

# The mathematical description of the electromagnetic transitory processes in synchronous electric machine as the shaft-generator. Principles and the methodology of the description

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## Abstract

On the contemporary ships wide applies the systems electromagnetic with the shaft-generator. The simulating investigations of the dynamics of the electromagnetic transitory processes in such system need the complex model mathematical study, describing the dynamics of objects in him contained. The following article is the **next** part of the final description of the mathematical model of the shipping system electromagnetics with the shaft-generator. The paper describes external conditions, which can influencing on the course of electromagnetic processes in the systems and there is giving exit principles of the description of the electromagnetic transitory processes in synchronous electric machine, which is working as the shaft-generator (PW).

## Introduction

Electric energy production cost on modern ships, especially specialist, takes an increasing part of the ship's daily operating costs.

Modern ships are characterized by the high stage of installed electric power. The growth in amount of the ship's electrical devices, in turn, affects the range and the stage of the automation of the power plant [1, 2]. It means sharper requirements for quality and costs of the electro-energy.

Electric energy takes the considerable part among various factors influencing the cost of the ship's exploitation. There is simply dependence between the quantity of electric receivers and both, installed electric power and the quantity of current generators included in the ship's power plant.

That is the reason why the ship-owners are so interested in the electro-energetic systems with the shaft-generator.

Simulation studies of transitory processes in such systems become the necessity [2]. This requires creating a complex mathematical model

for the whole electro-energetic system, not only for individual objects.

There is giving exit principles of the description of the electromagnetic transitory processes in synchronous electric machine, which is working as the shaft-generator (PW). The article presents description of the adopted methodology and requirements, which should be met by the model to obtain reliable results.

## Initial assumptions

Assumptions that should be met to create useful model description:

- the structure of model must contain all the important objects that have an impact on the transitory processes in the electro-energetic system;
- the most important object in a mathematical description of and electromagnetic system is the description of the shaft-generator (PW);
- this difficulty results from the fact that the shaft-generator combines two systems: the electric

system and the energetic system. Therefore, it is subject to coercions from both systems;

- the course of electromagnetic processes in the shaft-generator is affected by disturbance coming not only from the electrical grid, but from its driving unit, which is the main engine;
- model should include the influence of external conditions, i.e.: rolling of the sea, wind and its direction, ship's transverse and longitudinal swinging, ship's propeller ascending. This results in the change of the rotary speed of the main engine (SG);
- the main engine (SG) is perceived as the object regulating the shaft-generator rotational speed (PW);
- created model should be useful for ship with the adjusting propeller, as well as non-adjusting propeller;
- adoption of proper description methods of dynamic conditions of the principal components of the system, i.e.: main engine, auxiliary engines (AE); ship's power station and electric receipts with the division on static and dynamic receiver;
- possibility to analyze working conditions of ship's power station, i.e.: lonely work or parallel work of the ship's generators in various configurations; including cooperation with the shaft-generator (PW) and the port electrical grid;
- and mainly it should fulfill the criterion of credibility and liability. This condition imposes the necessity of use widely recognized and proven methodology of description.

Fulfillment of above assumptions will enable the study of electromagnetic transitory processes, which result will let to approach to conditions stepping out in real ship's systems.

Only methodology of description of electromagnetic processes inside the shaft generator is presented below (PW).

### **Specific of the work of the shaft-generator**

Advantages and disadvantages of the generator shafts are well known. However, the mathematical description of the dynamics is complex due to the specific of main engine drive and its location in the electro-energetic system. Getting the knowledge about the dynamic processes of shaft-generator and its influence on the work of the fleet by the experiments, is not only inconvenient, but also expensive. It forces us to build:

- laboratory workstation, which research results are greatly doubtful;
- or a prototype to test it on a ship, which is expensive and risky, what it is expensive in the

turn and burdened with the risk. After that, knowledge of certain symptoms is most needed at the stage of design work.

Therefore, the most purposeful is computer simulation [2], and this requires mathematical model. The shaft-generator (SG) is unit joining two physically different arrangements – electric net and the propulsive complex in the type-matter: main engine – main shaft (GW) – adjustable or non-adjustable propeller.

In result, the shaft-generator subject to two principal forces:

- from the side of the electric net, because of the change of active and passive electric power;
- from the side of the drive of shaft-generator, because of the change of the resistory moment on the shipping screw, what causes the labile rotary speed of the main shaft.

The first force is influences on the change of voltage of shaft-generator. The second influences both, on the tension and the frequency of the electric current. Summing up, we should affirm, that the dynamics of shaft-generator depends on the dynamic properties of the propulsive complex. This in turn, subjects to the influence of the whole sequence of external factors. One can here enumerate principal influences:

- condition of the sea and strength of the wind;
- transverse and longitudinal swing of the ship;
- level of immersion of the ship and its propeller;
- periodical raising out of the propeller of the water;
- the displacement the center of gravity of the ship called out by the loading etc.

The situation is being complicated by the fact, that one can't subordinate the work of the main engine to the exploration requirements of the shaft-generator. This results from the conditions of the safety of navigation. In effect, labile rotary speed of the main shaft causes labile frequency in the electric net, what is harmful.

Taking into account these circumstances sets certain requirements for the mathematical model, which contains the description of the shaft-generator.

### **The choice of the type of the ship adjustable propeller or steady propeller**

It is believed that the adjustable propeller system favors the use of shaft-generator. It is not so obvious. It depends on the specific solution and adopted criterion of propeller control. Propeller type selection is dictated by kind of ship and its destination (scheduled jobs). There is a considerable number of

solutions of adjustable propeller systems. Generally adjustable propeller systems can be divided into:

- separate steering of the stroke of the propeller spades ( $h$ ) and rotary speed ( $n$ );
- adjustment by the stroke of the propeller only ( $h \neq \text{const}$ ), near the stable rotary speed ( $n = \text{const}$ );
- simultaneously program control of both parameters ( $h \neq \text{const}$ ;  $n \neq \text{const}$ );
- automated, self-regulating systems.

While separate steering, operator gives both the parameters separately, with the support of suitable charts and diagrams, with regard to the conditions of navigation and the technical condition of the main engine. The last two systems from the principle enter the changes of both parameters. This results in a labile rotary speed of the main shaft. This situation is usually accompanied by a control based on the criteria:

- reaching optimum ship's speed;
- the optimum fuel consumption.

There is no doubt that, the hardest working conditions of shaft-generator take place on ship with steady propeller. This case should be considered and included in the mathematical model.

Only second type of adjustable propeller provides supportive conditions of work for the shaft-generator. This kind of steering the adjustable propeller had been chosen as the object of the description.

### Choosing the type of electrical machine as a shaft generator

From the standpoint of the shaft generator, the main engine serves as variable rotational speed's drive [1, 3]. This causes difficulties in maintaining constant voltage ( $U$ ) and frequency ( $f$ ) in electrical network. Voltage problem may be considered as solved because of the widespread use of excellent, electronic, very fast-acting voltage regulators. They are able to maintain constant voltage over a wide change of speed of the generator. The problem of frequency is still valid.

There are many solutions of shaft generator's systems [1, 2, 3, 4, 5, 6]. They differ from each other in the idea of work and in the way of stabilizing the frequency ( $f$ ) current and voltage. One can observe two main trends:

- stabilizing the rotational speed ( $n$ ) between the main shaft and the electric machine;
- stabilization of the frequency ( $f$ ) by electric methods if we allow fluctuations of the main shaft's rotational speed.

In light of Classification Societies laws, shaft generator can be only considered as a basic source of ship's electrical power if it is capable to long-term, trouble-free parallel cooperation with the traditional ship's generator [1, 2, 3, 4, 5, 6]. In fact, there is no such a universal solution that can satisfy all the requirements of the law. The possibility of such an investigation must be taken into account in the simulation model.

Synchronous electric machine is the main source of electric power of alternating, three-phase, sinusoidal current on ships all over the world. It is also used in 90% of linear alternators, which are currently in operation.

The alternative is to use an asynchronous machine but its commonly known, operating difficulties during working as a generator make that it is used reluctantly and only in a small, experimental solutions of linear alternators [3, 4, 5]. For this reason, described model assumes a synchronous, three-phase machine.

Description of electromagnetic phenomena in three-phase, synchronous machine should contain Park-Gorev's equations [2] with the following assumptions:

- synchronous machine should be treated as a two-phase machine in the coordinate of ( $q, d$ );
- coordinate of ( $q, d$ ) spins with a frequency of the rotor of shaft generator;
- equations should be written using relative units.

Then the number of differential equations will be limited this way, what will facilitate analysis of the results. Usage of relative units will allow to compare changes of different physical parameters.

### Description of the generator in the ship's power plant

Standard three-phase generator (ZP) of alternating current that is used in ship's power plants consists of:

- two machines, auxiliary Diesel's engine as drive and three-phase synchronous machine as a generator;
- two regulators: rotational speed regulator of the auxiliary engines (SP) and synchronous machine's voltage regulator.

In this case, the task of the auxiliary motor (SP) is completely different from the task of main engine (SG).

Its work is entirely subordinated to the needs of the generator. The only task is to maintain a constant rotational speed ( $n = \text{const}$ ) of the complex during widely balancing load of electric

power. Constancy of the rotational speed is equivalent to the constancy of frequency in ship's electrical network.

This task being fulfilled by Diesel's rotational speed regulator. The UG-8 Woodward's controller had been selected for this model because of its widespread use. In general, both the frequency of electric current and the amount of produced active electric power ( $P$ ) depends on rotational speed regulator's work.

When operating with a single unit in ship's network, it affects the frequency ( $f$ ) only. When operating in 'rigid' net, it affects only active electric power ( $P$ ). This task is being fulfilled by Diesel's rotational speed regulator. The UG-8 Woodward's controller had power ( $P$ ). With the parallel cooperation of two units of the same power, it affects both of these factors, depending on the circumstances. Description of the controller's dynamics will be equivalent to a description of dynamics of the auxiliary motor (SP).

Voltage regulator maintains voltage stability ( $U$ ) in the network, but also affects the reactive power ( $Q$ ). Adequate interaction depends-as noted above-on whether the unit is working alone, or it is in parallel work with another unit.

Electromagnetic processes in this generator, similarly like in the shaft-generator should be also described by Park-Gorev's equations [7, 8].

### The essence of description of transient processes in the shaft-generator (PW), using the Park-Gorev equation system

In the traditional description of transient processes, in a synchronous machine, a differential equation system (1) in a matrix form is used. The equations are written in fixed axis  $A, B, C$ , compliant with allocation of the phase windings on the perimeter of the stator of the machine.

$$\begin{aligned} -[\Psi] - R[\mathbf{I}] &= [\mathbf{U}] = -[\mathbf{U}_s] \\ \frac{d}{dt}[\Psi_r] - [\mathbf{R}_r][\mathbf{I}_r] &= [\mathbf{U}_r] \\ J \frac{d^2}{dt^2}\gamma - M &= M_M \end{aligned} \quad (1)$$

where:

$\Psi$  – the matrix of the streams of the coils of phase the stator  $A, B, C$ ;

$\Psi_r$  – the matrix of the streams of the rotor, suitably: the winding of the excitement  $\Psi_f$ , component the longitudinal and transverse fluxes of the damper windings  $\Psi_{rd}, \Psi_{rq}$ ;

$\mathbf{R}_r$  – the matrix of the active resistances of the rotor, suitably:  $R_f, R_{rd}, R_{rq}$

$\mathbf{I}$  – the matrix of currents of the phase stator  $I_A, I_B, I_C$ ;

$\mathbf{I}_r$  – the matrix of the currents of the rotor;

$\mathbf{U}$  – the matrix of the voltage of the phases of generator:  $U_A, U_B, U_C$ ;

$\mathbf{U}_s$  – the matrix of the voltage of the phases of net:  $U_{sA}, U_{sB}, U_{sC}$ ;

$\mathbf{U}_r$  – the matrix of the voltage of the rotor –  $U_f$  the winding of the excitement;

$M = \frac{dW}{dt}$  – the electromagnetic moment;  $W$  – the energy of the magnetic field;

$M_M$  – the mechanical moment (driving);

$J \frac{d^2}{dt^2}\gamma$  – the moment of the strengths of inertia;

$\gamma$  – angle between the axis of phase  $A$  of the stator and axis  $d$ .

The first equation determines the voltage balance in each phase of the stator. The second equation reflects the voltage balance in the excitation winding and the damper winding of the rotor. The third equation is an equation of the moments existing in the machine. All the values are given in physical units.

One has to notice that the ship's generators are salient pole machines and the equations contain many non-linearities. This leads to many difficulties, as most of coefficients of the self-inductance  $L$  and those of the mutual inductance  $M$  of the windings are periodic functions of angle  $\gamma$  of the position of the rotor whirling with period  $T = \pi$ .

Assuming a sinusoidal distribution in the machine air-gap, a linear dependence of this induction  $B$  on the magnetic field intensity  $H$  and the lack, it is possible to give the equations the form of linearly transformed Park-Gorev equations. These equations [7, 8] are much easier to apply and have constant coefficients (2):

$$\begin{aligned} \frac{d\psi_d}{dt} + \psi_q \frac{d\gamma}{dt} - RI_d &= U_d = -U_{sd} \\ \psi_d \frac{d\gamma}{dt} - \frac{d\psi_q}{dt} - RI_q &= U_q = -U_{sq} \\ \frac{d\psi_f}{dt} + R_f I_f &= U_f \\ \frac{d\psi_{rd}}{dt} + R_{rd} I_{rd} &= 0, \quad \frac{d\psi_{rq}}{dt} + R_{rq} I_{rq} = 0 \\ J \frac{d^2}{dt^2}\gamma + \frac{3}{2}(\psi_d I_q + \psi_q I_d) &= M_M \end{aligned} \quad (2)$$

The essence of the linear transformation of the original equations (1) consists in substituting the phase variables type  $Y_A, Y_B, Y_C$  with variables type

$Y_0, Y_d, Y_q$ . The variables are noted in a rectangular, whirling system of coordinates  $q, d$  rigidly linked to the electrical machine rotor. Axis  $q$  supersedes axis  $d$ .

In such circumstances, a synchronous three-phase machine becomes a diphasic electric machine. Its windings are located along axis  $d, q$  and perpendicularly to each other; a magnetic linking between the windings can be neglected [7, 8].

Finally, there are much fewer differential equations obtained. The equations do not contain any periodically changing coefficients and examination of transient electromagnetic processes in the machine becomes much easier.

On ships, electric systems have insulated generator neutral points, which fact helps to neglect the variables of neutral sequence  $Y_0$  and reduce even more the number the final equations (3):

$$\begin{aligned} \frac{d\psi_d}{dt} + \psi_q \frac{d\gamma}{dt} - RI_d &= U_d = -U_{sd} \\ \psi_d \frac{d\gamma}{dt} - \frac{d\psi_q}{dt} - RI_q &= U_q = -U_{sq} \\ \frac{d\psi_f}{dt} + R_f I_f &= U_f \\ J \frac{d^2\gamma}{dt^2} + \frac{3}{2}(\psi_d I_q + \psi_q I_d) &= M_M \end{aligned} \quad (3)$$

For the shaft-generator (PW), one may also neglect the effect of the damper, because the machine rotor is not able to change the rotational speed of the main engine (SG).

The complete deduction of the Park-Gorev equations is presented in item [7] of the reference list. Only the final forms of individual stages of the equation transformation are presented below.

### Selection of a system of relative factors

The ship's electroenergetic system is a set of object of various structures and modes of operation. It is necessary to notate the equations in relative units in order to be able to compare changes taking place in physically different parameters. Depending on adopted relative units, the final form of the equation system will be different. Hence, different forms of the Park-Gorev equation notation can be found in the literature [7, 8]. The following variants of the base units are relatively frequently used in notation of equations for the generator stator:

- the effective values of voltage and current  $I_{\sigma 1} = I_{\phi 1}$ . The power write down then  $S_{\sigma 1} = 3U_{\sigma} I_{\sigma}$ ;
- the effective values of voltage and current  $U_{\sigma 2} = U_{\phi n}$ ;  $I_{\sigma 2} = \sqrt{3} = \sqrt{3} I_{\sigma 1}$ . The power write down then  $S_{\sigma 2} = 3U_{\sigma 2} I_{\sigma 2}$ .

The relative unit system given below is based on values of the amplitudes of individual sinusoids:

For the stator circuits:

- the amplitude of the current nominal of the stator:  $I_{\sigma} = I_{\phi n} \sqrt{2}$ ;
- the amplitude of the nominal phase voltage of the stator:  $U_{\sigma} = U_{\phi n} \sqrt{2}$ ;
- magnetic flux:  $\psi_{\sigma} = U_{\sigma} / \omega_{\sigma}$ ;
- electrical impedance:  $Z_{\sigma} = U_{\sigma} / I_{\sigma}$ ;
- the apparent power three-phase:  $S_{\sigma} = \frac{3}{2} U_{\sigma} I_{\sigma}$ ;
- the rotatory moment answering the active power:  $S_{\sigma} = P_{\sigma}$ , near the synchronic angular speed  $\omega_s$ :  $M_{\sigma} = P_{\sigma} / \omega_s$ ;
- the synchronic angular speed:  $\omega_{\sigma} = \omega_s$ ;
- time, in which, at synchronous angular speed  $\omega_s$ , a rotation angle equal to one radian is achieved:  $t_{\sigma} = 1/\omega_s$  in seconds. This unit is frequently referred to as synchronous second or radian.

For the rotor circuits:

- electromotive forces SEM ( $e$ ) are adopted as the base units; acting separately during the idle motion of the generator, they generate equal voltage on the stator terminals.

By bringing the above mentioned units into the equation system (2), it is possible to express them either in the variable flux system ( $\Psi$ ), or in a system, where the variables are currents and SEM forces ( $i, e$ ).

### Equation system in variable currents and SEM forces, after applying the relative units

Transient, dynamic processes will be legible the best if they are expressed in variable currents and SEM forces ( $i, e$ ). After appropriate transformations of fluxes  $\Psi_j, \Psi_d, \Psi_q, \Psi_{rq}, \Psi_{rd}$  and application of the relative units, the following formulas are generated (4):

$$\begin{aligned} \Psi_d &= e_{af} - x_d i_d + e_{rd}, \quad \Psi_q = x_q i_q + e_{rq} \\ \Psi_f &= e_{af} - \frac{x_{ad}^2}{x_f} i_d + \frac{x_{ad}}{x_f} e_{rd} \\ \Psi_{rd} &= e_{rd} - \frac{x_{ad}^2}{x_{rd}} i_d + \frac{x_{ad}}{x_{rd}} e_{af} \\ \Psi_{rq} &= e_{rq} + \frac{x_{ad}^2}{x_{rq}} i_q \end{aligned} \quad (4)$$

By bringing the formulas (4) into the equations (2), a system of differential equations in the relative units is produced for the synchronous machine with a damper, noted in variables for currents and SEM forces ( $i, e$ ) (5):

$$\begin{aligned}
& p(e_{af} - x_d i_d + e_{rd}) + (x_q i_q + e_{rq}) p \gamma - r i_d = u_d \\
& (e_{af} - x_d i_d + e_{rd}) p \gamma - p(x_q i_q + e_{rq}) - r i_q = u_q \\
& e_{af} + T_{d0} p \left( e_{af} - \frac{x_{ad}^2}{x_f} i_d + \frac{x_{ad}}{x_f} e_{rd} \right) = u_f \\
& e_{rd} + T_{rd} p \left( e_{rd} - \frac{x_{ad}^2}{x_{rd}} i_d + \frac{x_{ad}}{x_{rd}} e_{af} \right) = 0 \\
& e_{rq} + T_{rq} p \left( e_{rq} + \frac{x_{aq}^2}{x_{rq}} i_q \right) = 0 \\
& T_J p^2 \gamma + [(e_{af} - x_d i_d + e_{rd}) i_q + (x_q i_q + e_{rq}) i_d] = M_M
\end{aligned} \tag{5}$$

where:

$p = d/\omega_s dt$  – the symbol of differentiating in relation to the synchronic time;

$x_q, x_d, x_{aq}, x_{ad}, x_f, x_{rq}, x_{rd}$  – the inductive passive resistance of the individual windings;

$T_{d0} = x_f / r_f$  – the constancy time of the circuit excitation near the windings of coils stator (the idle run of the machine);

$T_{rq} = x_{rq} / r_{rq}$ ;  $T_{rd} = x_{rd} / r_{rd}$  – the constancy time of the component of the damper windings, counted in radians;

$\gamma$  – angle between the axis of phase  $A$  of the stator and axis  $d$  of the co-ordinate of the rotor.

The applied indexes denominate respective components along the longitudinal and the transverse axes in the configuration of rectangular coordinates ( $q, d$ ), respectively for the stator, the rotor, the excitation winding and the damper.

The generated set of final equations (5) describes fully the transient electromagnetic processes occurring in the synchronous machine.

## Conclusions

The equation system, in the form presented by Gorev, can be generated by introducing also the damping decrements of the damper windings, coefficients of magnetic coupling and the rotor slip into the equations (5). Nevertheless, supplementation of the equations (5) with equations of the main engine dynamics will disclose the effect of the shaft-generator work on the transient processes in the ship's system to a satisfactory degree.

The Park-Gorev equations are presented in the literature in different forms. As stated above, it is an effect of adopting different systems of relative units and different configurations of rotating coordinates, rigidly linked to the rotor ( $d, q$ ) or ( $q, d$ ). As a result, differences in the final form of the equations appear. Therefore, it seems to be recommendable to discuss rather the methodology of the

Park-Gorev description than the Park-Gorev equations.

On the other hand, regardless the subtleties presented above, Park-Gorev equations are commonly believed to be the most precise mathematical model of dynamics of electromagnetic processes in alternated current machines.

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