

ASSESSMENT OF THE DEGREE OF FITTING THE TRANSPORT POTENTIAL OF THE TRANSPORTATION COMPANY TO A RANDOM DEMAND FOR TRANSPORT SERVICES

GUSTAW KONOPACKI

*Military University of Technology, Department of Cybernetics,
Institute of Information Systems*

Will be considered transport company exploiting the uniform in the sense of destination, means of transport such as tankers, with a total transport potential equal to a loading units (e.g. tones). The company operates in the transport market, where demand for transport services is random. The problem of fitting the transport potential of a transport company to the demand for such services is formulated. As measure of this fit assumes that the probability of a firm's transport potential is not being exceeded by demand over the given time horizon. Practically useful formulas for estimating such probability for cases where demand for transport services is described by stationary and non-stationary normal stochastic processes. The results are illustrated by numerical examples.

Keywords: transport potential, demand, stochastic process

1. Introduction

Transport plays a significant role in every economy. This is due to the fact that it performs important service functions for other branches of the economy. There is a close relationship between the development of transport systems and economic development. Efficient transport determines economic growth, while its underdevelopment is becoming a barrier to economic development.

Transport is a sector of the economy that meets the needs of moving people and goods. Participants in the transport market are suppliers and buyers of transport services, who represent the supply of these services and demand for them. The proper management of a transport company requires knowledge of issues related to the behavior of transport market participants, i.e. [17]:

- transport company as a provider of transport services (supply side),
- charge carriers as the recipients of these services (demand side).

The volume of transport service supply is determined by [17, 18]:

- economic conditions - mainly prices and costs in the transport sector,
- technical conditions - mainly the number, type and quality of means of transport and the condition and quality of infrastructure in the transport sector,
- market conditions - supply-side market structures that may limit or facilitate the activities of carriers (e.g. alliances),
- administrative conditions - regulations limiting the possibility of carrying out transport (e.g. axle load in road transport, customs regulations).

Demand for transport services arises from transport needs, which are defined as the willingness or the necessity to move people or cargo from one place to another by means of transport [18]. In general, there are two types of demand:

- potential demand, which is understood the natural demand for the goods or services,
- actual, effective demand, which is understood as the ability to buy a good or service at a certain time for a certain number of buyers.

One of the basic problems of transport company management is to ensure its continuous presence on the market of transport services. This is achieved mainly by ensuring the appropriate transport potential of the means of transport available, in line with the anticipated demand for transport services.

Failure to match the transportation potential of the transport company to the demand for transport services that will shape the market for these services in the time horizon foreseen by company will, of course, lead to the following two situations:

- reduce the company's competitiveness in the transport services market, and even exit the market when transport capacity is lower than demand,
- incur additional costs for non-use of transport potential on the demand for transport services.

One of the measure of matching the transportation potential of a transport company to the demand for transport services may be the probability that in the forecast time horizon the transport potential will not be exceeded by demand - in the first situation or transport potential is exceeded by demand - in the second situation.

Usually, the first of the aforementioned situations will have greater implications for the transport company and will therefore be considered further,

taking into account the random nature of demand for transport services and will not take into account the operational characteristics of transport as was the case in [5].

2. Description of the problem

Consider transport company using the same or very similar means of transport (e.g. tankers) is used to meet the demand for homogeneous transport services (e.g. fuel transportation). It is assumed that the means of transport need not be the same i.e. they do not have to have the same construction solutions and the same capacity. The transport potential of the transport company will be the total capacity of all currently available means of transport. This potential will be determined by a and treated as the ability of the transport company to provide transport services at most in this size. Transport potential can be either constant or random. It is also assumed that the demand for transport services is random and can be interpreted as a stationary or non-stationary stochastic process. Thus, temporary demand values, which are the realization of random variables, may both be larger than and be lower than value of the transport potential of the transport company.

These cases have a character of temporal mismatch of transport potential to demand, which occur only in finite time intervals and the length of which depends on the probabilistic characteristics of the process of formation the size of demand for transport services in case the performance characteristics of the means of transport are not taken into account, as is the case in this article.

Next will be considered the second case, which seems to give rise to more serious consequences for the transport company and as previously noted, the lack of ability to meet demand can lead to a undermine of the transport company's position in the transport services market and even to its exit of the market. Therefore, it seems necessary previous discernment the transport company about its ability to function in the transport services market, if it has transport potential of a . This knowledge can be useful for making decisions such as buying additional means of transport or changing the profile of providing transport services.

3. Demand for transport services of unchanging trend

3.1. Constant value of transport potential

It is presumed that the further demand for transport services can be described by means of continuous stochastic process $X(t)$ class CC. It is assumed also that process $X(t)$ is stationary, ergodic and differentiable in the mean-square sense [1–3, 6, 9, 13, 14, 16, 19]. Let m_x be the expected value of this process and $K_x(\tau) = \sigma_x^2 r(\tau)$ its correlation function, where $r(\tau)$ defines a normalized correlation function. The

problem to be considered is to determine the probability of exceedance by the demand for transport services (process $X(t)$) of the transport potential of transport company fixed at a constant level a . An exemplary implementation of demand for transport services as a function of time (exemplary implementation of the stochastic process $X(t)$) for the constant value of the transport potential is shown in Figure 1.

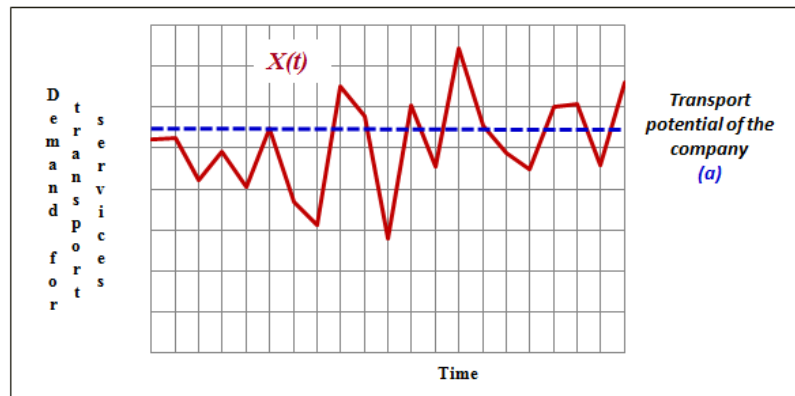


Figure 1. Exemplary implementation of the stochastic process $X(t)$ describing the demand for transport services in case of constant value of transport potential

This problem determining of the probability distribution of residence time of a stochastic process $X(t)$ over a defined threshold value a is computationally difficult. Fortunately, however, in practice, often enough to know the expected value of the residence time of a stochastic process over a defined threshold value, which greatly facilitates the analytical solution.

The discussed problem was considered in detail in [6], and practically useful calculation formulas were obtained for the normal stochastic process in the form of the following estimates of the probability of not exceeding the transport potential of the transport company by the demand for transport services in the T -period:

- lower estimation

$$P_0(a, T) \geq P_0^{\min} = \Phi\left(\frac{a - m_x}{\sigma_x}\right) - n_0 \cdot T \cdot \exp\left(-\frac{(a - m_x)^2}{2 \cdot \sigma_x^2}\right) \quad (1)$$

- upper estimation

$$P_0(a, T) \leq P_0^{\max} = \Phi\left(\frac{a - m_x}{\sigma_x}\right) \cdot \exp\left(-n_0 \cdot T \cdot \exp\left(-\frac{(a - m_x)^2}{2 \cdot \sigma_x^2}\right)\right) \quad (2)$$

where n_0 is the expected number of heights by process $X(t)$ of its expected value per unit of time, which can be taken as equal [12]:

$$n_0 = \frac{1}{2\sqrt{\pi}} \quad (3)$$

Estimating (1) and (2) can be use if T satisfies the following inequality:

$$T \leq \frac{\Phi\left(\frac{a-m_x}{\sigma_x}\right)}{n_0} \cdot \exp\left(\frac{(a-m_x)^2}{2 \cdot \sigma_x^2}\right) \quad (4)$$

Example 1

The transport company has the potential to transport equal to $a = 6000$ units (e.g. tons), while the demand for transport services market is described by a normal stationary stochastic process $X(t)$ of the expected value $m_x = 4000$ units loading and correlation function $K_x(\tau) = 640000 \exp(-2,56 \tau^2)$. Calculate the value of the lower (P^{min}) and upper (P^{max}) estimates of probability $P(a, T)$ not exceedances of the value of the transport potential of the company (the ability to meet the demand for transport services in its entirety) by demand in the time horizon $T = 1, 2, \dots, 15$ (e.g. months).

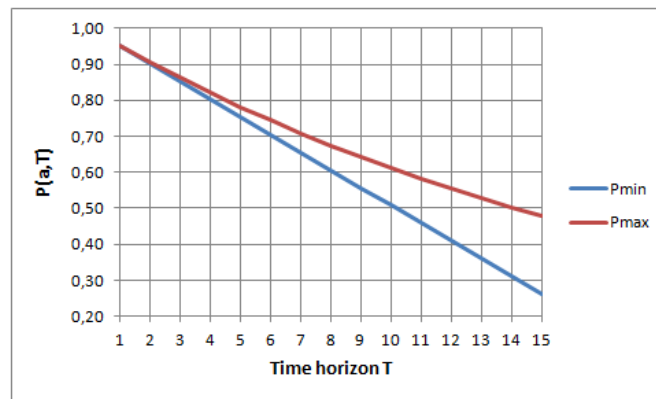


Figure 2. Graphical presentation of the lower (P^{min}) and upper (P^{max}) estimates for probability $P(a, T)$ on the basis of the data of Example 1

3.2. Random value of transport potential

The management of real businesses shows that the assumption of the constant transport potential of transport company is an optimistic assumption. Usually, this potential is shaped by the random factors characteristic of the process of exploitation of means of transport. It is therefore reasonable to consider the question of the degree of adaptation of the randomly changing transport potential to the random demand for transport services. Let the demand for transport services be described by the stochastic process $X(t)$ as in 3.1, but let the firm's transport potential be a random variable with the normal distribution $N(m_a, \sigma_a)$. Let the transport potential and demand for transport services not will be correlated with each other.

Exemplary implementation of demand for transport services as a function of time (exemplary implementation of the stochastic process $X(t)$) for the random transport potential is shown in Figure 3.

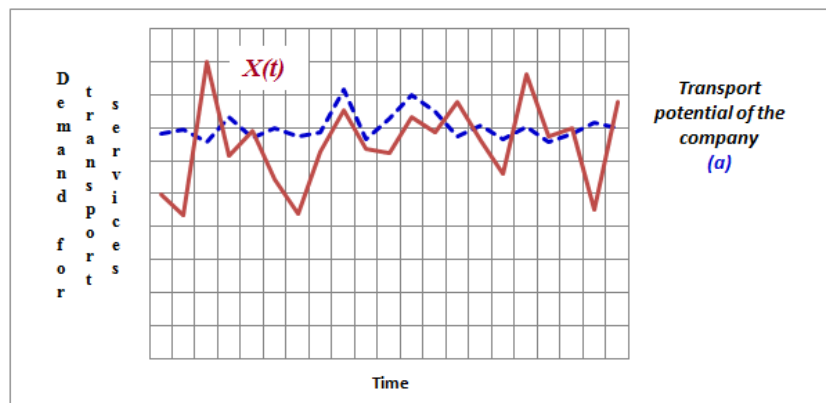


Figure 3. Exemplary implementation of the stochastic process $X(t)$ describing the demand for transport services in case the random transport potential.

In this case, the expected number of exceedance by the process $X(t)$ of the random value of transport potential is expressed in terms of:

$$n_a = n_0 \exp\left(-\frac{(m_a - m_x)^2}{2(\sigma_x^2 + \sigma_a^2)}\right) = \frac{1}{2\pi} \sqrt{\frac{r_0'' \sigma_x^2}{\sigma_x^2 + \sigma_a^2}} \exp\left(-\frac{(m_a - m_x)^2}{2(\sigma_x^2 + \sigma_a^2)}\right) \quad (5)$$

Using a similar approach as in point 3.1, a lower and upper estimate of the probability of not exceeding the value of the transport enterprise's transport potential value over a time interval of length T is obtained.

These are:

- lower estimation

$$P_0(a, T) \geq P_0^{min} = \Phi\left(\frac{m_a - m_x}{\sqrt{\sigma_x^2 + \sigma_a^2}}\right) - n_0 \cdot T \cdot \exp\left(-\frac{(m_a - m_x)^2}{2 \cdot (\sigma_x^2 + \sigma_a^2)}\right) \quad (6)$$

- upper estimation

$$P_0(a, T) \leq P_0^{max} = \Phi\left(\frac{m_a - m_x}{\sqrt{\sigma_x^2 + \sigma_a^2}}\right) \exp\left(-n_0 \cdot T \cdot \exp\left(-\frac{(m_a - m_x)^2}{2 \cdot (\sigma_x^2 + \sigma_a^2)}\right)\right) \quad (7)$$

Estimating (6) and (7) can be use if T satisfies the following inequality:

$$T \leq \frac{\Phi\left(\frac{m_a - m_x}{\sqrt{\sigma_x^2 + \sigma_a^2}}\right)}{n_0} \exp\left(-\frac{(m_a - m_x)^2}{2 \cdot (\sigma_x^2 + \sigma_a^2)}\right) \quad (8)$$

Example 2

The transport company has a transport potential that is a random variable with normal distribution $N(6000, 400)$ units loading, while the demand for transport services market is described by a normal stationary stochastic process $X(t)$ of the expected value $m_x = 4000$ units loading and correlation function $K_x(\tau) = 90000 \exp(-2,56\tau^2)$. Calculate the value of the lower (P^{min}) and upper (P^{max}) estimates of probability $P(a, T)$ not exceedances of the value of the transport potential of the company (the ability to meet the demand for transport services in its entirety) by demand in the time horizon $T = 1, 2, \dots, 15$ (eg months).

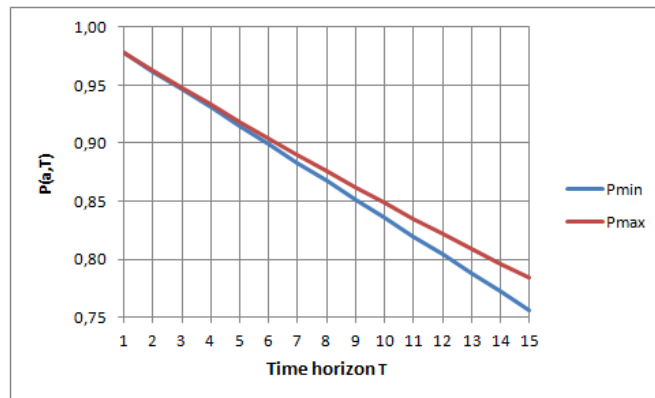


Figure 4. Graphical presentation of the lower (P^{min}) and upper (P^{max}) estimates for probability $P(a, T)$ on the basis of the data of Example 2

4. Demand for transport services of changing trend

The real market for transport services is usually characterized by demand, which is described by non-stationary continuous stochastic processes. This implies, in practice, that demand is shaped not only by random factors, but also by others that take the form of a trend. Further, formulas will be proposed to assess the fit of the transport potential of the company to the demand that is trended, which has a trend. These formulas - as in point 3 - will be express the probability of not exceeded the transport potential by the process describing the demand for transport services.

Estimation of the probability of not exceeded by the non-stationary stochastic process of the set threshold value a can be calculated from the following general formula:

$$P(a, t) \geq F_a(t_0) - \int_0^t n_a(\tau) d\tau \quad (9)$$

where: $F_a(t_0)$ - cumulative distribution function the coordinates of stochastic process at the initial moment t_0 , $n_a(\tau)$ - expected positive number exceedances of the threshold value a by the stochastic process in a unit of time.

In the general case, it is the great difficulty is an analytical calculation of the expected number of exceedances the threshold value by the stochastic process in a unit time. In [12] there are given formulas for determining this value for a normal non-stationary stochastic process.

Let us consider the case of demand, which can be described by the non-stationary stochastic process $Z(t)$ of the form:

$$Z(t) = X(t) + \varphi(t) \quad (10)$$

where: $X(t)$ - normal stationary stochastic process with zero expected value and correlation function of form $K_x(\tau) = \sigma_x^2 r(\tau)$, $r(\tau)$ - normalized correlation function, $\varphi(t)$ - some non-random monotonic function of time.

An exemplary realization of demand for transport services as a function of time (exemplary realization of stochastic process $Z(t)$ with liner function of trend) is shown in Figure 5.

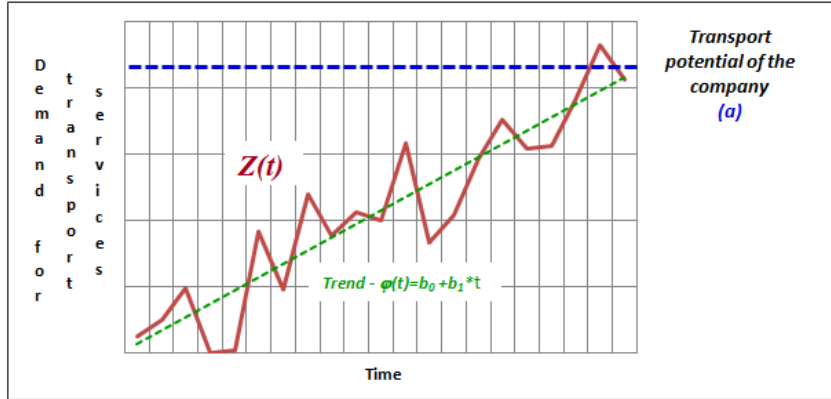


Figure 5. Exemplary implementation of the stochastic process $Z(t)$ with linear function of trend describing the demand for transport services

From (10) comes that the non-stationarity of process $Z(t)$ is conditioned by the function $\varphi(t)$, while its randomness - by the process $X(t)$. If $\varphi(t)$ is a monotonically increasing function, then the lower estimate of the probability of not exceedances by process $Z(t)$ of the threshold a can be estimated from the dependence:

$$P(a, t) \geq \Phi\left(\frac{a - \varphi(t)}{\sigma}\right) - \int_0^t n_a(\tau) d\tau \quad (11)$$

where

$$n_a(t) = \frac{1}{2\pi} \sqrt{-r''(\theta)} \cdot \exp\left(-\frac{1}{2} \left(\frac{a - \varphi(t)}{\sigma}\right)^2\right) \cdot \left(\exp\left(\frac{1}{2} \frac{(\varphi'(t))^2}{\sigma^2 r''(\theta)}\right) + \sqrt{2\pi} \frac{\varphi'(t)}{\sigma \sqrt{-r''(\theta)}} \Phi\left(\frac{\varphi'(t)}{\sigma \sqrt{-r''(\theta)}}\right) \right) \quad (12)$$

In the simplest case of the linear trend $\varphi(t)$, i.e. when

$$\varphi(t) = b_0 + b_1 t \quad (13)$$

the discussed probability can be estimated from the lower using the following formula:

$$P(a,t) \geq \Phi\left(\frac{a-b_0-b_1t}{\sigma}\right) - \left(\Phi\left(\frac{a-b_0}{\sigma}\right) - \Phi\left(\frac{a-b_0-b_1t}{\sigma}\right)\right) \cdot \left(\frac{\sigma}{b_1} \sqrt{\frac{-r''(\theta)}{2\pi}} \exp\left(\frac{b_1^2}{2\sigma^2 r''(\theta)}\right) + \Phi\left(\frac{b_1}{\sigma\sqrt{-r''(\theta)}}\right)\right) \quad (14)$$

Formula (37) can be used only if $P(a,t) \geq 0$, i.e. when $t \leq t^*$, which is the solution of the following equation:

$$\Phi\left(\frac{a-b_0-b_1t}{\sigma}\right) = \left(\Phi\left(\frac{a-b_0}{\sigma}\right) - \Phi\left(\frac{a-b_0-b_1t}{\sigma}\right)\right) \cdot \left(\frac{\sigma}{b_1} \sqrt{\frac{-r''(\theta)}{2\pi}} \exp\left(\frac{b_1^2}{2\sigma^2 r''(\theta)}\right) + \Phi\left(\frac{b_1}{\sigma\sqrt{-r''(\theta)}}\right)\right) \quad (15)$$

Example 3

The transport company has the potential to transport equal to $a = 3600$ units loading (e.g. tons), while the demand for transport services market is described by the non-stationary stochastic process $Z(t)$ as (30) i which trend is described by linear function as (36) with parameters: $b_0 = 3500$ and $b_1 = 5$ units loading. Process $X(t)$ is a normal stationary stochastic process of the expected value $m_x = 0$ units loading and correlation function $K_x(\tau) = 400\exp(-2,56\tau^2)$. Calculate the value of the lower (P^{min}) estimate of probability $P(a,T)$ not exceedances of the value of the transport potential of the company (the ability to meet the demand for transport services in its entirety) by demand in the time horizon $T = 1, 2, \dots, 15$ (e.g. months).



Figure 6. Graphical presentation of the lower (P^{min}) estimate for probability $P(a,T)$ on the basis of the data of Example 3

Unfortunately, for more complex relationships describing the volatility during the trend of demand for transport services, it is not possible to obtain satisfactory analytical formulas that allow us to estimate the probability of not exceedances by the demand of the transport potential of transport company. In these cases, the general dependence (11) and the dependence to determine the expected number of positive exceedances of the threshold value a by the stochastic process in a unit of time according to the formula $\varphi(t)$ are used. Detailed proposals in this regard are included in [12].

5. Conclusions

Points 3 and 4 dealt the question of the degree of adaptation of its transport potential to the demand in the transport services market which is a practical issue for any transport company. The assessment of this degree of fit allows the company to determine its position on the transport services market within the time horizon the specified by her, assuming that the demand for transport services is also shaped by random factors. Adopting this assumption has led to the need to treat transport demand as a certain stochastic process.

The article proposes that matching the transport potential to transport demand will be measured the value of the probability that demand will not exceed a certain threshold value which corresponds to the transport potential of the transport company. Two main scenarios that may occur in the transport services market are analyzed: first - demand for transport services is constant, second - demand for services has a time-varying trend, described by a real function. With additional assumptions, the demand for transport services in the first case was described by the stationary normal stochastic process CC class, while in the second case by the non-stationary normal stochastic process of the same class. Both cases were dealt with in points 3 and 4 respectively.

In point 3 two detailed cases about the transport potential are discussed. In the first it was assumed that the potential is invariable in time, while in the second that the potential is a random variable. The latter case is closer to reality, because transport potential is largely shaped by random factors such as:

- loss of means of transport due to failure (randomness), technical wear, economic inefficiency, etc.,
- purchase or hire additional means of transport.

The results obtained are illustrated by numerical examples that they are consistent with intuition.

Because of the extent of the problem and the degree of complexity, less attention has been paid to the other case when the demand for transport services is described by the non-stationary stochastic process (point 4). The degree of

difficulty of this issue is so large that failure to obtain for a wide class of non-stationarity detailed analytical expressions to estimate the probability that transport potential of transport company not exceeded by of non-stationary demand. The exhaustive analysis of this problem should include the following cases of reasons for the non-stationarity of demand for transport services, which caused by the existence of a trend described by:

- another function of real time than linear,
- by the random function.

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