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SELECTED MODELS FOR THE DESCRIPTION OF THE KINEMATICS OF CHANGES OF HEIGHT DIFFERENCES BETWEEN POINTS IN A GEODESIC NETWORK UNDER THE INFLUENCE OF MINING

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The article attempts to describe the progress of deformation of the surface of the Legnica-Głogów Copper Mining Area in the years 1967-2008. The state of deformation has been described with kinematical models of the displacement of points representing the area under research. It has been analysed whether there are possibilities of using a counter-propagation algorithm for estimating displacements of selected points for which an assumption has been made that during the research they were damaged or destroyed. The numerical procedures of the estimation of parameters of displacement models were carried out by means of traditional optimisation methods and neural networks.

Keywords: neural networks, reference system, model of vertical displacements.

1. INTRODUCTION

Geodesic measurements of deformations and displacements of objects make it possible to precisely represent their geometrical condition in real spatial dimensions and to predict changes of that condition in time. Results of geodesic measurements are particularly important for research on the influence of mining on an orogenic belt and the surface of terrain. Geodesic controlling consists in determining the dynamism of the phenomenon of the displacement of controlled points which are stabilised in the research area, where there are processes caused by a change in soil-water conditions or the displacement of land masses, which happens in the case of areas and objects damaged by mining.

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Apart from adequate measurement equipment geodesic controlling also requires the use of correct methods of processing results of experiment data in order to correctly estimate displacements. Estimating possible displacements consists in selecting points with important displacements and points maintaining stability during measurements [8]. Defining a reference system determined at points which satisfy the criterion of stability encounters difficulties in the case of the occurrence of deformations over a large area because of the influence of unavoidable measurements errors on results of measurements of displacements of points remaining at considerable distances from one another.

2. KINEMATICAL DISPLACEMENT MODELS

The process of deformation of the surface of terrain in the Legnica – Głogów Copper Mining Are has been presented by means of functional models of kinematical networks, expressed in a general form as

$$\mathbf{H}(\mathbf{x},t)=\mathbf{L}(t) \tag{1}$$

where:

$\mathbf{H}(\mathbf{x},t)$ - component of a vector function expressing changes in height differences between the points (i,j) at the moment t,

$\mathbf{x}=[\mathbf{a}_i]\in R^n$ - vector of parameters,

$\mathbf{L}(t)=[\mathbf{l},t]\in R^m$ - observation vector,

t - real variable (time),

R^n - space of parameters,

R^m - space of measurements $(m>n)$.

Replacing a discrete set of observations of changes of height differences in a scalar form with time functions, we will analyse displacements obtained from two non-linear tendencies including time in the form [6]:

$$\mathbf{H}_1(\mathbf{x},t)=\alpha_1+\alpha_2 t+\alpha_3 t^2 \tag{2}$$

$$\mathbf{H}_2(\mathbf{x},t)=\alpha_1+\alpha_2\exp(-\alpha_3 t) \tag{3}$$

The first of the abovementioned models is a linear model, and the second, which has an exponential form, is a non-linear model. At this point it should be added that during the research topological properties of a geodesic network (defects, redundancy) together with the elimination of aberrant observations by means of a Huber function were included in each of the abovementioned models being a basis for synthetic characteristics of the displacements observed.

A basic problem in the process of determining displacements and analysing deformations is the problem of estimating a set of reference points which remain stable in terms of a random influence of measurement errors in the time interval of the research. Bearing in mind an empirical evaluation of such points, a reference system is an insignificantly flexible system, which in this paper was defined in two stages, namely [2]:

- stage I – on the basis of the minimisation of the sum of component modules of a vector of displacements obtained from equalisation with minimum restrictions on rates of freedom,
- stage II – on the basis of a criterion of an increment of the square of the norm of the vector of corrections to the observations.

Most optimisation tasks are non-linear tasks, the solution of which for research purposes requires iterative methods in almost all the cases. A numerical solution to the task of a kinematical network consists in identifying parameters of a kinematical model of geometrical components of the network which have been observed. Approximation procedures used in this case include a requirement for a minimum sum of the squares of corrections to the observations, resulting from the minimisation of the objective function

$$\sum_{i \in n} \mathbf{v}^2 = E(\mathbf{x}) = \sum_{(i,j) \in n} \left[A^{ij}\left(\alpha^i, \alpha^j, t\right) - \Delta h^{ij}(t) \right]^2 \tag{4}$$

where:

$A^{ij}\left(\alpha^i, \alpha^j, t\right) = \Delta h^j\left(\alpha^j, t\right) - \Delta h^i\left(\alpha^i, t\right)$ - component of a vector of non-linear functions which expresses a change in height differences between the points (i, j) in the time t,

n - set of points (i, j), for which the height differences Δh^{ij} were measured in the time t,

$\sum \mathbf{v}^2$ - square of the norm of the vector of corrections.

For a discretionary set of observations, the solution to the task of identifying, by means of approximation, parameters of the kinematical model (2) of changes in height differences observed is defined by a set of observation equations written in the form of vectors as

$$\mathbf{F}[\mathbf{x}(\boldsymbol{\alpha}, t)] = \Delta\mathbf{h}(\boldsymbol{\alpha}, t) + \mathbf{v}(\boldsymbol{\alpha}, t) \tag{5}$$

Because of a linear character of the task of multinomial approximation of parameters with the use of the method of the least squares, a linearized system of equations of corrections assumes the form

$$\mathbf{A}\Delta\mathbf{x} - \Delta\mathbf{h} = \mathbf{v} \tag{6}$$

where:

$$\mathbf{A} = \frac{\partial F\left(\mathbf{x}_o\left(\boldsymbol{\alpha}, t\right)\right)}{\partial \mathbf{x}_o\left(\boldsymbol{\alpha}, t\right)} \tag{7}$$

$\mathbf{x}_o$ - approximate value of the coordinates of the vector of parameters.

From the solution of the system of normal equations

$$\mathbf{A}^T \mathbf{A}\Delta\mathbf{x} = \mathbf{A}^T \Delta\mathbf{h} \tag{8}$$

we will obtain:

- estimator $\bar{\mathbf{x}}$ of the vector of parameters $\mathbf{x}$ with accuracy characteristics,
- coordinates of the vector of corrections $\mathbf{v}$,
- scaling parameter $m_0 = \sqrt{\frac{\sum \mathbf{v}^2}{m-n}}$,
- $m-n$ - number of rates of freedom of the observation.

A task of identifying parameters of kinematical models can be solved by means of neural networks, which in a number of applications play the part of a general approximator of functions of several variables. In the process of estimating parameters of the models in question an optimising neural network of a circumferential structure was used (fig. 1), which working as a one direction and recurrent network, solved the system of linear equations (6) [1, 7].

The solution of the system (6) by means of a gradient optimisation method, with the imposition of a condition of the least squares on the vector of corrections, leads to the solution of the system of differential equations [3]

$$\frac{d\mathbf{x}}{dt} = -\eta\nabla E(\mathbf{x}) = -\eta\mathbf{A}^T\left(\mathbf{A}\mathbf{x} - \Delta\mathbf{h}\right) \tag{9}$$

where:

η - learning ratio of the neural network assuming values from the range $\eta \in (0,1)$,

$\nabla E(\mathbf{x})$ - vector of the objective function (energy function) (4).

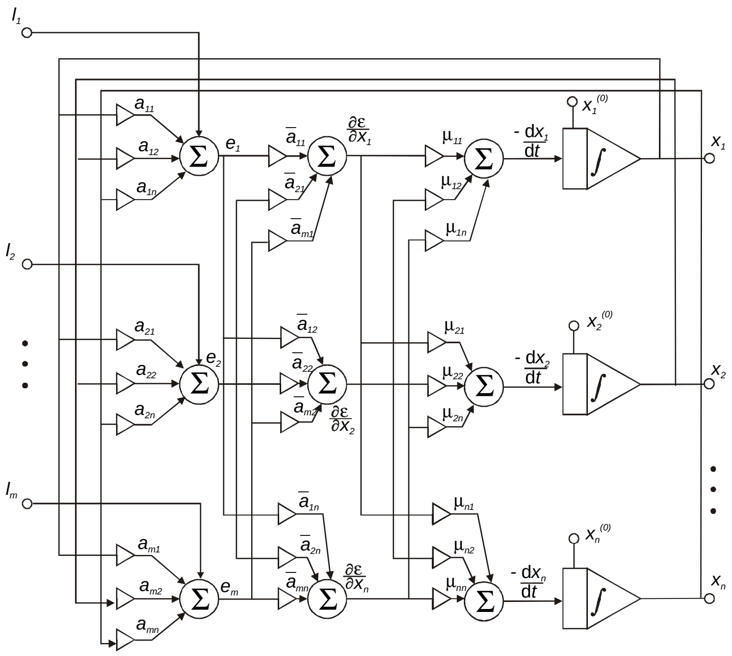

Fig. 1. Optimising a neural network of a circumferential structure [7]

The model of a kinematical network in an exponential form (3) is an implicit function which contains the transcendental function $y = e^{x}$. The solution of transcendental equations can be achieved by means of numerical iterative methods, because their solution can not be expressed by means of infinitesimal functions. In order to determine the estimators $\breve{\alpha}_i$ and the parameters α_i $(i = 1,2,3)$ we will use a mean square solution, which will be obtained from the minimisation of the objective function (4). The numerical realisation of the process of estimation of parameters of the model was carried out sequentially by means of division into a linear and a non-linear model. The algorithm consists of two subsequent iterative algorithms [2]:

- estimation of the parameters $\alpha_1, \alpha_2 \in R^{2n}$ of the linear model (for $\alpha_3 = const$, i.e. $\breve{\alpha}_1, \breve{\alpha}_2 = \alpha_1, \alpha_2(\alpha_3)$),
- estimation of the parameters $\alpha_3 \in R^{n}$ of the non-linear model (for $\breve{\alpha}_1, \breve{\alpha}_2 = const$, i.e. $E(\alpha_1, \alpha_2(\alpha_3), \alpha_3)$).

This course of action reduces the scope of the minimisation and increases the effectiveness and reliability of the determination of a real minimum of the objective function.

In the first case for $\alpha_3 = const$, from a formal point of view, the values of parameters estimated α_1, α_2 can be obtained by means one of methods of linear algebra, and the non-linear estimator $\breve{\alpha}_3$ will be obtained e.g. by means of the procedure of the greatest fall with the use of a polynomial spline function of the second degree, used for the approximation of an objective function in a specific section and the determination of a gradient vector.

3. COUNTER-PROPAGATION NEURAL NETWORK

Among a number of types of neural networks, self-organising networks play an important role, the advantage of which is the ability to classify an input vector disturbed with noise and other disturbances. A self-organising neural network, which shows the ability to represent the function $y_i = f(x_i) + \varepsilon$ z R^n as R^m (ε - the influence of the distortion of the coordinates of the vector $\mathbf{x}$), is a counter-propagation network described by Hecht – Nielsen [4, 5] (fig. 2). Algorithms using this network have good abilities to learn a representation of the vector-vector type. The network has the ability of hetero and auto association (approximation in both directions), and as a classifier, it gives a favourable answer even if the input vector is not complete.

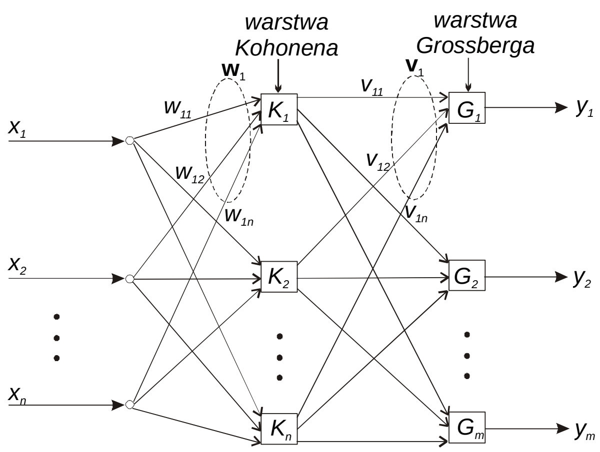

Fig. 2. Simplified structure of a Hecht – Nielsen neural network

It results from fig. 2 that the network consists of two layers: a Kohonen layer and a Grossberg layer, with a different number of neurons in particular layers. The operation of a Hecht – Nielsen network can be divided into two modes:

- the replication mode, in which we apply the vector $\mathbf{x}$ at the input, and at the output we obtain the vector $\mathbf{y}$,

- the learning mode, in which the vector $\mathbf{x}$ is accompanied by the assigned values $\mathbf{d}$ of the vector $\mathbf{y}$, and values of the vectors of weights $\mathbf{w}$ and $\mathbf{v}$, which will make it possible to represent the vector $\mathbf{x}$ as the vector $\mathbf{d}$, are searched for.

The operation of the Kohonen layer takes place in the learning mode without supervision and requires an initial normalisation of the input vector $\mathbf{x}$ according to the dependence

$$x_i = \frac{x_i}{\sqrt{\sum_{j=1}^{n} x_j^2}} \tag{10}$$

Neurons in this layer generate the weighed sum of signals

$$net_i = \sum_{j=1}^{n} w_{ij} x_j \tag{11}$$

and achieve a particular rate of activation. A neuron with the highest net_i value, which is assigned a state equal 1, wins the competition, and the state of the other vectors equals 0. In consequence of the competition won a particular neuron updates its weights according to the relation

$$w_{ij}(t+1) = w_{ij}(t) + \eta\left[x_{ij} - w_{ij}(t)\right] \tag{12}$$

where t and $t+1$ denote subsequent learning cycles, and η is a learning ratio from the range $(0,1)$.

At the second stage of the operation of the Hecht – Nielsen network, the Grossberg layer is trained in a supervised learning mode. For each pair $(x_1, y_1), \ldots, (x_m, y_m)$ only one neuron from the Kohonen layer is active, and weights coming from it are updated according to the rule

$$v_{ik}(t+1) = v_{ik}(t) + \mu\left[d_i - v_{ik}(t)\right] \tag{13}$$

where μ is a learning ratio, usually a very small one, which still decreases as learning progresses. As a result of the operation of the algorithm the values of weights v_{ik} change in the direction of the answer $y_i = v_{ik}$, corresponding to the value assigned d_i. In this case the network works as an approximator of the function $\tilde{\mathbf{y}} = f(\mathbf{x})$, which is a copy of the relation $\mathbf{y} = f(\mathbf{x})$. The mean error of the approximation for a single vector, with the assumption of an even distribution of probability of the components of the vector $\tilde{\mathbf{y}}$, is [3]

$$m = 2 - 2\sum_{i=1}^{m} v_i \tilde{y}_i \tag{14}$$

In the replication mode the network, which belongs to the class of auto-associating networks, plays the role of a function of the reciprocal association of vectors. This is so, because when we put the coordinates of the input vector x_i in the place of the values assigned d_i, we will obtain a vector $\mathbf{x}$ in the form of a vector of weights $\mathbf{v}$. If the vector $\mathbf{x}$ is an incomplete vector, then the algorithm accepts this state of affairs and generates a complete answer as a representation of a particular interpolation function. This characteristic of the algorithm was used by the authoress to determine in the network under research the values of displacements of points which were assumed to have been destroyed during the measurements.

4. NUMERICAL EXAMPLE

In order to determine displacements of measurement points located in the Legnica – Głogów Copper Mining Area results of five measurement campaigns from the years 1967-2008 were analysed. The choice of campaigns resulted from the possibility of the widest representation in terms of observations carried out. In the area under discussion of about 75000 ha 118 measurement points were located, connected with one another with 125 observations. A diagram of a measurement control network has been presented in fig.3.

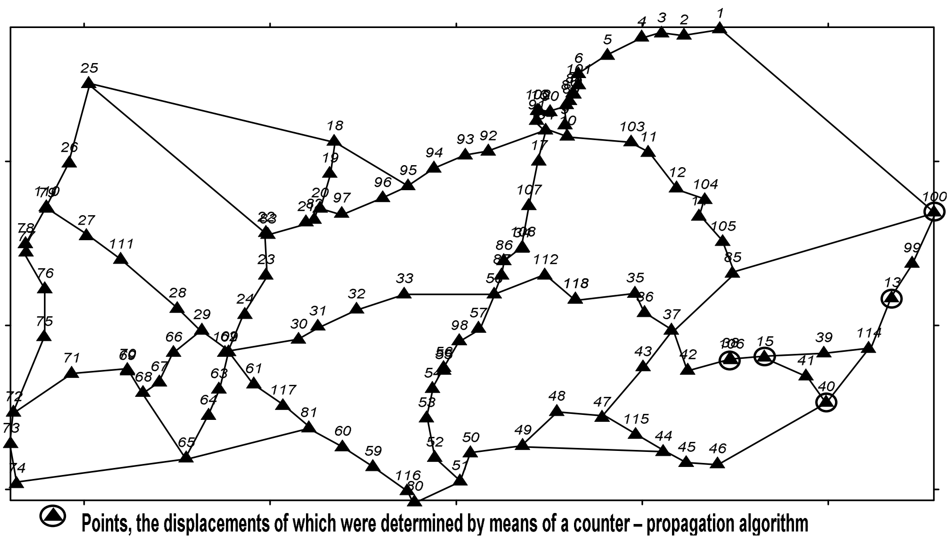

Fig. 3. Diagram of a measurement control network

As a result of the calculations in a linear and non-linear aspect values of vertical displacements of measurement points were obtained for a pre-defined reference system. The estimation of parameters of each of the two models under analysis was carried out by means of two optimisation algorithms. In order to solve the task of identification of parameters of the kinematical models (2) and (3) by means of approximation a neural network was used (fig.1), and the procedure of the least squares was used as a second method of estimating parameters of the model (2). Parameters of the model (3) were estimated by means of a hybrid method with a division into a linear and a non-linear model with the use of the method of the greatest fall.

Characteristics of the accuracy of the abovementioned methods of representation of functional models in the form of a mean approximation error are as follows:

1. linear model (2):
   - neural network $m_0 = 1{,}4$ mm
   - method of the least squares $m_0 = 1{,}4$ mm
2. non-linear model (3):
   - neural network $m_0 = 2{,}0$ mm
   - hybrid method $m_0 = 1{,}9$ mm

In time intervals of the research the displacement values determined are between +84,5 mm to -3851,3 mm, and the maximum speed of displacements is on the level of 88 mm/year. The displacements with the line of steady fall and speed including the range of maximum values have been illustrated respectively in fig.4 and 5.

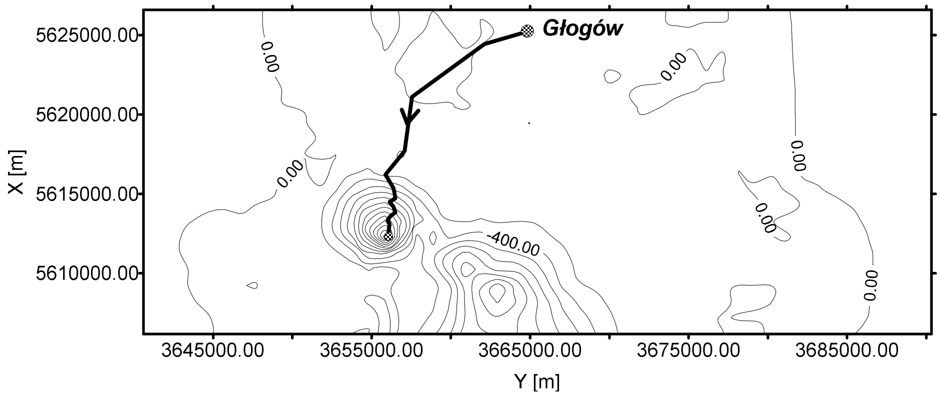

Fig. 4. Kinematical displacement model obtained for the period of time 1967 – 2008 with the line of the greatest fall

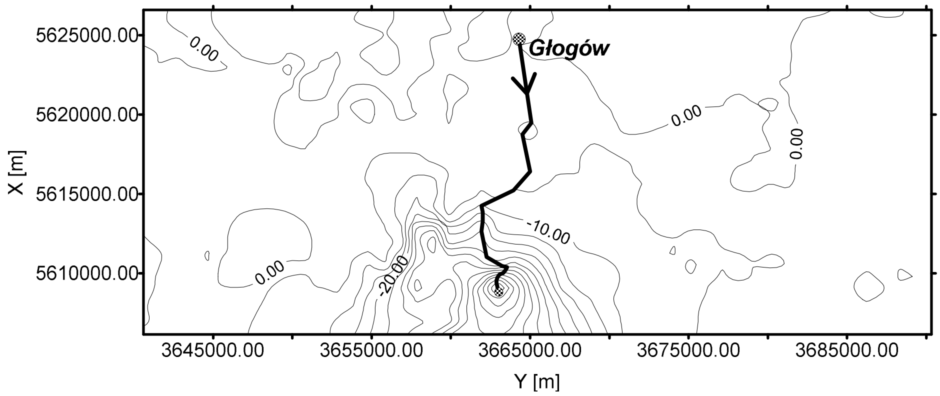

Fig. 5. Speed of displacements of controlled points in mm/year with the line of the greatest displacement speeds

In the case of an incomplete data set (damage or destruction of measurement points), a Hecht – Nielsen neural network based on counter-propagation can be used to generate a relatively correct vector of displacements. A result of the use of a counter-propagation algorithm has been presented for one of the measurement sequences of 12 points, with the assumption that as a result of damage or destruction 5 points were unavailable for measurement in the years 2004 – 2008. The mean error of the approximation calculated from the formula (14), for the measurement period 1967 – 2004 was 0,37, and for the measurement period 1967 – 2008 - 0,74. Particular values of displacements obtained from the measurement for the model (2) and reproduced by means of the counter-propagation algorithm have been presented in table 1, and represented graphically in fig.6 and fig. 7.

Table 1. Values of displacements obtained by means of the counter-propagation algorithm

Point number	Displacements obtained from measurements in particular measurement periods [mm]				Displacements reproduced [mm]	
	1967-1998	1967-2000	1967-2004	1967-2008	1967-2004	1967-2008
13	-0,87	-1,45	-1,76	-2,34	-1,61	-2,28
15	-10,14	-12,08	-14,24	-16,08	-14,04	-16,22
40	-7,24	-8,94	-12,08	-15,26	-11,48	-13,57
86	-9,78	-11,84	-14,23	-16,17	-14,29	-16,08
100	-0,95	-1,15	-1,12	-1,10	-1,34	-1,46

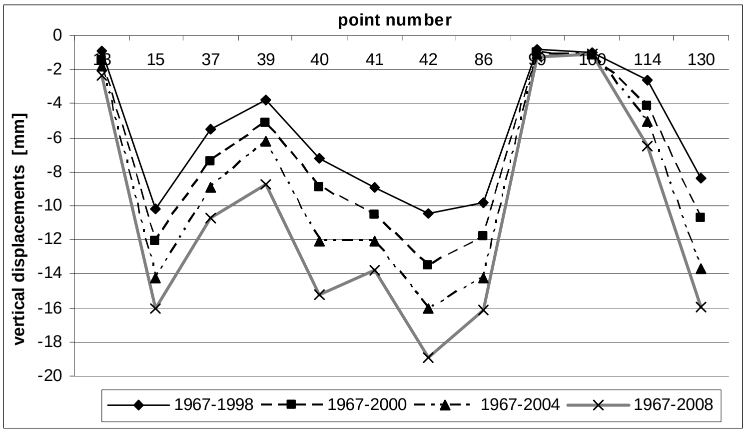

Fig. 6. Vertical displacements of measurement [mm]

While comparing the values of displacements obtained from the measurement with the displacements reproduced by means of the Hecht – Nielsen network, it is possible to notice that the smallest differences can be found in the case of points 13 and 86, for which values of the speed of displacements within the whole measurement period are small. The greatest difference - 1,69 mm occurs at point No. 40. This situation is most likely caused by a considerable speed of settlement of this point in the years 2004 and 2008, in comparison to the speed of displacements obtained in the previous years (cf. fig. 6).

The values of vertical displacements reproduced for selected controlled points by means of the method of counter – propagation have been presented graphically in fig. 7.

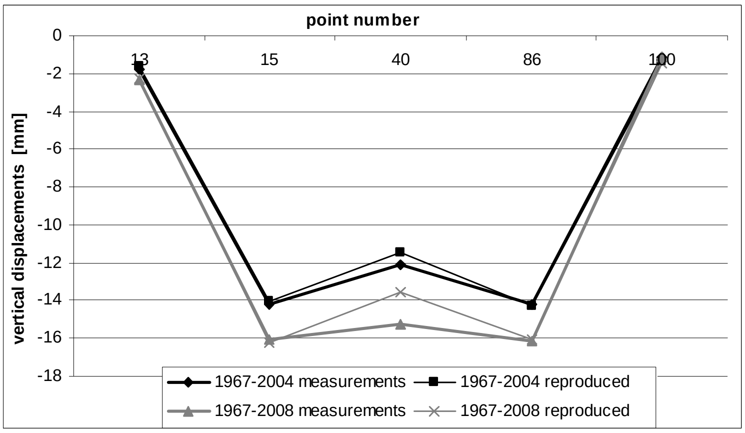

Fig. 7. Values of displacements of points No. 13, 15, 40, 86, 100 for the periods of time 1967 – 2004 and 1967 – 2008 reproduced by means of counter propagation

Prediction of values of displacements by one step for the measurement period 1967 – 2010 has been presented graphically in fig.8. It should be noticed that values of displacements determined on the basis of an exponential curve increased at the most by 92 mm for point *P* marked in fig. 8.

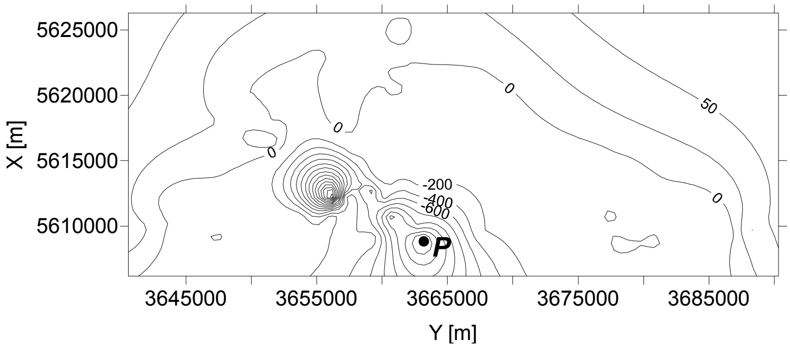

Fig 8. Prediction of displacements for the area under discussion in the year 2010

5. CONCLUSIONS

Values of vertical displacements of measurement points located in an area influenced by mining presented in the paper make it possible to say that displacements obtained by means of kinematical models, the parameters of which were estimated by means of a neural network do not differ in terms of quality from displacements obtained by means of traditional methods of optimisation. By means of a counter – propagation neural network it is possible to reproduce, with a particular level of approximation, displacements of points which were damaged or destroyed during the measurements. However, it is necessary to notice that the speed of changes occurring influences the accuracy of displacements reproduced. If the speed is steady during the whole time of measurements, then the accuracy of the reproduction of displacements is much higher than in the case of rapid accelerations of the settlement process. Research into vertical displacements of the surface of terrain caused by mining, together with exogenous factors, justifies the adoption of non-linear models of movement, because the cause and effect relationships between reactions of variables occurring in nature are non linear relationships.

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WYBRANE MODELE OPISU KINEMATYKI SIECI GEODEZYJNEJ WYSOKOŚCIOWEJ POD WPŁYWEM EKSPLOATACJI GÓRNICZEJ

S t r e s z c z e n i e

W treści artykułu podjęto próbę opisu przebiegu deformacji powierzchni terenu obszaru Legnicko – Głogowskiego Okręgu Miedziowego w latach 1967 – 2008. Stan deformacji został opisany modelami kinematycznymi przemieszczeń punktów reprezentujących badany obszar. Przeprowadzono rozważania dotyczące możliwości wykorzystania algorytmu kontrpropagacji do oszacowania przemieszczeń wybranych punktów, dla których przyjęto założenie, że w trakcie prowadzonych badań punkty zostały uszkodzone bądź zniszczone. Procedury numeryczne estymacji parametrów modeli przemieszczeń realizowano za pomocą tradycyjnych metod optymalizacji i sieci neuronowych.