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APPLICATION OF THE PARIS FORMULA WITH M=2 AND THE VARIABLE LOAD SPECTRUM TO A SIMPLIFIED METHOD FOR EVALUATION OF RELIABILITY AND FATIGUE LIFE DEMONSTRATED BY AIRCRAFT COMPONENTS

UPROSZCZONA METODASZACOWANIA NIEZAWODNOŚCI I TRWAŁOŚCI ZMĘCZENIOWEJ ELEMENTÓW KONSTRUKCJI STATKU POWIETRZNEGO Z WYKORZYSTANIEM WZORU PARISA DLA M=2 I ZMIENNEGO WIDMA OBCIĄŻENIA*

The presented paper is the follow-up to the study, where the method for assessment of the fatigue life of a structural component was outlined with consideration of the variable spectrum of loads and with use of the Paris formula for m ≠ 2. Due to the different nature inherent to analytic forms of solutions for the Paris equations with their exponential parameter m = 2, that special case is the subject of a separate analysis. This paper also uses the transformation of a real spectrum with variable values of fatigue cycles into a homogenous spectrum with weighted cycles. The method was developed that uses the transformed spectrum to evaluate fatigue life for a selected component of the aircraft structure when the component suffers from an initial crack. The method for modeling of the crack length expansion uses a differential equation that is then subjected to transformations to obtain a partial differential equation of the Fokker-Planck type, which has a particular solution, explicitly the length density function for the crack of the component in question. That length density function served subsequently to determine reliability and fatigue life of a structural component where the crack length expanded from the permissible value l_d *to the critical threshold* l_{kr}.

Keywords: *fatigue of structures, reliability, fatigue life, random spectrum of loads.*

Prezentowany artykuł jest uzupełnieniem pracy, w której przedstawiono metodę oceny trwałości zmęczeniowej elementu konstrukcji dla zmiennego widma obciążenia z wykorzystaniem wzoru Parisa dla m≠2. Ze względu na odmienność postaci analitycznych rozwiązań dla wykładnika równania Parisa m=2, ten szczególny przypadek rozwiązań został przedstawiony w niniejszym opracowaniu. Pokazany został sposób przekształcenia widma rzeczywistego o zmiennych wartościach cykli w widmo jednorodne o cyklach ważonych. Wykorzystując widmo przekształcone opracowano metodę oceny trwałości zmęczeniowej wybranego elementu konstrukcji statku powietrznego z początkowym pęknięciem. Do modelowania przyrostu długości pęknięcia wykorzystano równanie różnicowe, z którego po przekształceniu otrzymano równanie różniczkowe cząstkowe typu Fokkera-Plancka. Rozwiązaniem szczególnym tego równania jest funkcja gęstości długości pęknięcia elementu. Wykorzystując następnie funkcję gęstości długości pęknięcia określono niezawodność i trwałość zmęczeniową elementu konstrukcji dla pęknięcia narastającego do wartości dopuszczalnej l_d *mniejszej od wartości krytycznej* l_{kr}.

Słowa kluczowe: *zmęczenie konstrukcji, niezawodność, trwałość zmęczeniowa, losowe widmo obciążenia.*

1. Introduction

Assessment of the fatigue life for components that are ‘operated’ under variable load spectrum is really troublesome to formulate analytical relationships. Thus, it is a main subject ofworld-wide scientific research [1–6, 16, 18]. However, the assessment of the fatigue life for components that are ‘operated’ under variable load spectrum is crucial to manage flight safety forcivilian and military aircraft. Therefore, there is a necessity to find simplified methods, that could be practically applied in aviation transport [7, 9, 13–15, 18]. In this paper the simplified method is used that has already been disclosed in [17]. The applied simplification consists in transformation of the variable spectrum of loads to a homogenous one with weighted cycles.

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

This paper is the follow-up to the previous study [17] where the simplified method for assessment of the fatigue life of an aircraft structural component was outlined with consideration for the variable spectrum of loads and with use of the Paris formula for m ≠ 2. The forms of analytic solutions for this problem differ from each other depending on the exponent parameter for the Paris formula, i.e. whether m ≠ 2or m = 2. It is why this study is dedicated to the case when the exponent in the Paris formula m = 2.

It is assumed that the length of the initial crack within a structural component is l_0 and then the crack expands due to the effect of a load with a variable spectrum up to the length of that is still permissible and safe as being less than the critical length of l_{kr}. It is also assumed that the expansion rate of the crack is subject to a deterministic rule defined by the Paris equation [8]:

$$\frac{dl}{dN} = C\left(\Delta K\right)^m, \tag{1}$$

where:

ΔK – variation range for the coefficient of stress intensity factor,

C, m – material-dependent constants,

N – the variable the represents the number of load cycles of a structural component.

For the case in question, i.e. when m = 2, the formula(1) adopts the following form:

$$\frac{dl}{dN} = C\left(\Delta K\right)^2. \tag{2}$$

2. Determination of the crack expansion rate for m = 2 and for transformed spectrum of loads applied to a structural component

Transformation of a real load spectrum with variable load values into a homogenous spectrum with weighted cycles is based on the following assumptions:

1) Each component of an aircraft is operated under variable loads during the aircraft missions;
2) The spectrum of loads affecting the aircraft component during a standard mission is available. The load is a multiplication of a standard cycle;
3) It is assumed that the available standard load makes it possible to calculate:
   - the total number of load cycles during a single flight,
   - the spectrum comprises L thresholds with the maximum values of stresses $\sigma_1^{\max}, \sigma_2^{\max}, \ldots, \sigma_L^{\max}$;
4) For the analyzed spectrum the repetition numbers of the maximum stress threshold is the following: σ_1^{max} occurs n_1 times, σ_2^{max} occurs times, …, σ_L^{max} occurs n_L times; Therefore, for the entire flight the repetition numbers of the predefines stress threshold amounts to $N_c = \sum_{i=1}^{L} n_i$;
5) The minimum values for the predefined stress thresholds is calculated with the use of the following formula:

$$\sigma_{i,\acute{s}r}^{\min} = \frac{\sigma_{i,1}^{\min} + \sigma_{i,2}^{\min} + \ldots + \sigma_{i,n_i}^{\min}}{n_i}, \text{ where } i = 1,2,\ldots,L;$$

6) The values of maximum σ_i^{max} and minimum $\sigma_{i,\acute{s}r}^{min}$ stress values for operation cycles with frequencies P_i of their occurrences are summarized in Table 1;

Table 1. Maximum σ_i^{max} and minimum $\sigma_{i,\acute{s}r}^{min}$ stress values for operation cycles with frequencies P_i of their occurrences

σ_i^{max}	σ_1^{max}	σ_2^{max}	…	σ_i^{max}	…	σ_L^{max}
$\sigma_{i,\acute{s}r}^{min}$	$\sigma_{1,\acute{s}r}^{min}$	$\sigma_{2,\acute{s}r}^{min}$	…	$\sigma_{i,\acute{s}r}^{min}$	…	$\sigma_{L,\acute{s}r}^{min}$
P_i	$P_1 = \frac{n_1}{N_c}$	$P_2 = \frac{n_2}{N_c}$	…	$P_i = \frac{n_i}{N_c}$	…	$P_L = \frac{n_L}{N_c}$

7) The asymmetry coefficients for operation cycles are summarized in Table 2.

Table 2. Asymmetry coefficients $\hat{R}_i$ for operation cycles with U_i factors that take into account the impact of these coefficients on the cracking rates

i^{th} cycle	1	2	…	i	…	L
$\hat{R}_i$	$\hat{R}_1$	$\hat{R}_2$	…	$\hat{R}_i$		$\hat{R}_L$
U_i	U_1	U_1	…	U_i	…	U_L

where:

$\hat{R}_i = \frac{\sigma_{i,\acute{s}r}^{\min}}{\sigma_i^{\max}}$, $U_i = \propto_1 + \propto_2 \hat{R}_i + \propto_3 \hat{R}_i^2$, $\propto_1, \propto_2, \propto_3$ – empirical coefficients [11, 12].

8) Ranges for stress variations are calculated by the formula:

$$\Delta\sigma_i = \sigma_i^{\max} - \sigma_{i,\acute{s}r}^{\min}$$

and summarized in Table 3.

Table 3. Ranges for stress variations $\Delta\sigma_i$ with frequencies P_i of their concurrencies

cycle types	1	2	…	i	…	L
$\Delta\sigma_i$	$\Delta\sigma_1$	$\Delta\sigma_2$	…	$\Delta\sigma_i$		$\Delta\sigma_L$
P_i	P_1	P_2	…	P_i	…	P_L

9) Considering the effect exercised by overload cycles onto expansion of cracks (Table 4)

$$\Delta\sigma_{i,ef} = C_i^P \Delta\sigma_i$$

where:

C_i^P– coefficients that represents retardation of the crack expansion after occurrence of overload cycles [10].

Table 4. Variation ranges $\Delta\sigma_{i,ef}$ for effective stress with consideration of overload cycles

cycle types	1	2	…	i	…	L
coefficients	C_1^P	C_2^P	…	C_i^P		C_L^P
$\Delta\sigma_{i,ef}$	$\Delta\sigma_{1,ef}$	$\Delta\sigma_{2,ef}$	…	$\Delta\sigma_{i,ef}$	…	$\Delta\sigma_{L,ef}$

For the foregoing assumptions the relationship (1) with regard to the rate of crack development assumes the following form:

$$\frac{dl}{dN} = C\pi^{\frac{m}{2}} \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^m \right) M_k^m l^{\frac{m}{2}}, \tag{3}$$

where:

M_k – the parameter that indicates the location of the crack within the structural component and its dimensions with respect to dimensions of the overall component [8].

Having considered all the possible load cycles the relationship (3) adopts the form:

$$\frac{dl}{dN} = C\pi M_k^2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) l, \tag{4}$$

Where: $i = 1,2,\dots,L$

The relationship (4) can be expressed as a function of time or, more precisely, the function of an aircraft flying time. For this purpose it is assumed that:

$$N = \lambda t, \tag{5}$$

where:

λ – intensity (frequency) of occurrence of load cycles in a structural component;

N – number of load cycles;

t – overall flying time of an aircraft.

For the case in question $\lambda = 1/\Delta t$, where Δt stands for duration of the fatigue cycle for the specific component. The easiest way to determine the Δt parameter is the use the following equation:

$$\Delta t = \frac{T}{N_c}, \tag{6}$$

where:

T – average duration of a standard flight of an aircraft and assumed for determination of a load cycle,

N_c – number of load cycles within a standard load spectrum.

After the foregoing substitutions and transformations the formula (4) adopts the following form:

$$\frac{dl}{dt} = \lambda C\pi M_k^2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) l. \tag{7}$$

The formula (7) makes it possible to calculate the rate of crack expansions for the homogenous spectrum with weighted cycles of a single type.

3. Determination of the density function for a crack length as a function of time (flying time)

Let $U_{l,t}$ stand for the probability that the crack length of a component is l for the overall flying time t of an aircraft. The difference equation for the foregoing assumptions adopts the following form [7, 18]:

$$U_{l,t+\Delta t} = (1 - \lambda\Delta t) U_{l,t} + \lambda\Delta t U_{l-\Delta l,t}, \tag{8}$$

where:

Δl – expansion of the crack length during a single equivalent cycle.

The value of the crack length expansion, calculated on the basis of the equation (7) amounts to:

$$\Delta l = \lambda C\pi M_k^2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) l\Delta t. \tag{9}$$

The equation (8) can be rewritten in the functional form:

$$U(l, t+\Delta t) = (1 - \lambda\Delta t) U(l,t) + \lambda\Delta t U(l - \Delta l, t). \tag{10}$$

where:

$U(l, t)$ – the density function for the crack length after expiring of the t total flying time expressed in flying hours;

$(1-\lambda\Delta t)$ – probability that no equivalent load cycle occurs during the time interval with the length of Δt;

$\lambda\Delta t$ – probability that an equivalent load cycle occurs during the time interval with the length of Δt.

The equation (10) can be converted into a partial differential equation. For that purpose the following approximations are made:

$$\left. \begin{aligned} U(l, t+\Delta t) &\cong U(l,t) + \frac{\partial U(l,t)}{\partial t}\Delta t \\ U(l-\Delta l, t) &\cong U(l,t) - \frac{\partial U(l,t)}{\partial l}\Delta l + \frac{1}{2}\frac{\partial^2 U(l,t)}{\partial l^2}(\Delta l)^2 \end{aligned} \right\}. \tag{11}$$

After substitution of (11) for (10) the following formula is obtained:

$$\frac{\partial U(l,t)}{\partial t} = -\lambda \frac{\partial U(l,t)}{\partial l}\Delta l + \frac{1}{2}\lambda(\Delta l)^2 \frac{\partial^2 U(l,t)}{\partial l^2} \tag{12}$$

where:

$$\Delta l = \lambda C\pi \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) M_k^2 l\Delta t.$$

Since, $\lambda\Delta t = 1$, then:

$$\Delta l = C\pi \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) M_k^2 l. \tag{13}$$

Let:

$$C\pi M_k^2 = C_2, \tag{14}$$

$$\Delta l = C_2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) l. \tag{15}$$

Substitution of (15) for (12) leads to the following equation:

$$\frac{\partial U(l,t)}{\partial t} = -\lambda \frac{\partial U(l,t)}{\partial l} C_2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) l + \frac{1}{2}\lambda \left(C_2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) l\right)^2 \frac{\partial^2 U(l,t)}{\partial l^2}. \tag{16}$$

The solution of the equation (7) should be substituted for the crack length in the equation (16):

$$\frac{dl}{dt} = \lambda C_2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) l,$$

$$\int_{l_0}^{l} \frac{dx}{x} = \int_0^t C_2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) dt,$$

$$l = l_0 e^{\lambda C_2 \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^2 \right) t}. \tag{17}$$

Where, according to the formula (14),

$$C_2 = C\pi M_k^2 .$$

With consideration of (17), coefficients of the equation (16) can be expressed in the following way:

$$\alpha(t) = \lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right) l_0 e^{\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t} \tag{18}$$

$$\beta(t) = \lambda\left[C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right) l_0 e^{\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t}\right]^2 =$$

$$= \lambda C_2^2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)^2 l_0{}^2 e^{2\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t} . \tag{19}$$

For m=2, the equation (16) with coefficients in the form of the relationships (18) and (19) is as follows:

$$\frac{\partial U(l,t)}{\partial t} = -\alpha(t)\frac{\partial U(l,t)}{\partial l} + \frac{1}{2}\beta(t)\frac{\partial^2 U(l,t)}{\partial l^2} . \tag{20}$$

The particular solution for the equation (20) is as follows [7, 18]:

$$U(l,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(l-B(t))^2}{2A(t)}} \tag{21}$$

where:

$B(t)$ – the average increment in the crack length for the overall flying time t calculated as:

$$B(t) = \int_0^t \alpha(t)dt . \tag{22}$$

$A(t)$ – variance for the average increment in the crack length for the overall flying time t calculated as:

$$A(t) = \int_0^t \beta(t)dt . \tag{23}$$

Computation of the integral (22):

$$B(t) = \int_0^t \alpha(t)dt = \lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right) l_0 \int_0^t e^{\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t} dt =$$

$$= \lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right) l_0 \frac{1}{\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)} \cdot e^{\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t}\Bigg|_0^t =$$

$$= l_0\left(e^{\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t} - 1\right). \tag{24}$$

Computation of the integral (23):

$$A(t) = \int_0^t \beta(t)dt = \lambda C_2^2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)^2 l_0^2 \int_0^t e^{2\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t} dt =$$

$$= \frac{\lambda C_2^2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)^2 l_0^2}{2\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)} \cdot e^{2\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t}\Bigg|_0^t =$$

$$= \frac{1}{2} C_2 l_0^2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)\left(e^{2\lambda C_2\left(\sum_{i=1}^{L} P_i U_i\left(\Delta\sigma_{i,ef}\right)^2\right)t} - 1\right). \tag{25}$$

Where, according to the formula (14),

$$C_2 = C\pi M_k^2 .$$

4. Determination of reliability and fatigue life for a selected structural component of an aircraft

The diagram of a growing risk of a catastrophic hazard due to the crack of a structural component is shown in Fig. 1.

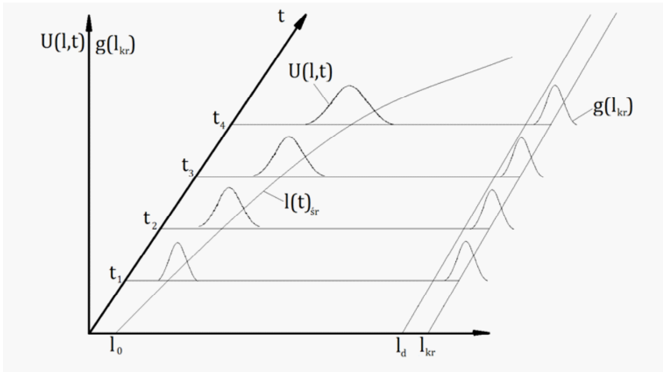

Fig. 1. The diagram of a growing risk of a catastrophic hazard due to the crack of a structural component [18]

The component is deemed damaged when the current length of a crack l exceeds the value of a critical threshold l_{kr} or is equal thereto. Thus

$$l - l_{kr} \geq 0.$$

Where both l and l_{kr} are exemplifications of random variables $\hat{L}_t$ and L_{kr}. Therefore,

$$\varkappa = \hat{L}_t - L_{kr} . \tag{26}$$

The function of a random variable density $\varkappa$ is calculated from the relationship:

$$f(\varkappa)_t = \int_0^\infty g(l-\varkappa)U(l,t)dl . \tag{27}$$

Therefore, the probability of the damage of a structural component is expressed by the relationship:

$$Q_t^{'} = P\{L_t - \hat{L}_{kr} \geq 0\} = \int_0^\infty f(\varkappa)_t d\varkappa . \tag{28}$$

Finally, the reliability of a component can be calculated by means of the function:

$$R(t) = 1 - \int_0^\infty f(\varkappa)_t d\varkappa . \tag{29}$$

Reliability of structural components can be also calculated in another manner. The critical length of cracks is to be determined by means of a stress intensity coefficient in the following form:

$$K = M_k \sigma\sqrt{\pi l} . \tag{30}$$

The coefficient that is determined by the relationship (30) becomes the critical parameter K_c when critical length l_{kr} and critical stress σ_{kr} are reached. This critical parameter is referred to as the cracking resistance of a material:

$$K_c = M_k \sigma_{kr} \sqrt{\pi l_{kr}} \,. \tag{31}$$

Hence, after a simple transformation:

$$l_{kr} = \frac{K_c^2}{M_k^2 \sigma_{kr}^2 \pi} \,.$$

By substitution of (31) and incorporation of a safety factor, one can calculate the maximum permissible (safe) length of a crack:

$$\overline{l}_d = \frac{K_c^2}{k M_k^2 \sigma_{kr}^2 \pi} \,, \tag{32}$$

where:

k – safety factor.

With consideration of the initial length l_0 of a crack one can calculate the maximum permissible increment of the crack length l_d with use of the following formula:

$$l_d = \overline{l}_d - l_0 \,. \tag{33}$$

Next, the formula (33) is used to find out the reliability of a structural component:

$$R(t)_{l_d} = \int_{-\infty}^{l_d} U(l,t)\,dl \tag{34}$$

Normalization of the integrand in the equation (34) leads to the following expression:

$$R(t)_{l_d} = \int_{-\infty}^{-\infty \frac{l_d - B(t)}{\sqrt{A(t)}}} U(z,t)\,dz \,, \tag{35}$$

where:

$$z = \frac{l - B(t)}{\sqrt{A(t)}} \,,$$

whilst $B(t)$, $A(t)$ are expressed by the relationships (24) and (25).

For the assumed reliability level, the upper limit for the integral (35) can be looked up in the tables for normal distribution. It enables to establish the relationship:

$$Q_{l_d} = \frac{l_d - B(t)}{\sqrt{A(t)}} \tag{36}$$

where:

Q_{ld} – the upper limit for the integral (35), for that limit the integral value is equal to $R(t)_{l_d}$.

Resolving of the equation (36) enables to calculate the value of the overall flying time (the desired lifetime of a structural component) that guarantees that the assumed reliability level is achieved.

5. Final remarks with a numerical example

To illustrate the newly developed method the following example shows the way to calculate expansion rates for the average length of a crack in a component made of steel with specific material properties and exposed to the effect of a real load spectrum. The calculations were carried out for the spectrum of loads with variable amplitudes after having the load spectrum transformed in the manner that is explained in Section 2. The original load spectrum corresponds to real load affecting the component [7]. The characteristic parameters of the transformed load used for further investigations as summarized in the Table 5 below. Table 5 shows boundary ranges $\Delta\sigma_i$ for stress variations in the cycle within the presumed load thresholds i together with the frequencies P_i of their occurrence as well as coefficients that take into account the impact of the cycle asymmetry on crack expansion.

For the defined model material the following values of coefficients related to materials were assumed for calculations:

$$m = 2,$$
$$C = 5 \cdot 10^{-9} \,.$$

The presented example assumes that the initial length of the crack within the component is l_0=10 mm, whilst the maximum permissible length of the crack was calculated with the use of the equation (32) and it equals to $\overline{l}_d = 25mm$. It was also assumed for calculations that the coefficient that reflects retardation of the crack expansion after occurrence of overload cycles C_I^{P}=1, whereas the coefficient that takes into account the impact of the cycle asymmetry on crack expansion is defined by the empirical formula U_i=0,55+0,33$\hat{R}_i$+0,12$\hat{R}_i^2$ Alteration of the M_k coefficient in pace with expansion of the crack has been considered in the process of numerical computations according to the formula:

Table 5. Characteristic parameters for the transformed spectrum of loads

Load threshold *i*	1	2	3	4	5	6	7
Number of cycles	1	5	4	10	30	50	140
σ_i^{max} [MPa]	186	159	141	129	112	93	72
$\sigma_{i,śr}^{min}$ [MPa]	–28	–13	8	17	23	27	27
$\hat{R}_i$ coefficient	–0,1505	–0,0818	0,0567	0,1317	0,2053	0,2903	0,375
Stress range $\Delta\sigma_{i,ef}$ [MPa]	214	172	133	112	89	66	45
U_i factor	0,5030	0,5238	0,5691	0,5955	0,6228	0,6559	0,6906
Share of the threshold in the spectrum (frequency of occurrence) P_i	0,0042	0,0208	0,0167	0,0417	0,125	0,2083	0,5833

$$M_k = 1 + 0{,}128\left(\frac{l}{b}\right) - 0{,}288\left(\frac{l}{b}\right)^2 + 1{,}525\left(\frac{l}{b}\right)^3, \qquad (37)$$

where:

l – current length of the crack;

b – width of the component towards the direction where the crack expands.

Then, the transformed equation (24) that expresses the average crack length was used to establish, based on the equation (5), the relationship between that crack length and the number of the load cycles N:

$$B(N) = l_0 \cdot (e^{C\cdot\pi\cdot M_k^2\cdot\left(\sum_{i=1}^{L} P_i\cdot U_i\cdot\left(\Delta\sigma_{i,ef}\right)^2\right)\cdot N} - 1). \qquad (38)$$

The foregoing equation made it possible to calculate an increase in the average crack length from the initial value of $l_0 = 10$ mm to the maximum permissible limit $\overline{l_d} = 25mm$, where the relationship was sought between the crack length and the N number load cycles. The variation of the average crack length as the function of load cycle numbers is shown in Fig. 2.

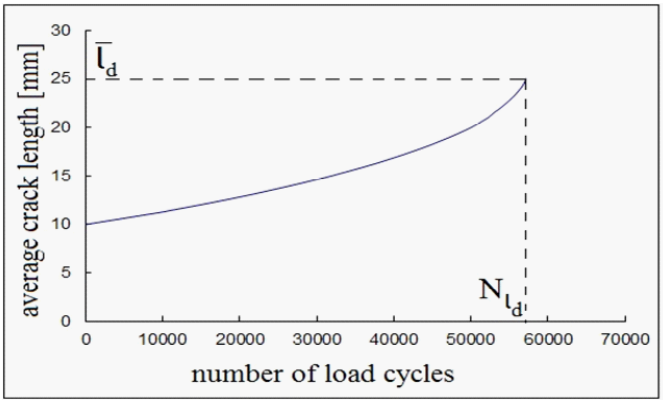

Fig. 2. Increase in the average crack length as the function of the number of load cycles

On the exclusive basis of calculations related to the growth of the average length of fatigue cracks $B(N)$ it is easy to find out that the maximum permissible crack length $\overline{l_d} = 25mm$ is achieved after $N_{l_d} = 57115$ of load cycles. However, the comprehensive calculations of the fatigue life for a specific component take also into account the probabilistic factors, therefore the variance $A(N)$of the crack length as described by the formula (25) must be additionally included. For that purpose the equation (36) is used and it depends on the number of load cycles N established on the basis of the equation (5):

$$Q_{l_d} = \frac{l_d - B(N)}{\sqrt{A(N)}}. \qquad (39)$$

For the assumed reliability level $R(N)_{l_d}^{\ *} = 0{,}99958$ the upper limit $Q_{l_d} = 3{,}34$ for the integral (35) can be looked up in tables of normal standard distribution. Having resolved the above equation the number of load cycles $N_{l_d} = 56750$ is obtained, which is the fatigue life of the examined component with consideration of probability factors.

The advantage of the foregoing method lies in the fact that the method takes account of physical phenomena that are associated with the variable spectrum of loads. It must be kept in mind that this study reveals the method that is suitable solely in the case when the material of the structural component exhibits appropriate features. These properties are conventionally reflected as the material constant that occurs as the exponent m=2 in the Paris formula. The values of material constants that are involved in the method (except for the presumed m=2 parameter) can be either found out from experiments or estimated on the basis of operational data for expansion of cracks, where the method of moments or the trustworthiness function (e.g. the C coefficient of the Paris equation) are applied to calculations. When the fatigue life is to be determined for such a structural component where assumption of the exponent m≠2 for the Paris equation is justified, the method already disclosed in [17] should be applied.

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