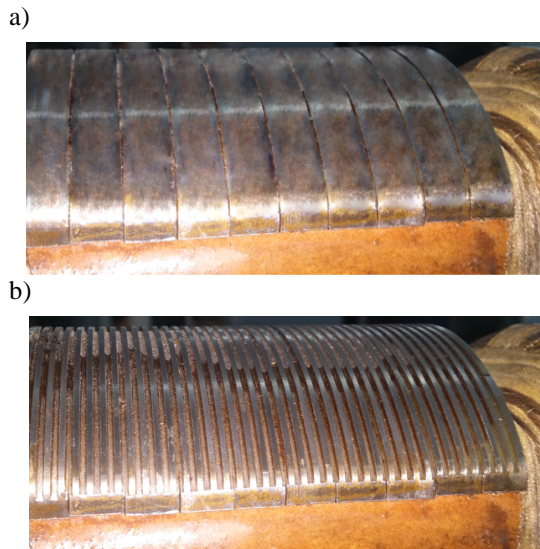


Fig. 6. View of the pole shoe surfaces of two salient pole rotors a) solid, b) solid with radial incisions

Figure 7 shows a comparison of registered no-load characteristics of the examined salient pole synchronous generator with following rotor structures: solid with radial incisions and factory rotor made of insulated electrotechnical sheets.

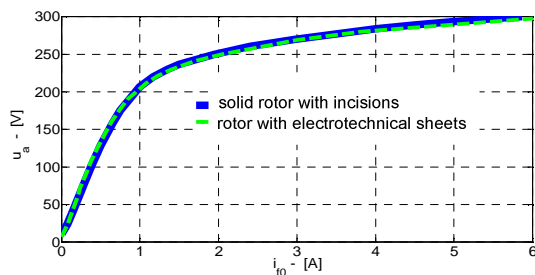


Fig. 7. Comparison of the two registered no-load characteristics of the examined salient pole synchronous generator with the solid rotor with radial incisions and with rotor made of insulated electrotechnical sheets

From comparison (Fig. 7) it can be concluded that the no-load characteristics are very similar. Solid rotor structure due to the small initial length of the air gap and a significant influence of eddy currents in the area of the pole shoe causes major deformations of induced voltage in the stator windings especially for the generator with the solid rotor without skew. In Figure 8 are presented registered waveforms of the induced stator voltages, the field current of the 5.5 kVA salient pole synchronous generator for the case of powering the field winding by  $U_f =$

const (cyan waveform - Channel 2 - Fig. 8c) with following rotor structures:

- solid without skew (200 V/div),
- solid without skew with radial incisions (100 V/div),
- with insulated electrotechnical sheets with skew (100 V/div).

Experimental investigations for the same RMS value 230 V of the fundamental voltage are carried out. As shown in [6, 7] this way of powering the field winding causes the voltages induced in the stator windings to have the lowest content of higher harmonics.

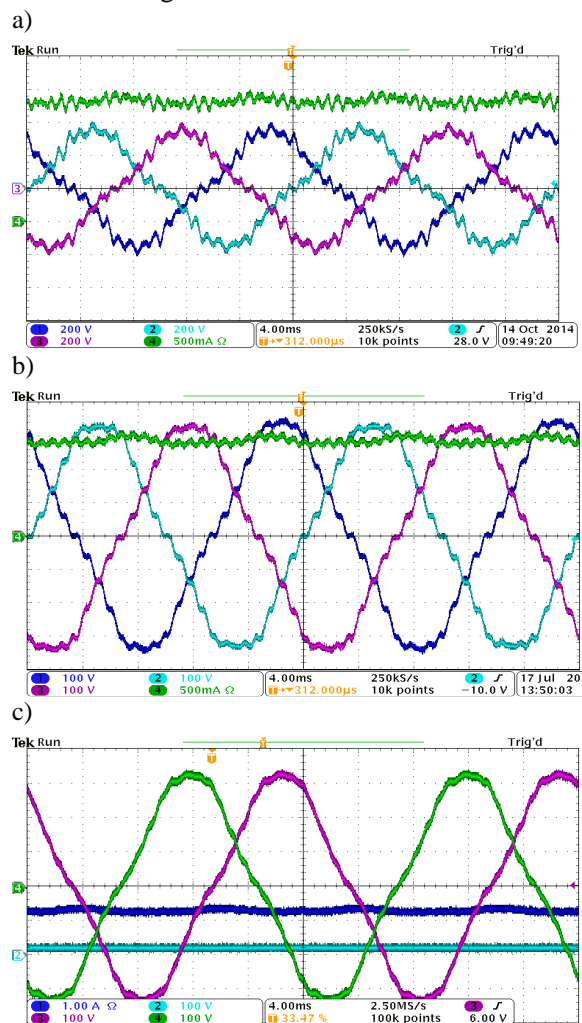


Fig. 8. Registered waveforms of the induced stator voltages under no-load conditions for the 5.5 kVA salient pole synchronous generator with a) solid rotor without skew (200 V/div), b) solid rotor with radial incisions without skew (100 V/div), c) rotor made of electrotechnical sheets with skew (100 V/div)



Figure 9 shows a comparison of the individual harmonic distortion (*IHD*) in the induced stator voltages (Fig. 8) with respect to the voltage fundamental component due to Fourier analysis.

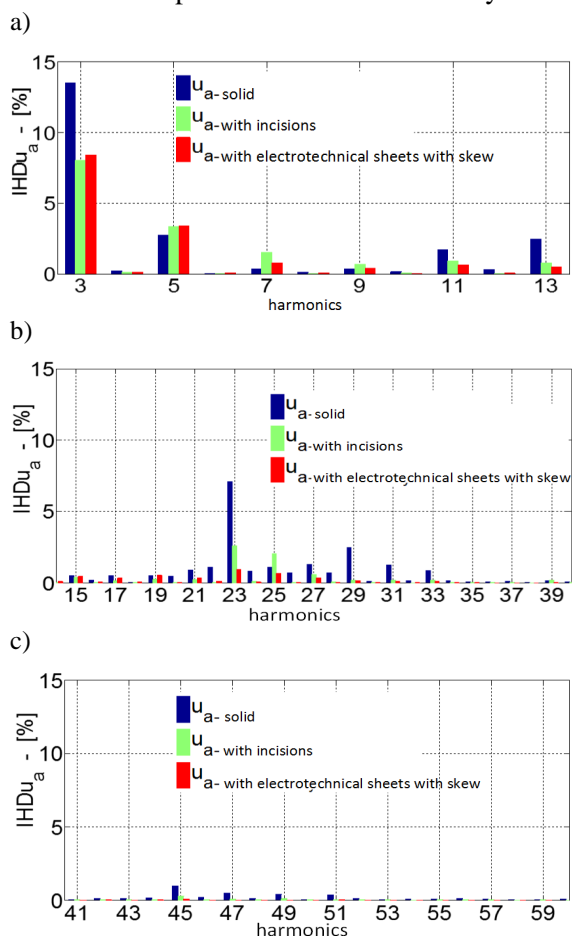


Fig. 9. Comparison of the higher harmonics in the induced stator voltages of the salient pole synchronous generator with three rotors a) from 3rd to 13th order, b) from 15th to 39th order, c) from 41st to 59th order

The total harmonic distortion  $THDu_a$  (Fig. 9) counted up to 60th harmonic for generator with:

- solid rotor without skew (index – *S*)  $THDu_{a-S} = 16.38\%$ ,
- solid rotor with radial incisions (index – *SwRI*) without skew  $THDu_{a-SwRI} = 9.58\%$ ,
- with electrotechnical sheets with skew (index – *ES*)  $THDu_{a-ES} = 9.29\%$ .

#### 4. Conclusions

The paper presents a comparison of higher harmonic contents in induced stator voltages of low power 5.5 kVA salient pole synchronous generators with solid rotor without skew, with solid rotor and radial incisions without skew and with the rotor made of electrotechnical sheets with skew. Based on the comparison of

$IHDu_a$  and  $THDu_a$  following conclusions are drawn:

- introduction of the radial incisions on the solid surface of the pole shoe increases the equivalent air gap length, which according to (3) and Figure 3 reduces the amplitude of the magnetic flux density into the zone of stator slot openings,
- higher harmonics occurring in the normal component of the magnetic flux density are  $\nu$ -times lower in the induced voltages (with the most dominant 3rd harmonic),
- introduction of the radial incisions on the solid surface of the pole shoe increases by 32% the value of the 3rd harmonic in component of the normal magnetic flux density but its participation is reduced by 43% in the induced voltages,
- introduction of radial incisions on the solid surface of the pole shoe causes similar spectrum of higher harmonics in the induced stator voltages as for the generator with rotor made of electrotechnical sheets.

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