

A novel kinematic model for a functional spinal unit and a lumbar spine

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Purpose: The aim of this paper is to present the novel model for the functional spinal unit and spine designed as a rigid mechanism and solve it with methods commonly used in robotics. *Method:* The structure of the intervertebral joint is analyzed with special attention paid to elements defining the displacements in the joint. The obtained mechanism is then numerically solved using a constraint equations method. *Results:* The input data set for the simulation is prepared using the 3D scan of the lumbar spine. The simulation results show that the intervertebral joint mechanism can satisfy the ranges of flexion, lateral bending and axial rotation as compared with literature data. It is also possible to study complex, coupled displacements of the lumbar spine segment. *Conclusions:* Structural analysis of the functional spinal unit with methods common in robotics can eventually lead to better understanding of stabilizing and guiding mechanisms. The proposed mechanism can be used as a reference in the study of spine guidance. It can reproduce the angular displacements of the actual functional spine unit. It is also possible to expand the model to facilitate the analysis of a lumbar spine segment.

Key words: spine stabilization, spine guiding mechanism, structural analysis

1. Introduction

Proper model of the functional spinal unit (FSU) or the spine can provide much needed insight into operation planning of the spine segment damaged through illness, mechanical injury and into spine stabilization or guidance. Two methods are commonly used to describe the human spine. The first of them is the finite element method (FEM). The method is perfectly suited for complex models. It can help design elements that could substitute a disc or ligaments as shown in [5], [27], aid the process of operation planning and give deeper insight into common injuries of the spine [2], [11], study the effect of spine stabilization [16], [23], [36] or research the effect of external orthotic devices as presented in [10]. Finally, even elements like a spinal cord, which are often omitted in mechanical models in order to simplify them, might be considered [12].

On the other hand, while FEM modeling is very versatile and can provide data on different external and internal devices aiding a spine, it is very hard to design a new device using only FEM. It is also difficult to obtain a relatively simple but accurate model of a spine or spinal segment. For this purpose, a multi-body system method might be used.

The functional spinal unit (FSU) consists of two vertebrae connected with various passive elements (ligaments, a disc). Each vertebra is also connected to multiple active elements (muscle system). Many different, general models were proposed in the literature. Probably the most common one is the model, where the FSU's passive elements are replaced with one spherical or revolute joint [1], [26]. Those simple models are very useful, however, they do not represent the structure of the intervertebral joint well and the spherical/revolute joint coordinates have to be estimated. Models using a stiffness matrix to describe the intervertebral joint are also very popular [8], [14],

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[28]. On the other hand, many researchers tried to provide complex, spatial models of the FSU. In those models the ligaments and the disc are substituted with appropriate systems of flexible elements [22]. The models in which muscular activity is considered are often prepared and solved in software such as OpenSim or Anybody due to their complexity [7], [24], [37]. There are also several less general models prepared for special purposes. Examples of such models are as follows: a spine model that consists of three solid elements connected with revolute joints [31] used in research concerning road accidents, a spine model using spline approach for vibrational simulation [33], a simplified model used for the study of effects of continuous passive motion on the lumbar spine in seating [15] or a cyclist's dynamical lumbar spine model [20].

The aim of this paper is to find a novel, simple mechanism for the FSU of the lumbar spine and the lumbar spine based on the structure of the intervertebral joint. The mechanism should be able to reproduce the basic, angular FSU displacements while having relatively simple structure. Thus, the mechanism could be used to aid people with spinal injuries where one or more FSU is no longer able to properly guide the body.

2. Methods

It is notable that facet joints and passive elements of the spine (ligaments) define and limit the intervertebral displacements in the FSU [3], [6]. Surface analysis of the facet joints leads to a conclusion that their contact can be described with sphere-sphere

contact pair. If linear displacement of the spheres is negligible, this contact pair can be substituted with a link of length equal to the distance between spheres. The link is connected to the vertebrae with spherical joints.

For the purpose of structural simplicity the complex system of ligaments can be substituted with one ligament that has a great influence on possible displacements of the vertebrae. Anterior longitudinal ligament was chosen as a substitute ligament after consideration of many different ligaments. In the model the ligament is considered as a fixed-length link connected to vertebrae with spherical joints.

The proposed FSU model is a spatial, parallel mechanism with three degrees of freedom, Fig. 1. There are three angular, input variables that correspond to flexion/extension, lateral bending and axial rotation. The linear intervertebral displacements are coupled with angular ones. The mechanism contains only rigid elements. Flexible elements representing other ligaments and the disc can also be considered to facilitate static and dynamic analyses. Platform mechanisms of this type are widely known in robotics [13]. It is worth noting that the addition of another leg to the mechanism would limit its degrees of freedom to two. The angular displacements would no longer be independent.

The model of the lumbar spine L1-L5 will also be considered in this paper, Fig. 2. The lumbar spine model consists of 4 functional spinal unit models (L5-L4, L4-L3, L3-L2, L2-L1). It is assumed that the vertebra L5 is a basis of the mechanism.

As the simplified model of the FSU and the lumbar spine is established position and displacement analysis can be performed.

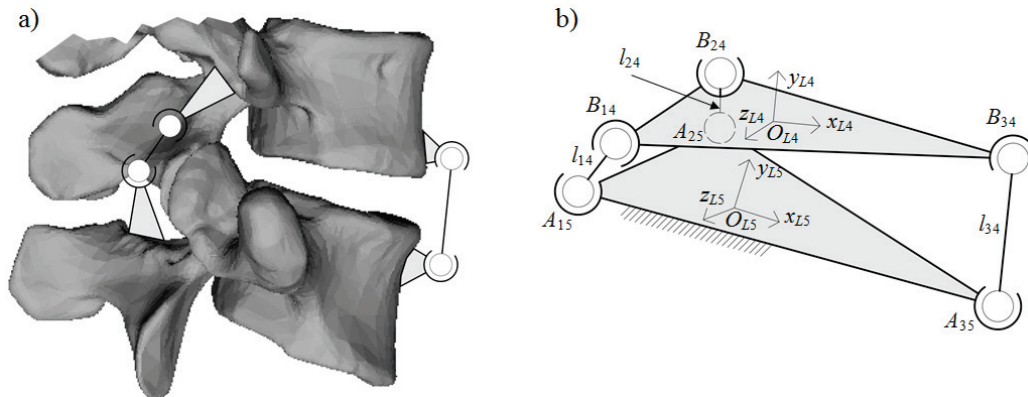


Fig. 1. (a) The FSU L5-L4 with elements of the proposed model, (b) the parallel model of the FSU where:
 $A_{15}, A_{25} (B_{14}, B_{24})$ – the center of the sphere estimated from the surface of the upper (lower) facet joint in the vertebra L5 (L4),
 $A_{35} (B_{34})$ – the substitute ligament upper (lower) attachment to the vertebra L5 (L4), l_{i4} – the distance between A_{i5} and B_{i4} ,
 $\{x_{Li} y_{Li} z_{Li}\}$ – the vertebra Li reference frame ($i = 1, 2$)

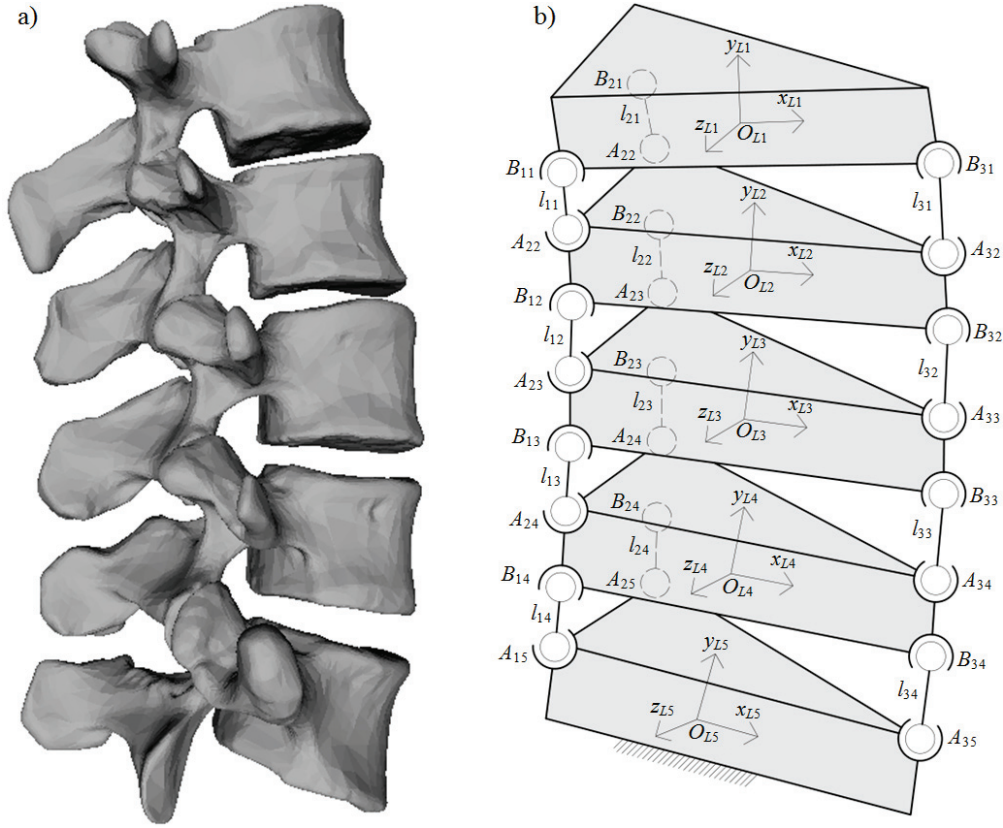


Fig. 2. (a) The 3D scan of the lumbar spine geometry, (b) the hybrid model of the lumbar spine, where:
 $A_{1j}, A_{2j} (B_{1j}, B_{2j})$ – the center of the sphere estimated from the surface of the upper (lower) facet joint in the vertebra L_j ,
 $A_{3j} (B_{3j})$ – the substitute ligament upper (lower) attachment to the vertebra L_j ,
 l_{ij} – the distance between $A_{i(j+1)}$ and B_{ij} ($i = 1, 2, 3, j = 1..4$), $\{x_{Li}, y_{Li}, z_{Li}\}$ – the vertebra L_i reference frame ($i = 1..5$)

2.1. The parallel model of the FSU

Position and displacement analysis of the presented FSU model using a constraint equations method leads to a system of 3 nonlinear equations (written for L5-L4 vertebra)

$$\begin{cases} l_{14} = \| \mathbf{a}_{15}^{L5} - \mathbf{R}_{45} \mathbf{b}_{14}^{L4} - \mathbf{p}_{45}^{L5} \| \\ l_{24} = \| \mathbf{a}_{25}^{L5} - \mathbf{R}_{45} \mathbf{b}_{24}^{L4} - \mathbf{p}_{45}^{L5} \|, \\ l_{34} = \| \mathbf{a}_{35}^{L5} - \mathbf{R}_{45} \mathbf{b}_{34}^{L4} - \mathbf{p}_{45}^{L5} \| \end{cases} \quad (1)$$

where \mathbf{a}_{i5}^{L5} – the position vector of the A_i point s in the L5 reference frame ($i = 1, 2, 3$), \mathbf{b}_{i4}^{L4} – the position vector of the point B_{i4} in the L4 reference frame ($i = 1, 2, 3$), l_{i4} – the distance between points A_{i5} and B_{i4} ($i = 1, 2, 3$), \mathbf{p}_{45}^{L5} – the position vector from the L4 reference frame origin O_{L4} to the L5 reference frame origin O_{L5} in the L5 reference frame, \mathbf{R}_{45} – the rotation matrix from the L4 reference frame to the L5 reference frame. These equations assure constant length of mechanism's legs in every position.

A sequence of rotations is assumed after [30] and presented earlier in [21]

$$\mathbf{R}_{45} = \begin{bmatrix} c\alpha c\gamma + s\alpha s\beta s\gamma & -s\alpha c\gamma + c\alpha s\beta s\gamma & -c\beta s\gamma \\ s\alpha c\beta & c\alpha c\beta & s\beta \\ c\alpha s\gamma - s\alpha s\beta c\gamma & -s\alpha s\gamma - c\alpha s\beta c\gamma & c\beta c\gamma \end{bmatrix} \quad (2)$$

where $s\alpha = \sin\alpha$, $c\alpha = \cos\alpha$.

If the rotation matrix is written in the following form

$$\mathbf{R}_{45} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}, \quad (3)$$

then values of individual angles can be calculated using the following formulas

$$\beta = \text{atan2}(r_{23}, \pm \sqrt{1 - r_{23}^2}), \quad (4)$$

$$\alpha = \text{atan2}\left(\frac{r_{21}}{\cos\beta}, \frac{r_{22}}{\cos\beta}\right), \quad (5)$$

$$\gamma = \operatorname{atan2}\left(\frac{-r_{13}}{\cos \beta}, \frac{r_{33}}{\cos \beta}\right). \quad (6)$$

In terminology used in orthopaedics and traumatology the following applies: α – flexion/extension, β – axial rotation, γ – lateral bending.

The system of equations (1) can be solved numerically if the values of angles α , β , γ are assumed. Then coordinates of the translation vector \mathbf{p}_{45}^{L5} can be obtained from equation (1).

2.2. The hybrid model of the lumbar spine

As stated before the lumbar spine model includes 4 FSU mechanisms connected in a serial way. Each pair of vertebrae has to be solved separately using the following systems of equations

$$\begin{cases} l_{11} = \|\mathbf{a}_{12}^{L2} - \mathbf{R}_{12}\mathbf{b}_{11}^{L1} - \mathbf{p}_{12}^{L2}\| \\ l_{21} = \|\mathbf{a}_{22}^{L2} - \mathbf{R}_{12}\mathbf{b}_{21}^{L1} - \mathbf{p}_{12}^{L2}\| \\ l_{31} = \|\mathbf{a}_{32}^{L2} - \mathbf{R}_{12}\mathbf{b}_{31}^{L1} - \mathbf{p}_{12}^{L2}\| \\ l_{13} = \|\mathbf{a}_{14}^{L4} - \mathbf{R}_{34}\mathbf{b}_{13}^{L3} - \mathbf{p}_{34}^{L4}\| \\ l_{23} = \|\mathbf{a}_{24}^{L4} - \mathbf{R}_{34}\mathbf{b}_{23}^{L3} - \mathbf{p}_{34}^{L4}\| \\ l_{33} = \|\mathbf{a}_{34}^{L4} - \mathbf{R}_{34}\mathbf{b}_{33}^{L3} - \mathbf{p}_{34}^{L4}\| \end{cases} \quad \begin{cases} l_{12} = \|\mathbf{a}_{13}^{L3} - \mathbf{R}_{23}\mathbf{b}_{12}^{L2} - \mathbf{p}_{23}^{L3}\| \\ l_{22} = \|\mathbf{a}_{23}^{L3} - \mathbf{R}_{23}\mathbf{b}_{22}^{L2} - \mathbf{p}_{23}^{L3}\| \\ l_{32} = \|\mathbf{a}_{33}^{L3} - \mathbf{R}_{23}\mathbf{b}_{32}^{L2} - \mathbf{p}_{23}^{L3}\| \\ l_{14} = \|\mathbf{a}_{14}^{L5} - \mathbf{R}_{45}\mathbf{b}_{14}^{L4} - \mathbf{p}_{45}^{L5}\| \\ l_{24} = \|\mathbf{a}_{24}^{L5} - \mathbf{R}_{45}\mathbf{b}_{24}^{L4} - \mathbf{p}_{45}^{L5}\| \\ l_{34} = \|\mathbf{a}_{34}^{L5} - \mathbf{R}_{45}\mathbf{b}_{34}^{L4} - \mathbf{p}_{45}^{L5}\| \end{cases} \quad (7)$$

where \mathbf{a}_{ij}^{Lj} – the position vector of the point A_{ij} in the L_j reference frame, \mathbf{b}_{ij}^{Lj} – the position vector of the point B_{ij} in the L_j reference frame, l_{ij} – the distance between points $A_{i(j+1)}$ and B_{ij} , \mathbf{p}_{ij}^{Lj} – the position vector from the L_i reference frame origin to the L_j reference frame origin in the L_j reference frame, \mathbf{R}_{ij} – the rotation matrix from the L_i reference frame to the L_j reference frame.

In the next step, transformation matrices \mathbf{T}_{12} , \mathbf{T}_{23} , \mathbf{T}_{34} , \mathbf{T}_{45} can be obtained

$$\mathbf{T}_{12} = \begin{bmatrix} \mathbf{R}_{12} & \mathbf{p}_{12}^{L2} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \dots, \mathbf{T}_{45} = \begin{bmatrix} \mathbf{R}_{45} & \mathbf{p}_{45}^{L5} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (8)$$

where \mathbf{T}_{12} – the transformation matrix from the L_1 reference frame to the L_2 reference frame.

It is possible to assume any vertebra as the basis of the mechanism. If the vertebra L_5 is assumed to be the basis of the model then matrices \mathbf{T}_{15} , \mathbf{T}_{25} , \mathbf{T}_{35} have to be calculated

$$\mathbf{T}_{15} = \mathbf{T}_{45}\mathbf{T}_{34}\mathbf{T}_{23}\mathbf{T}_{12}, \quad \mathbf{T}_{25} = \mathbf{T}_{45}\mathbf{T}_{34}\mathbf{T}_{23}, \quad \mathbf{T}_{35} = \mathbf{T}_{45}\mathbf{T}_{34}, \quad (9)$$

where \mathbf{T}_{ij} – the transformation matrix from the L_i reference frame to the L_j reference frame.

Now, position and orientation of every vertebra with respect to the vertebra L_5 can be computed.

3. Results

In this paragraph the results concerning the positions and displacements of the FSU and the lumbar spine segment are presented. The data set used in simulations is given below.

Input data set can be obtained from a 3D spine geometry scan. In this paper, a 3D scan prepared by BodyParts3D, © The Database Center for Life Science licensed under CC Attribution-Share Alike 2.1 Japan is used. Coordinates of points are read using Netfabb Basic. Reference frames are assumed for 5 vertebrae (L_1 - L_5). Then spheres are estimated from facet joint surfaces for every pair of vertebrae. In the next step, attachment centers of the anterior longitudinal ligament to each vertebrae are assumed (paper [4] is used as reference). Finally, spheres and attachment centers are transformed to the vertebrae reference frames.

The input data set for the displacement analysis contains (for vertebrae: L_5 - L_4 – other pairs, L_4 - L_3 , L_3 - L_2 , L_2 - L_1 , are similar): \mathbf{b}_i^{L4} ($i = 1, 2, 3$) the position vector of the point B_i in the vertebra L_4 reference frame, \mathbf{a}_i^{L5} ($i = 1, 2, 3$) the position vector of the point A_i in the vertebra L_5 reference frame and distances l_{14} , l_{24} , l_{34} . The assumed 21 parameters for the FSU L_5 - L_4 are listed below (distances and coordinates of points in [mm])

$$l_{14} = 9.88, \quad l_{24} = 9.88, \quad l_{34} = 21.69,$$

$$\mathbf{a}_1^{L5} = \begin{bmatrix} -38.89 \\ 20.64 \\ 13.65 \end{bmatrix}, \quad \mathbf{a}_2^{L5} = \begin{bmatrix} -38.89 \\ 20.64 \\ -13.65 \end{bmatrix}, \quad \mathbf{a}_3^{L5} = \begin{bmatrix} 19.86 \\ 16.51 \\ 0.00 \end{bmatrix},$$

$$\mathbf{b}_1^{L4} = \begin{bmatrix} -34.28 \\ -7.86 \\ 20.67 \end{bmatrix}, \quad \mathbf{b}_2^{L4} = \begin{bmatrix} -34.28 \\ -7.86 \\ -20.67 \end{bmatrix}, \quad \mathbf{b}_3^{L4} = \begin{bmatrix} 18.18 \\ 0.51 \\ 0.00 \end{bmatrix}.$$

The input data set also contains a starting position of the mechanism derived from the 3D scan. This position is used as a starting solution for the systems of equations (1) and (7) in all simulations performed in the next part of the paper.

3.1. Simulation results

In the following section the results obtained for the L5-L4 FSU model are presented. Positions and displacements of the vertebra were computed using data set presented above.

Simulation results for FSU flexion are presented below. It is worth noting that flexion is coupled with

translation in the sagittal plane as in reality. There is no additional displacement in other anatomical planes of the body.

The axial rotation is connected with significant linear displacement along axis z of the base vertebra. Displacements occurring in the sagittal plane are negligible.

The lateral flexion is connected with significant linear displacement along axis z of the base vertebra

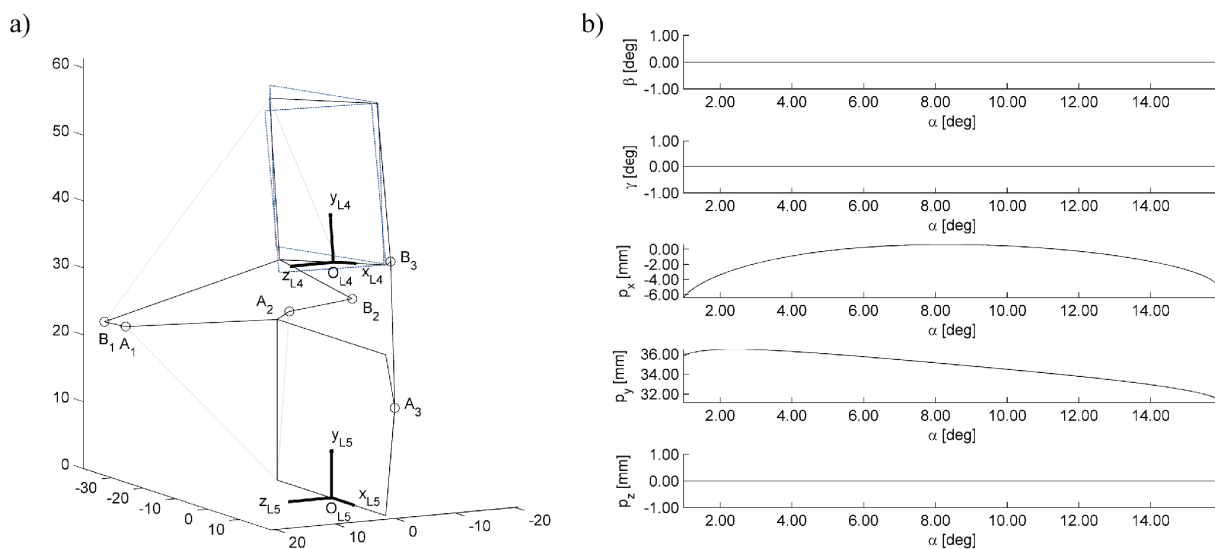


Fig. 3. Simulation results – flexion/extension: (a) the FSU mechanism in three positions (the starting position is drawn with thick line), (b) graphs of β , γ – orientation angles, p_x, p_y, p_z – coordinates of the position vector of the L4 reference frame in regard to the L5 reference frame as functions of the flexion/extension angle α

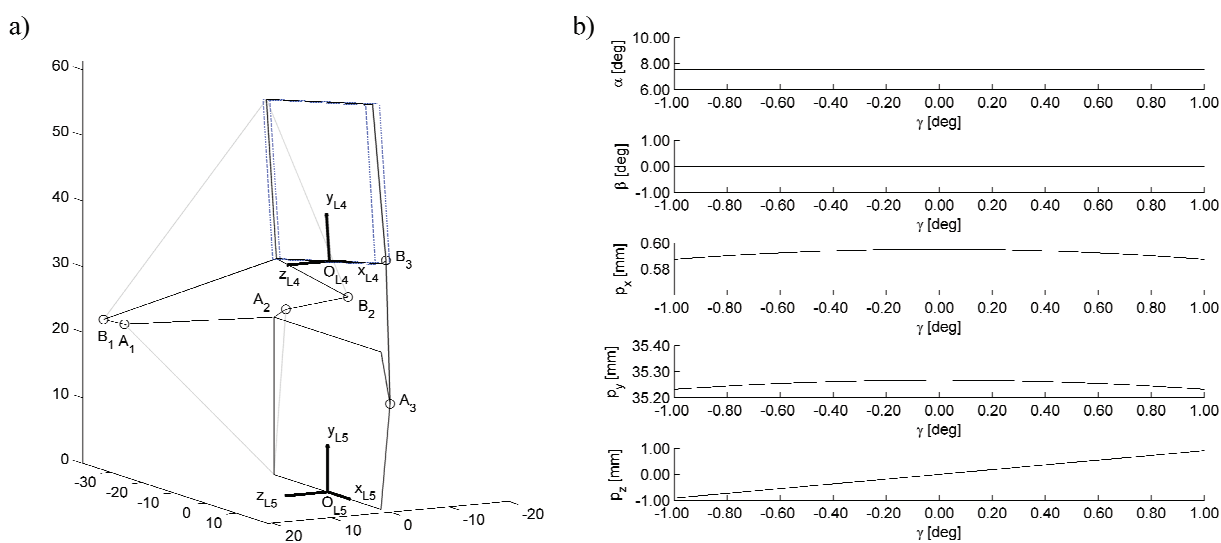


Fig. 4. Simulation results – axial rotation: (a) the FSU mechanism in three positions (the starting position is drawn with thick line), (b) graphs of α , β – orientation angles, p_x, p_y, p_z – coordinates of the position vector of the L4 reference frame in regard to the L5 reference frame as functions of the axial rotation angle γ

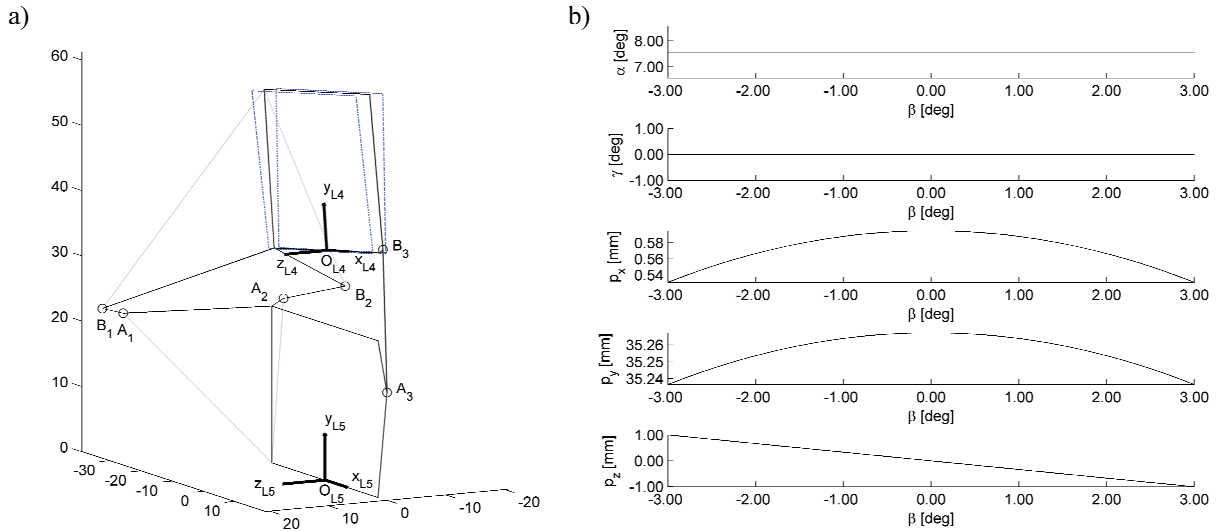


Fig. 5. Simulation results – lateral bending: (a) the FSU mechanism in three positions (the starting position is drawn with thick line), (b) graphs of α, γ – orientation angles, p_x, p_y, p_z – coordinates of the position vector of the L4 reference frame with regard to the L5 reference frame as functions of the lateral bending angle β

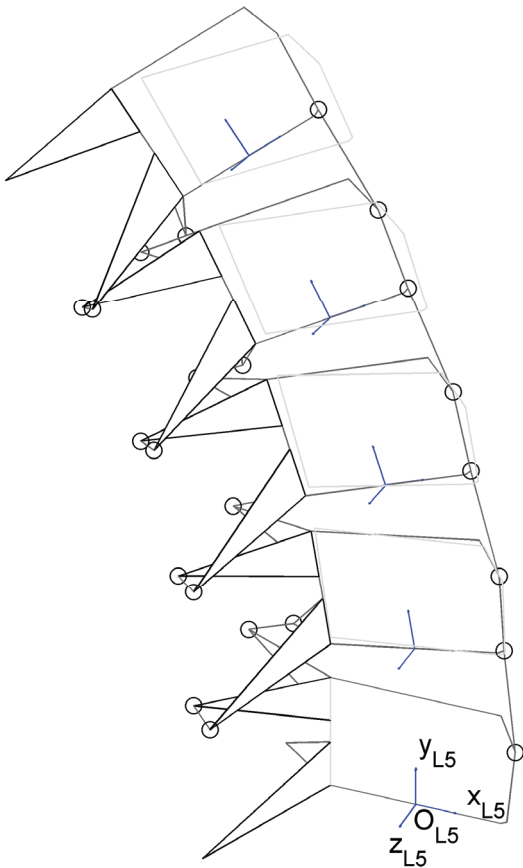


Fig. 6. Simulation results – extension with lateral bending for the lumbar spine L5-L1; the hybrid mechanism is drawn in the initial (fine line) and final position

as shown in Fig. 5. There is no notable displacement in the sagittal plane.

Results obtained from the simplified model can be improved by using parameter estimation. In order to perform this procedure a set of different positions of the spine is required. The procedure is beyond the scope of this paper.

The proposed model is capable of satisfying the ranges of flexion, lateral bending and axial rotation as presented in [34].

It is worth mentioning that any type of complex coupled displacement can be simulated using the model as seen in Fig. 6.

4. Discussion

The approach for modeling a spine fragment using parallel, rigid mechanism is still rather unique. There is a limited number of published works in which a spine is considered as a complex hybrid, parallel mechanism, which stands for several parallel mechanisms connected in a serial way. This method has been successfully applied to various body joints. In [30], a complex spatial model of a knee joint is presented. This mechanism contains two nonsymmetrical platforms (the femur and the tibia), connected with two point contact pairs (sphere-sphere contact type, where the spheres correspond to the tibial condyles and the femoral condyles) and four legs: three of which are passive of S-S type (the cruciate ligaments and the collateral ligament, where: S – the spherical joint) and one active of S-S-C type (the patellar ten-

don and the patella with C – the cylindrical joint). The muscle system of the actual joint has been simplified and substituted with one linear actuator. A simplified spherical model of the knee has also been presented by the same research group [29]. The model does not represent the structure of the knee well, however, it is capable of reproducing that joint displacements. A spatial model of the ankle has been proposed in [17]. Its structure is similar to that of the knee joint model [30].

Recently, an increasing interest can be seen in mechanisms that can serve as a replacement of a damaged intervertebral joint or as a robotic spine module. The FSU model where one spherical joint substitutes the structure of the intervertebral joint is proposed in [1], [26]. While these models provide good results it would be very difficult to use them as a basis in spine guidance research (where spine guiding mechanism means a mechanism that could be added to an existing, damaged FSU and help reproduce the displacements of a healthy intervertebral joint). In [25], a complex, hybrid mechanism of trunk and waist is presented. The mechanism is perfectly suited for robotic systems. In its current state, due to size and differences in structure, it is not possible to use it to aid a human spine. An interesting solution is shown in [32], where a simple spatial mechanism is used as prosthesis of a disc. Simulations show that during a flexural displacement the ligament and facet force are reduced when compared to common ball-socket joint mechanism. However, the disc would have to be removed to use it. Slightly different approach is presented in [18], where the presented mechanism could serve as a robotic neck system. The authors emphasize the fact that this mechanism has a low motion noise to mimic a human neck. Its structure is similar to that of the FSU. The compression spring corresponds to a disc and cables to muscles and ligaments. While perfect for humanoid robots, this solution is not feasible when human body is considered. In [9], a compliant, parallel mechanism for the intervertebral joint is presented. The mechanism contains three legs, two of which are UPU type (U – universal joint, P – prismatic joint) and one is a flexible, central rod. According to the authors, the mechanism can be used in the trunk of a humanoid robot. On the other hand, the mechanism presented in this paper is based on the structure and dimensions of an actual intervertebral joint. It is capable of reproducing the three angular displacements of the FSU. These displacements are independent. The structure of the mechanism is simple and its links are rigid. This makes it an interesting choice as a intervertebral joint replacement, aid or a humanoid robot spine. The mechanism does not

contain an element that corresponds to a disc. Thus, it seems possible to add it to an existing, damaged intervertebral joint noninvasively. Since the structure of the mechanism is very simple, the optimization procedure could be used to limit the ranges of selected angular displacements, which could be useful in treatment of medical conditions (such as stretch or tear of ligaments).

When comparing the possible displacements obtained from the model to that of the actual FSU, it has to be noted that the ranges vary individually [35] as every spine is different and so are its dimension ratios. The model, simulated using the presented data set, allows analyses for 15.00 deg flexion/extension range, while starting position is at 7.57 deg flexion. These results are consistent with the data presented in [34], where flexion/extension range is 16.00 deg for the L4-L5 FSU. Furthermore, as seen in Fig. 3, this displacement occurs only in the sagittal plane as there is no change in p_z , β , γ values. A similar flexion/extension range can be observed in the disk prosthesis mechanism [32]. The flexion range in the elasto-static model of the intervertebral joint [22] is only 5.20 deg. The lateral bending range obtained from the model is 6.00 deg, while starting position is at 0.00 deg. In [34], the lateral bending range for the FSU L4-L5 is also 6.00 deg. The linear displacement in the sagittal plane is negligible, as the range of p_x and p_y is 0.02 mm. In the case of axial rotation, the range obtained from the model is 2.00 deg, while starting position is at 0.00 deg. The experimental axial rotation range presented in [34] is also 2.00 deg for the L4-L5 FSU. The linear displacements p_x and p_y could be negligible, too. Only the p_z is significant as its range is 2.00 mm. The angular displacements are independent, but complex, spatial displacements, where two or three angles change, are also possible as seen in Fig. 6.

It is also worth mentioning that there are no numerical difficulties with computation of the mechanism positions. Clinical verification of the model seems expedient. The model can be further completed by adding flexible elements, which would facilitate static and dynamic analyses. Considering the results obtained from the simulations the approach proposed in this paper seems worth exploring.

5. Conclusions

The novel intervertebral joint mechanism has been presented. Positions and angular displacements of the proposed mechanism have been analyzed using the

constraint equations method. Numerical simulations proved the effectiveness of the proposed model. It might be useful in research concerning kinematic ability of the FSU and treatment of a damaged intervertebral joint.

The FSU model was further expanded by adding 3 additional FSU models to it in a serial way and thus creating the lumbar spine model. The lumbar spine model can be suitable for studies regarding various methods of spine stabilization. It is very simple to limit possible, relative displacements in the selected vertebrae. If other passive elements of the FSU are considered the lumbar spine model can also facilitate static and dynamic analyses.

Analysis of the kinematic structure of the FSU with methods commonly used in robotics may lead to better stabilizing mechanisms. The proposed mechanism can be used as a reference in the study of spine guidance and possibly in substituting damaged joints – its structure is relatively simple and it is easy to solve. It can also reproduce the displacements of the actual FSU in three degrees of freedom (flexion, lateral bending and axial rotation). The presented mechanism might also be used in a spinal module in humanoid robots, as it provides continuous motion during complex displacements where two or three angular values change.

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