

# Scaling laws for the FE solutions of induction machines

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**Abstract:** In this paper a scaling approach for the solution of 2D FE models of electric machines is proposed. This allows a geometrical and stator and rotor resistance scaling as well as a rewinding of a squirrel cage induction machine enabling an efficient numerical optimization. The 2D FEM solutions of a reference machine are calculated by a model based hybrid numeric induction machine simulation approach. In contrast to already known scaling procedures for synchronous machines the FEM solutions of the induction machine are scaled in the stator-current-rotor-frequency-plane and then transformed to the torque-speed-map. This gives the possibility to use a new time scaling factor that is necessary to keep a constant field distribution. The scaling procedure is validated by the finite element method and used in a numerical optimization process for the sizing of an electric vehicle traction drive considering the gear ratio. The results show that the scaling procedure is very accurate, computational very efficient and suitable for the use in machine design optimization.

**Key words:** evolutionary strategy, finite element method analysis, induction machine, induction motor, loss calculation, multi-objective optimization, scaling laws

## 1. Introduction

Energy optimization performed for example by an improvement in the efficiency of electrical equipment is the global trend today [1]. In developed countries the industrial induction machines (IMs) are the major consumers of electric energy and globally account for about 40% of overall power consumption [2].

To lead manufacturers to design and build more efficient IMs the European Union specified the new premium efficiency standard (IE3) for IMs operated at 50 Hz or 60 Hz by the IEC



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60034-30 standard [3]. Since January 1, 2015 this standard is valid for IMs with a rated power of 7.5 to 375 kW and since January 1, 2017 for IMs with a rated power of 0.75 to 375 kW. The standard also reserves an IE4 class for the future.

Due to the low-cost, ruggedness and fault tolerance, frequency-inverter-driven IMs are used as the main workhorse in the rising market of electrical and hybrid drive trains [4]. Here too, however, the main aim is to reduce the energy consumption or, in other words, to improve the efficiency of the drive train and particularly of the IM [15]. Moreover, in electric or hybrid vehicles minimum cost and weight are further goals [15]. Proficient manufacturers have the experience to design high premium efficient electrical machines. But only mathematical optimization can handle the complexity of the relations between the machine's geometry and its performance and nonlinearity [5] in a short time. With the mathematical optimization tool the limits can be pushed toward more efficient designs. For the design of high efficient IMs, particularly for frequency-inverter-driven ones, a detailed loss analysis is required. This requires local and temporal highly resolved nonlinear field computation and can be performed in the post processing of nonlinear transient finite element simulations of the magnetic circuit [16].

For the IM, the finite element method (FEM) takes a large number of simulation time steps to build up the machine's rotor flux matrix [16]. Hence, using the time-consuming FEM in a mathematical optimization procedure would end up in an extremely time-consuming calculation and therefore is not suitable. Von Pfingsten, Nell and Hameyer [6, 16] proposed a hybrid simulation approach for the IMs 2D finite element (FE) calculation that drastically decreases the simulation time by shortening the transient build-up of the rotor flux. Nevertheless, this hybrid approach, that needs about 2 000 core hours for an efficiency map, is still not sufficient for the application in a mathematical optimization procedure.

Due to the fact that the FEM and other numerical methods are very time-consuming scaling laws are a popular method in physics and engineering. They are often used in numerous examples. Wood [7] described the general scaling laws for electromagnetic systems. His work was motivated by the constraint of the system's thermal stability. By using the electromagnetic and thermal diffusion equation, as well as the momentum and kinematic equation Hsieh and Kim [8] presented a detailed derivation of scaling laws for electromechanical systems.

## 2. Scaling laws of electrical machines

Žarko, Stipetič and Ramakrishnan published several papers about the scaling laws for synchronous machines (SM). In [17] and [9] the procedures of radial and axial geometrical scaling and of rewinding for the SM are introduced. In [18] the efficiency maps of the SM are calculated by using a scalable saturated flux linkage and loss model of the SM. In [15] and [19] the scalable SM models are used to find the optimal sizing of a SM traction motor. Moreover, Žarko presented a method to design a premium efficiency IM, using scaling laws for its equivalent circuit parameters in [2]. He assumed that the temperature rise in the slot of the original and the scaled motor are similar.

Another method to improve the IM efficiency classes, using the method to scale the core axial lengthening, was described by Alberti, Bianchi, Boglietti and Cavagnino in [10]. The influence of the rotor diameter and the length on the rating of IMs was presented by Bone in [11]. His scaling

laws for IMs are not as exact as the ones derived in this paper because the field solutions change. Nonetheless, they are well applicable for machine designers. In this paper a more sophisticated approach for a mathematical optimization of squirrel cage IMs is discussed. It is based, on the one hand, on the hybrid simulation approaches for induction machine calculation from von Pfingsten, Nell and Hameyer [6, 16], and on the other hand, on the scaling laws for the IMs proposed in this paper and in [20]. The procedure of this method is illustrated in Fig. 1. The stator-current-slip-frequency-operation-planes ( $I_1$ - $f_2$ -planes) of a reference machine design are calculated with the hybrid simulation approach of von Pfingsten, Nell and Hameyer [6, 16].

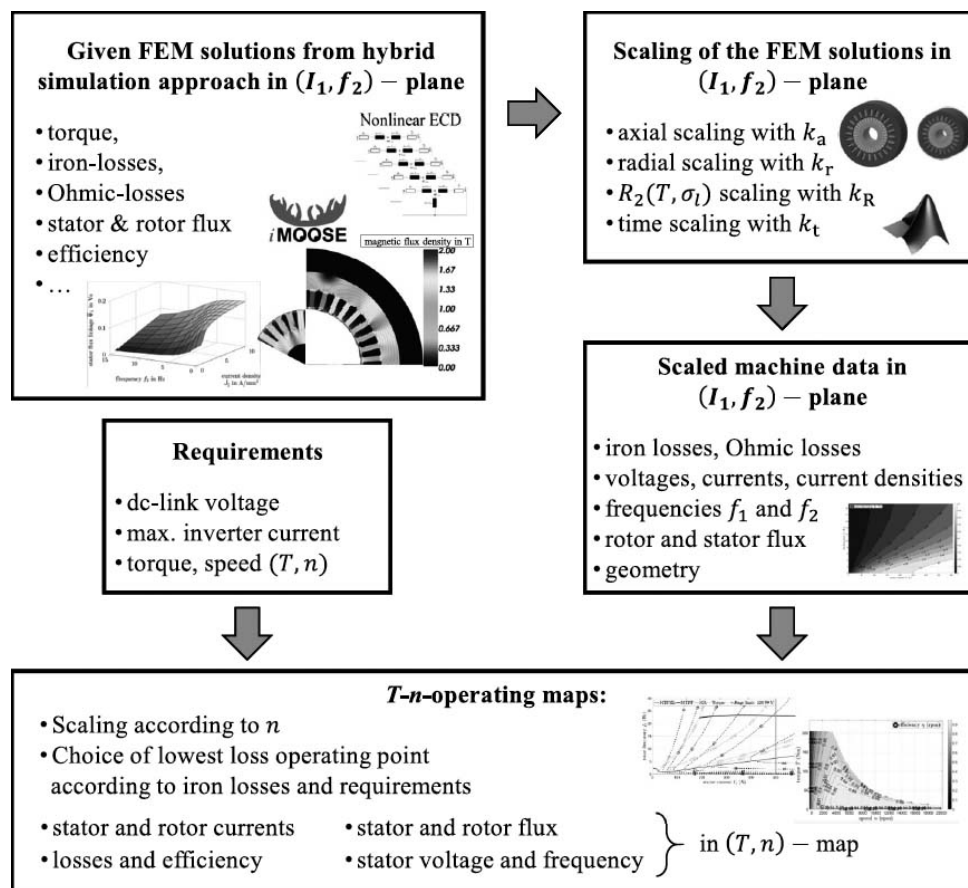


Fig. 1. Machine scaling scheme

To obtain a new scaled motor design the solutions are scaled in the  $I_1$ - $f_2$ -plane without changing the field solution of the IM. Taken into account requirements, such as dc-link voltage, maximum inverter current, and the operation strategy, such as Maximum Torque Per Ampere (MTPA) or Maximum Torque Per Electrical Losses (MTPELs), the  $I_1$ - $f_2$ -planes are transformed to torque-speed-maps ( $T$ - $n$ -maps).

### 3. Modeling of an induction machine

#### 3.1. IM operating points in terms of the $I_1$ - $f_2$ -plane

The fundamental wave T-equivalent-circuit of an IM is presented in Fig. 2. It demonstrates the allocation of the stator current  $I_1$  into the magnetizing current  $I_\mu$  and the rotor current related to the stator side  $I_2^S$ . Moreover, it illustrates that all reactances and the rotor resistance  $R_2^S$  are proportional to the synchronous angular frequency  $\omega_1$ . According to von Pfingsten, Nell and Hameyer in [6, 21] and [16] the allocation of the stator current  $I_1$  into the magnetizing current  $I_\mu$  and the rotor current  $I_2^S$  is independent of the stator frequency  $f_1$  by subtracting the voltage drop on the stator resistance  $R_1$ . As a result, this allocation only depends on the rotor frequency  $f_2$  and the saturation of the main inductance  $L_M$ . The saturation has to be considered in highly utilized traction drives and occurs at high values of  $I_\mu$  that is reached at low values for  $f_2$  and high values for  $I_1$  [21]. Therefore, the current allocation of  $I_1$  into  $I_\mu$  and  $I_2^S$  only depends on the amplitude of the stator current  $I_1$  and the rotor current frequency  $f_2$ , as long as the rotor resistance and the inductances are assumed to be constant. All torque speed operating points of an IM with a constant rotor resistance  $R_2^S$  can be mapped in the  $I_1$ - $f_2$ -plane.

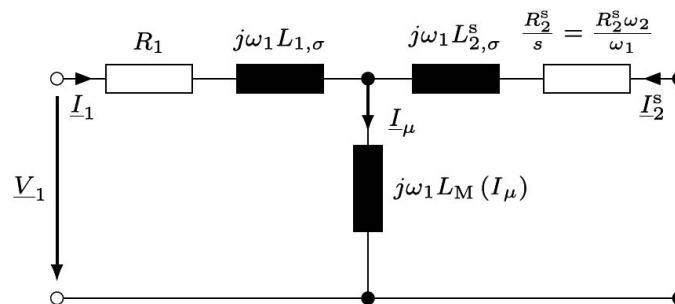


Fig. 2. Equivalent circuit diagram of a squirrel cage induction machine

#### 3.2. Induction machine calculation applying the hybrid simulation approach

To accelerate the FE calculation of the IM, the hybrid simulation approach presented in [6] and [16] is used. With the hybrid simulation approach, the entire  $I_1$ - $f_2$ -plane and T- $n$ -map, respectively, can be calculated 50% faster than with the transient FEM. In combination with the hereafter introduced scaling scheme it provides a fast procedure to calculate and scale IMs that leads to the possibility of the use for numerical optimization. The main aspects of it will be described in the following.

In the first step, a non-linear no-load static finite element analysis (FEA) with one simulation time step is conducted  $k = 1, \dots, K$  times, where  $k$  marks a certain saturation state [6, 16]. The stator current  $I_1$  is the only excitation. For each non-linear no-load FEA the inductance matrix  $\mathbf{L}(k)$  of the IM is extracted in accordance with [12]. From these extracted matrices  $\mathbf{L}(k)$  the rotor current  $I_2^S$  is calculated with the analytical fundamental wave equations derived from the equivalent circuit diagram in Fig. 2 for every saturation state  $k$ . With the stator current vector  $\vec{I}_1$  and the saturation dependent rotor current vector  $\vec{I}_2^S$ , the stator flux linkage vector  $\vec{\Psi}_1(k)$  for every

saturation state  $k$  is calculated. By comparing the amplitude of this stator flux linkage and the no-load stator flux linkage of the no-load FEA  $\vec{\Psi}_{1,nl}(k)$  the valid saturation state is found. With numerical interpolation, the value of the rotor current for each operating point in the  $I_1$ - $f_2$ -plane is found and used as the excitation for the second, now transient, FE simulation [6, 16]. The calculation steps are performed in the  $I_1$ - $f_2$ -plane with the fixed stator frequency  $f_1$  and fixed rotor resistance  $R_2^S$ , as well as the rotor conductivity  $\sigma_2$ , respectively.

By considering an operation strategy such as MTPEL and by scaling the loss power of the IM according to different synchronous speeds  $f_1$ , as described in [4], the  $I_1$ - $f_2$ -plane is transformed to the T- $n$ -map. A variation of the rotor resistance  $R_2^S$  by temperature can be considered with the scaling laws of the rotor resistance due to the temperature variations described in section 4.6.

#### 4. Scaling laws for induction machines

The previous scaling laws for IMs introduced by Bone in [11] are not exact due to the fact that the field solution is changed. The IM scaling laws of Žarko in [2] deal with scaling the IM's equivalent circuit parameters assuming equal temperature rise in the slots of the original and reference motor. For the SM, Stipetič, Žarko and Popescu derived scaling laws that consider the same field solution. This is the basis for the IM scaling laws in this paper. In the following the scaled parameters are marked with ( $'$ ).

##### 4.1. Geometrical scaling

The geometrical scaling in cylindrical systems is subdivided into radial and axial scaling with the radial scaling factor  $k_r$  and the axial scaling factor  $k_a$ . The effect of the geometrical scaling is pictured in Fig. 3 and described with

$$\rho' = \rho k_r, \quad (1)$$

$$l' = l k_a, \quad (2)$$

$$A'_{\text{cross}} = A_{\text{cross}} k_r^2, \quad (3)$$

$$A'_{\text{surface}} = A_{\text{surface}} k_r k_a, \quad (4)$$

$$V' = V k_r^2 k_a, \quad (5)$$

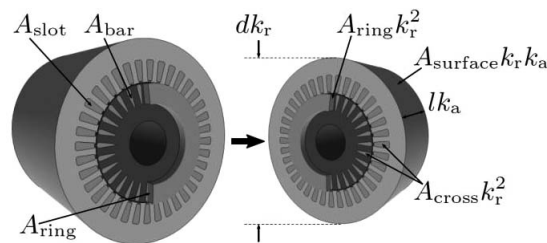


Fig. 3. Scaled machine parameters

where  $l$  is the axial length,  $\rho$  is the radial distance to the center,  $A_{\text{cross}}$  is the cross-section areas,  $A_{\text{surface}}$  is the radial surface area and  $V$  is the volume of the active part of the machine. The cross-section of the short-circuit ring also increases with  $k_r^2$  and is independent of  $k_a$  to preserve a constant relation to the bar cross-section. The conductor cross-section also changes with  $k_r^2$ . For the end windings it is assumed that the conductor length increases quadratically with  $k_r$  since the conductor cross-sections increase, thus the axial extent becomes larger and the arc length increases linearly with  $k_r$ .

#### 4.2. Centrifugal force

The rotation or the angular velocity  $\omega$  respectively of the rotor of an IM causes a centripetal force, orthogonal to the motion and towards the center of the rotor. The centripetal force

$$F_p = m\rho\omega^2 \quad (6)$$

is proportional to the mass of the rotor  $m$  and the radial distance to the center  $\rho$ . The fictitious centrifugal force that is directed away of the rotation axis, is a reaction to the centripetal force. The centripetal force causes a tension

$$\sigma = \frac{m\rho\omega^2}{A}, \quad (7)$$

where  $A$  is the area at which the force acts. To avoid plastic deformations in the materials of the rotor at high rotational speeds the tension in the materials must not exceed the maximum tension  $\sigma_{\text{max}}$ . Therefore, the angular velocity has to be limited to

$$\omega_{\text{max}} = \sqrt{\frac{\sigma_{\text{max}}A}{m\rho}}. \quad (8)$$

With the geometric scaling relations in (4) and (1) and the mass, scaled with  $k_a k_r^2$ , the scaling law for the maximum rotor speed

$$n'_{\text{max}} = n_{\text{max}} \frac{1}{k_r} \quad (9)$$

is derived.

#### 4.3. Scaling of the electrical and magnetic parameters

Due to the fact that the magnetic permeability  $\mu$  is in a non-linear relation to the magnetic field strength  $\vec{H}$ , one assumption of scaling the FE solutions is that the magnetic field strength distribution inside the IM does not change. Therefore,

$$\vec{H}'(\rho', \varphi') = \vec{H}(\rho, \varphi) \quad (10)$$

is applicable.

In accordance with Ampère's law

$$\vec{J}' = \nabla' \times \vec{H}' = \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho'} (\rho' H'_\varphi) - \frac{\partial H'_\rho}{\partial \varphi'} \right] \vec{e}_z, \quad (11)$$

with the polar coordinates  $\rho$  and  $\varphi$ , as well as the unit vector  $\vec{e}_z$  the scaling dependence of the electric current density

$$\vec{J}' = \frac{1}{k_r} \nabla \times \vec{H} = \frac{1}{k_r} \vec{J} \tag{12}$$

follows.

In [8] it is described that a scaling factor  $k_{t1}$  has to be used to satisfy that the magnetic flux density  $B$  is kept constant  $B(\rho, \varphi) = B'(\rho', \varphi')$  and therefore the equality of the magnetic field strength (10) is satisfied. With the assumption that the geometrical dimensions are clearly shorter than the magnetic wave length Ampère’s circuital law with Maxwell’s addition is simplified to Ampère’s law:

$$\nabla \times \vec{H} = \sigma \vec{E}. \tag{13}$$

With Faraday’s law of induction

$$\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \tag{14}$$

and mathematical transformations

$$k_{t1} \frac{\partial \vec{B}'}{\partial t'} + k_r^2 \nabla' \times \frac{1}{\sigma} \left( \nabla' \times \frac{\vec{B}}{\mu} \right) = 0 \tag{15}$$

is derived, where  $\sigma$  is the electric conductivity and  $\vec{E}$  is the electric field strength. The assumption that the magnetic field strength, and therefore, the magnetic flux density do not change during scaling, leads to

$$k_{t1} = k_r^2. \tag{16}$$

Hence, the time scaling factor  $k_{t1}$  corresponds to the square of the radial scaling factor  $k_r$ . As a result, all time depending parameters have to be scaled by the total time scaling factor  $k_t$ , which is the product of the first time scaling factor  $k_{t1}$  and the second one  $k_{t2}$ . The second time scaling factor is a result of the rotor resistance scaling and will be discussed in detail in section 4.5 and 4.6. The time scaling leads to the proportionality of the reactances to the reciprocal time scaling factor shown in Fig. 4.

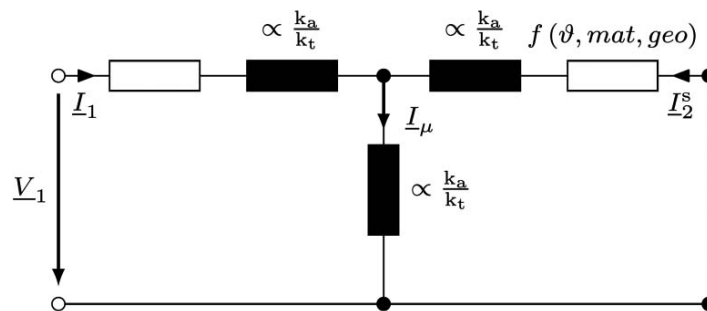


Fig. 4. Scaling factor dependencies of the elements of the equivalent circuit diagram of a squirrel cage induction machine

The magnetic flux  $\Psi$ , which is proportional to the inductance, scaled with  $k_a$  and the current, scaled with  $k_r$ , changes with  $k_a k_r$ :

$$\Psi' = k_r k_a \Psi. \quad (17)$$

From (3) and (12) it follows that the current changes with  $k_r$ :

$$I' = k_r I. \quad (18)$$

The machine's torque  $T_{\text{elec}}$  is dependent on the flux and current. Hence, the torque is scaled in accordance to:

$$T'_{\text{elec}} = \frac{3}{2} p (I_{q,1} k_r \Psi_{d,1} k_r k_a - I_{d,1} k_r \Psi_{q,1} k_r k_a) = T_{\text{elec}} k_r^2 k_a. \quad (19)$$

#### 4.4. Stator resistance scaling

The stator resistance of an IM has to be scaled separately according to the winding head and the axial length of the stator iron  $l_{\text{Fe}}$ , that is independent of the radial scaling  $k_r$  and proportional to  $k_a$ . It is assumed that the conductor length in the winding head  $l_{\text{WH}}$  increases linearly with  $k_r$ , since firstly, the conductor cross-sections increase and thus the axial expansions of the winding head become larger and secondly, the arc length increases linearly with  $k_r$ . The conductor cross-sections  $A_{\text{wire}}$  themselves increase with  $k_r^2$ . This leads to the scaled stator resistance:

$$R'_1 = 2N_1 \frac{l_{\text{Fe}} k_a + l_{\text{WH}} k_r}{\sigma_1 A_{\text{wire}} k_r}, \quad (20)$$

where  $\sigma_1$  is the conductivity of the winding material and  $N_1$  is the number of stator slots.

#### 4.5. Rotor resistance scaling due to geometric variations

The equivalent circuit of the IM in Fig. 2 shows that the rotor resistance related to the stator side  $R_2^S$  and therefore the rotor resistance  $R_2$  has a major impact on the machine's behavior. In contrast to the scaling of the SM done by Žarko, Stipetič and Ramakrishnan in IMs the scaling of the rotor resistance is an important step. In addition to the varied rotor resistance due to the geometrical scaling the resistance can vary because of temperature, material and electric conductivity variations, respectively. The consequence of the geometrical scaling has to be considered separately in terms of the bar resistance  $R_{\text{bar}}$  and the resistance of the short-circuit ring  $R_{\text{ring}}^*$ . The resistance of the rotor bar can be described with

$$R_{\text{bar}} = \frac{l_{\text{Fe}}}{\sigma_2 A_{\text{bar}}} \propto \frac{k_a}{k_r^2}, \quad (21)$$

where  $l_{\text{Fe}}$  is the active length of the IM,  $A_{\text{bar}}$  is the area of the rotor bar and  $\sigma_2$  is the conductivity of the rotor conductors, and it is proportional to  $\frac{k_a}{k_r^2}$ . The resistance of a short circuit ring segment  $\Delta R_{\text{ring}}$  can be described by:

$$\Delta R_{\text{ring}} = \frac{2\pi r_{\text{ring}}}{\sigma_2 A_{\text{ring}} Q_2} \propto \frac{k_r}{k_r^2} = \frac{1}{k_r}, \quad (22)$$



where  $r_{\text{ring}}$  describes the middle radius of the short-circuit ring,  $A_{\text{ring}}$  is the area of the short-circuit ring and  $Q_2$  is the rotor bar number. According to [13] this resistance can be transformed to an equivalent series resistance  $\Delta R_{\text{ring}}^*$  described by

$$\Delta R_{\text{ring}}^* = \Delta R_{\text{ring}} \cdot \frac{1}{\left(2 \sin\left(\frac{\pi p}{Q_2}\right)\right)^2} \propto \frac{1}{k_r}, \quad (23)$$

with the number of pole pairs  $p$ , which is used in the 2D FEM. As outlined in section 4.1 the cross-section area of the short-circuit ring increases with  $k_r^2$  and is independent of the axial scaling. Therefore,  $\Delta R_{\text{ring}}^*$  is proportional to the inverse of the radial scaling factor  $k_r$  as in (23). The addition of the bar resistance and the equivalent series resistance of the short-circuit ring lead to the total resistance of the rotor

$$R_2 = R_{\text{bar}} + 2\Delta R_{\text{ring}}^*. \quad (24)$$

By defining a compensating conductivity

$$\sigma_{2,\text{comp}} = \frac{l_{\text{Fe}}}{A_{\text{bar}} R_2} \propto \frac{k_a}{k_r^2 \cdot k(R_2)}, \quad (25)$$

it leads to a rotor resistance scaling factor

$$k_{R1} = \frac{\sigma_{2,\text{comp}}}{\sigma'_{2,\text{comp}}} = \frac{\frac{1}{R_{\text{bar}} + 2\Delta R_{\text{ring}}^*}}{\frac{k_a}{k_r^2 \left( R_{\text{bar}} \frac{k_a}{k_r^2} + 2\Delta R_{\text{ring}}^* \frac{1}{k_r} \right)}}, \quad (26)$$

which can be simplified to

$$k_{R1} = \frac{\sigma_{2,\text{comp}}}{\sigma'_{2,\text{comp}}} = 1 + \left( \frac{k_r}{k_a} - 1 \right) \kappa_2, \quad (27)$$

with

$$\kappa_2 = \frac{1}{\frac{l_{\text{Fe}}}{\pi_{\text{ring}}} \frac{A_{\text{ring}}}{A_{\text{bar}}} Q_2 \sin^2\left(\frac{\pi p}{Q_2}\right) + 1}$$

as the rotor geometry constant that is defined for the unscaled machine. The scaling factor  $k_{R1}$  describes the variation of the rotor resistance due to a variation in the relation of the axial length and radial length, expressed by  $\frac{k_r}{k_a}$  of the machine. If the relation is kept constant  $k_{R1}$  is equal to one.

#### 4.6. Rotor resistance scaling due to rotor conductivity variations

The scaling of the rotor resistance in accordance to the scaling of the compensating rotor conductivity leads to further possibilities of rotor resistance scaling. The rotor conductivity and

resistance, respectively, can vary due to a variation of the material or by temperature. Hence, a second rotor resistance scaling factor

$$k_{R2} = \frac{\sigma_2}{\sigma_{2,\text{new}}} \frac{1 + \alpha_{\text{new}}(\vartheta_{\text{sim,new}} - \vartheta_{\text{ref,new}})}{1 + \alpha(\vartheta_{\text{sim}} - \vartheta_{\text{ref}})} \quad (28)$$

is introduced. It is dependent on the old and new conductivity,  $\sigma_2$  and  $\sigma_{2,\text{new}}$ , the old and new temperature coefficients  $\alpha$  and  $\alpha_{\text{new}}$ , the old and new reference temperatures  $\vartheta_{\text{ref}}$  and  $\vartheta_{\text{ref,new}}$  and the old and new simulation temperatures  $\vartheta_{\text{sim}}$  and  $\vartheta_{\text{sim,new}}$ . The resulting scaling factor for the rotor resistance can be calculated by the total scaling factor of the rotor resistance  $k_R$  and leads to the scaled rotor resistance:

$$R'_2 = R_2 k_R, \quad \text{with } k_R = k_{R1} \cdot k_{R2} \cdot \frac{k_a}{k_r^2}. \quad (29)$$

In (29) the first part of the total rotor scaling factor  $k_{R1}$  considers a variation in the relation of the axial to the radial length of the machine. The second part  $k_{R2}$  considers a variation in the temperature or the conductivity of the rotor bars and rings. The last part  $\frac{k_a}{k_r^2}$  considers the geometric variation of the rotor bars due to an axial and radial scaling of the IM. Fig. 4 shows that the rotor resistance is dependent on the temperature, material and geometry of the IM. To satisfy (10) the allocation of  $I_2^S$  and  $I_\mu$  must not vary. As long as the rotor resistance is kept constant and the calculation of the machine is done in the  $I_1$ - $f_2$ -plane, as described in section 3.1, (10) is valid. With a variation in  $R_2$  the allocation changes. To ensure the same current allocation, even in the event of a variation in the rotor resistance, the condition that the ratio of the axial to the time scaling factor must correspond to the total rotor resistance scaling factor

$$\frac{k_a}{k_t} \stackrel{!}{=} k_R \quad (30)$$

has to be fulfilled. As a result the time scaling is supplemented by a second time scaling factor

$$k_{t2} = \frac{k_a}{k_R k_{t1}} = \frac{1}{k_{R1} k_{R2}}. \quad (31)$$

Thus, the total time scaling factor results in

$$k_t = k_{t1} k_{t2}. \quad (32)$$

With these rotor scaling factors it is possible to scale the IM due to a variation of the rotor resistance. This variation can be a result of a variation in the conductivity of the rotor conductor by different materials, such as copper or aluminum, or by their different qualities, as well as by an alternating temperature. Differences in the machine's behavior due to a varying quality of the rotor bar material can be calculated very fast with the proposed scaling process. In addition, a variation of the rotor resistance due to the skin effect can be taken into account by using analytical formula to recalculate the rotor resistance. This scaling process can also be used in combination with a thermal model of the IM to simulate the machine in different operating points with varying temperature conditions.

#### 4.7. Scaling the number of turns of the winding

In addition to a geometric scaling of the IM, a variation in the number of turns of the stator winding  $N_W$  can be useful. In an axially extended machine the induced voltage increases with  $k_a$ . As a result, the maximum voltage of the machine is reached for lower speed and thus the corner point is at lower speed. To compensate this, the induced voltage can be reduced by reducing the number of turns of the stator winding. This can be done in the post processing of the FE simulation but a scaling factor of the number of turns of the winding  $k_N$  enables shorter computation times as one complete execute of the post processing procedure. Since the number of turns is an integer number  $k_N$ , it is defined with:

$$N'_W = N_W k_N, \quad (33)$$

$$k_N = \frac{N'_W}{N_W}. \quad (34)$$

The proportionalities for the scaling of the number of turns of the winding are collected in Table 1.

Table 1. Parameter proportionalities for the scaling of the number of turns of the stator winding

Parameter	Variable	$\propto$
Number of turns of stator	$N_W$	$k_N$
Transformation ratio	$a$	$k_N$
Stator wire cross sectional area	$A_{\text{wire}}$	$\frac{1}{k_N}$
Stator current	$I_1$	$\frac{1}{k_N}$
Stator flux density	$B$	$k_N$
Stator resistance	$R_1$	$k_N^2$
Stator copper losses	$P_{L,\text{ohm},1}$	1
Stator voltage	$V_1$	$\frac{1}{k_N}$

#### 4.8. Scaling of the IM losses

The losses of an IM can be distinguished in Ohmic losses  $P_{L,\text{ohm}}$  and iron losses  $P_{L,\text{Fe}}$ . The Ohmic losses of the rotor  $P_{L,\text{ohm},2}$  are proportional to  $k_r^2 k_R$  as in:

$$P'_{L,\text{ohm},2} = P_{L,\text{ohm},2} k_r^2 k_R. \quad (35)$$

The scaled Ohmic losses of the stator  $P'_{L,\text{ohm},1}$  have to be calculated with the scaled stator current  $I'_1$  and the scaled stator resistance  $R'_1$ . This leads to the scaled Ohmic losses of the stator:

$$P'_{L,\text{ohm},1} = 3 \cdot I_1'^2 R'_1, \quad (36)$$

$$P'_{L,\text{ohm},1} = 3 \cdot \left( I_1 \frac{k_r}{k_N} \right)^2 \cdot 2N_1 \frac{l_{\text{Fe}} k_a + l_{\text{WH}} k_r}{\sigma_1 A_{\text{wire}} k_r} k_N^2. \quad (37)$$

The iron losses, in turn, can be separated into hysteresis, eddy current and excess losses [21]. All three parts have a different dependency on the frequency  $f$ . Hence, different scaling relations for the three iron loss components exist. The scaled iron loss power density  $p'_{L,Fe}$  is:

$$p'_{L,Fe} = \left( k_{\text{hyst}} B^\alpha \frac{f}{k_t} + k_{\text{eddy}} B^2 \frac{f^2}{k_t^2} + k_{\text{excess}} B^{1.5} \frac{f^{1.5}}{k_t^{1.5}} \right), \quad (38)$$

where  $k_{\text{hyst}}$ ,  $k_{\text{eddy}}$  and  $k_{\text{excess}}$  are the hysteresis loss, eddy current loss and excess loss coefficient, respectively. With the iron loss power density and the iron mass  $m_{Fe}$ , scaled with  $k_r^2 k_a$ , the scaled iron loss power  $P'_{L,Fe}$  is:

$$P'_{L,Fe} = p'_{L,Fe} m_{Fe} k_r^2 k_a. \quad (39)$$

All scaling laws for the IM are summarized in Table 2.

Table 2. Scaling factors for the machine's parameters

Parameter	Variable	$\propto$
Length	$l$	$k_a$
Lateral surface	$A_{\text{surface}}$	$k_a k_r$
Cross sectional area	$A_{\text{cross}}$	$k_r^2$
Volume	$V$	$k_a k_r^2$
Magnetic field strength	$H$	1
Magnetic field density	$B$	1
Magnetic field linkage	$\Psi$	$k_a k_r$
Current density	$J$	$\frac{1}{k_r}$
Current	$I$	$k_r$
Time	$T$	$k_t$
Frequency	$f$	$\frac{1}{k_t}$
Speed	$n$	$\frac{1}{k_t}$
Torque	$T$	$k_a k_r^2$
Voltage	$V$	$\frac{k_a k_r}{k_t}$
Inductance	$L$	$k_a$
Reactance	$X$	$\frac{k_a}{k_t}$
Rotor resistance	$R_2$	$k_R$
Mechanical power	$P_{\text{mech}}$	$\frac{k_a k_r^2}{k_t}$
Mechanical power density	$p_{\text{mech}}$	$\frac{1}{k_t} = \frac{k_R}{k_a}$

#### 4.9. Validation of the scaling laws

To validate the scaling laws for IMs a reference motor is calculated with the FEM in the first path via the hybrid simulation approach described in section 3.2. In the second path, the geometry and the rotor resistance of the IM are scaled and an FE simulation is performed with the scaled machine. The FE solutions of the scaled IM are rescaled in the  $I_1-f_2$ -plane to the parameters of the reference machine regarding the procedure described in Fig. 1. Finally, the solutions in the  $I_1-f_2$ -plane are transformed to the T- $n$ -map, considering the same requirements. The procedure is shown in Fig. 5. The scaling factors in this simulation are  $k_r = 1.2$ ,  $k_a = 1.1$  and  $k_{R2} = 1/1.05$ .

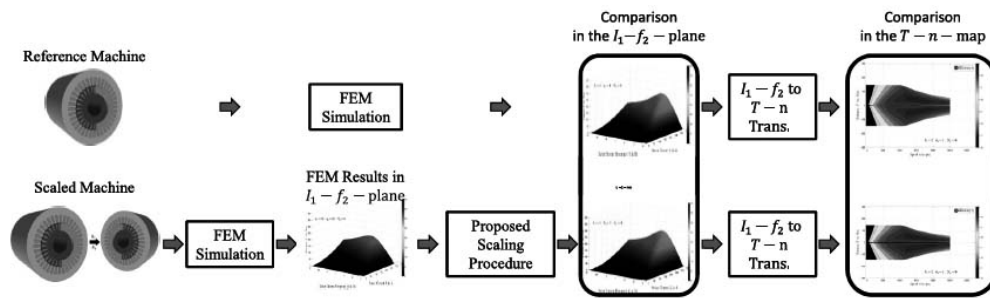


Fig. 5. Validation procedure of IM scaling

The results of the FE solutions of the reference machine (first path) and the rescaled FE solutions of the scaled machine (second path) are compared. The comparison is done in the  $I_1-f_2$ -plane and the T- $n$ -map as described in Fig. 5. The calculated deviation of the loss power in the  $I_1-f_2$ -plane has a maximum error of  $0.2 \cdot 10^{-3}\%$  that proves the correctness of the proposed scaling scheme. The comparison of the reference loss power and the loss power of the IM rescaled in the  $I_1-f_2$ -plane and transformed to the T- $n$ -map is presented in Fig. 6. It also shows a very accurate performance of the proposed scaling procedure.

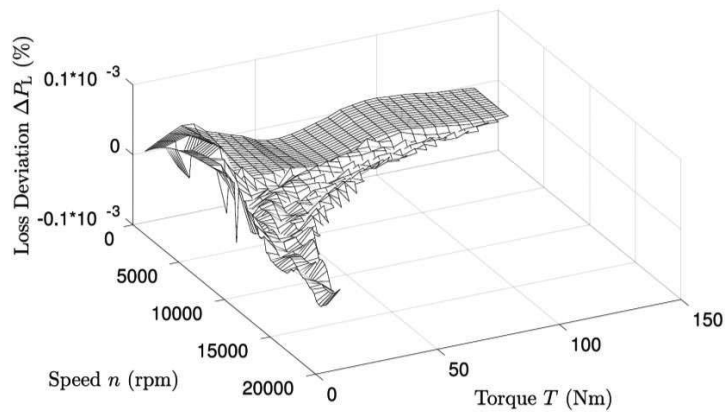


Fig. 6. Total loss deviation of the reference and the rescaled FE solutions in the T- $n$ -map

In Fig. 7 the total losses transformed to the T-*n*-map of a reference IM machine and in Fig. 8 the total losses for a scaled machine, with the scaling factors  $k_r = 1.2$ ,  $k_a = 1.1$  and  $k_{R_2} = 1/1.05$ , are shown. For the transformation to the T-*n*-map the same requirements, such as the maximum frequency, the maximum inverter current and dc-link voltage and the operation strategy MTPEL, are taken into account. The maximum speed of the reference machine is set to 20 000 rpm which results in a maximum surface velocity of the rotor of 110 m/s. By increasing the radial dimension of the IM the speed limit has to be scaled according to (9). Thus, the maximum speed in Fig. 8 is about 16 666 rpm. In Fig. 9 the percentage deviation of the magnetic flux density  $\Delta B$  in % of a reference IM and an IM whose geometrical dimensions are scaled with  $k_r = 1.2$  and whose rotor bar conductivity is scaled with  $k_{R_2} = 1/1.2$  are shown. Here, the input parameter, such as the stator current  $I_1$ , the stator frequency  $f_1$  or the rotor current frequency  $f_2$  are not scaled resulting in a non-constant field distribution. Therefore,  $\Delta B$  reaches values of more than 50%. Fig. 10 shows the percentage flux density deviation in the case that the input parameter are scaled due to the proposed scaling laws in Table 2. The maximum value of  $\Delta B$  is 0.02%.

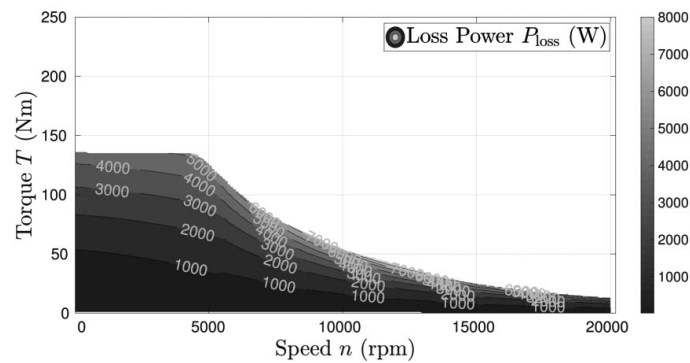


Fig. 7. Total losses of the reference IM machine

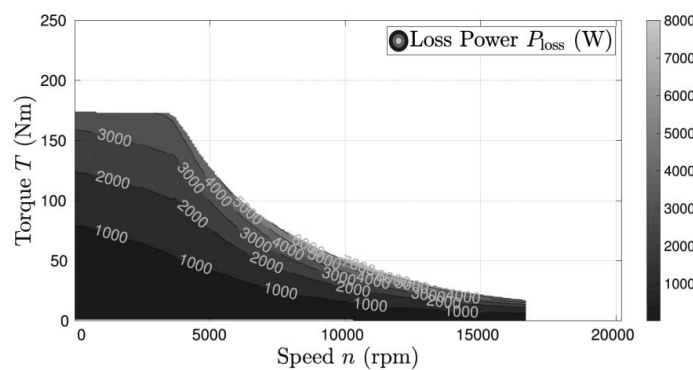


Fig. 8. Total losses of the scaled machine calculated with the scaling laws

It shows that with the proposed scaling laws the assumption of a constant flux distribution is fulfilled. For the calculation of  $\Delta B$  the magnetic flux density  $B$  of the reference machine in each point  $(\rho, \varphi)$  is compared with the magnetic flux density of the scaled machine ( $B'$ ) in each point  $(\rho', \varphi')$  and mapped into the coordinates of the reference machine  $(\rho, \varphi)$ .

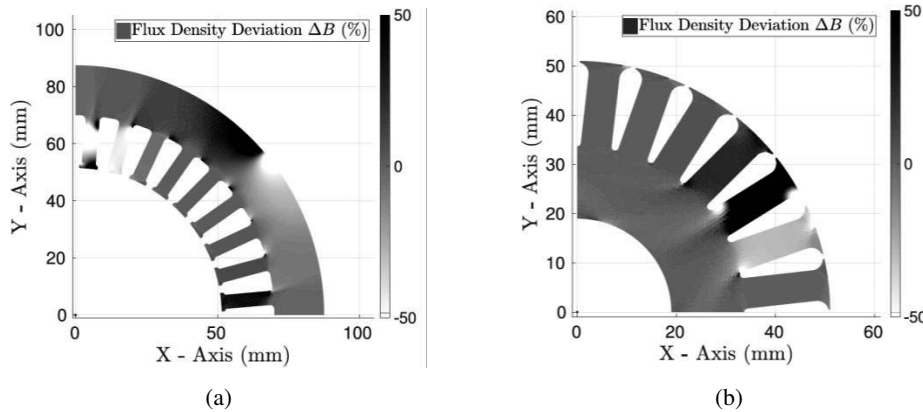


Fig. 9. Deviation of the flux density  $\Delta B$  in % for the stator (a) and rotor (b) of the reference and scaled IM without scaling of electrical parameters

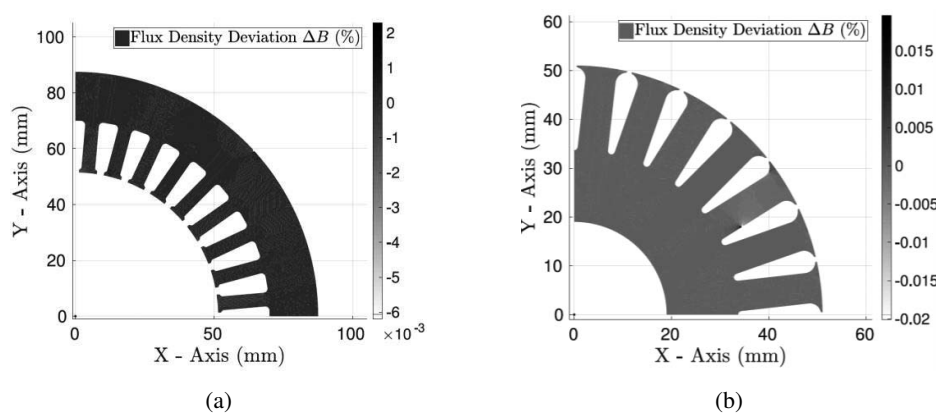


Fig. 10. Deviation of the flux density  $\Delta B$  in % for the stator (a) and rotor (b) of the reference and scaled IM with scaling of electrical parameters

## 5. Optimization of an induction machine for traction application

Due to its very fast performance, the proposed machine scaling scheme can be used in a numerical optimization procedure. In [22] Stipetič and Žarko present an overview of the methodology using mathematical optimization procedures to achieve an optimal design of an electrical machine. They suggest a metaheuristic algorithm, such as Evolution Strategy or Differential Evolution, for the complex electrical machine design. In [19], mixed integer distributed ant colony optimization is used to optimize a traction drive with a permanent magnet motor, calculated by using the FEM and geometrical scaling. In [14] multi-objective optimization in combination with the scaling laws of an SM is used to find the optimal size of the traction motor, as it is done in [15], with the gear ratio as an additional design variable.

### 5.1. Methodology

To show the potential and usability of the proposed IM scaling scheme an IM in an electric vehicle is optimized. Changes of the machine configuration, such as the number of stator slots, are not considered to focus on the radial and axial scaling. The gear ratio, the axial length and the radius of the IM are used as the design parameters. The objective function that is minimized in the optimization process, considers the costs of the machine and the produced loss energy of the IM in the worldwide harmonized light vehicles test procedure (WLTP). Here, the costs of the machine and the loss energy are weighted with different factors. The Evolutionary Strategy is used as an optimization strategy. The procedure of it is shown in Fig. 11.

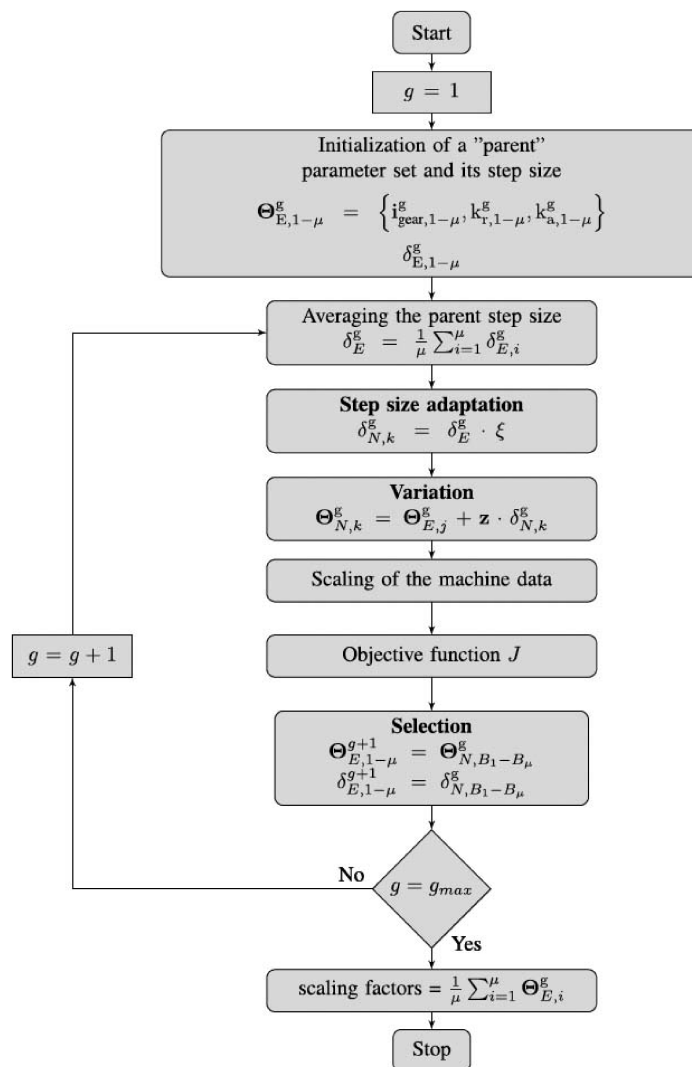


Fig. 11. Evolutionary strategy



It starts with the initialization of the parameter set  $\Theta_{E,1-\mu}^g$  and the step size of the parameters  $\delta_{E,1-\mu}^g$ , where  $\mu$  is the number of the parents and  $g$  the generation number. For the initial parameters the radial and axial scaling factor are set to  $k_r = k_a = 1$  and the gear ratio is set to  $i_{\text{gear}} = 8$ . The initial step size is set to a fourth of the initial parameter values. The step size is averaged in the second step and adapted by a logarithmic distributed random number. With the normal distributed random number  $z$  and the step size the new parameter sets  $\Theta_{N,k}^g$ , named progenies, are calculated in the variation step. With the new parameters the machine is scaled, the gear ratio changed and the objective function is calculated. In the selection the best progenies become the new parents parameter and the calculation of the next generation is started. The process ends after a certain number of generations.

## 5.2. Results

The results of the design optimization with the Evolutionary Strategy is shown in Fig. 12. It shows the machines with different parameters of the radial scaling factor  $k_r$ , the axial scaling factor  $k_a$  and the gear ratio  $i_{\text{gear}}$  that results in the minimum objective function. All different parameter sets are marked with a gray point. The optimum parameter sets are marked by black dots. It can be seen that the optimization algorithm varies the three variable parameters in a wide range and converge fast towards an optimum region. The calculation of the objective function for every single parameter set, including the scaling of the IM's FE solutions, in Matlab takes a processor time of ca. 0.8 s, using an Intel(R) Core(TM) i7-6500U CPU @ 2.5 GHz and an 8 GB RAM.

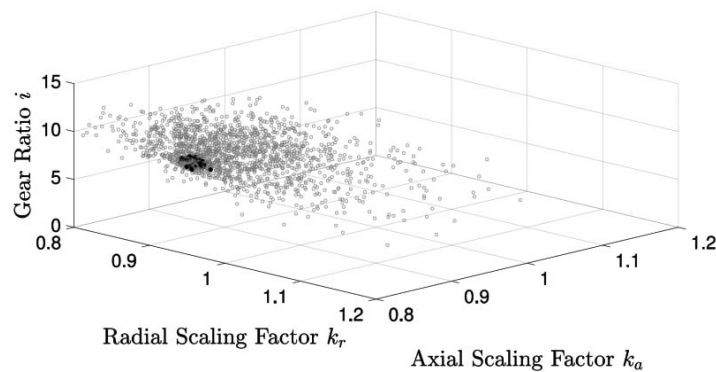


Fig. 12. Pareto front of the proposed optimization process

## 6. Conclusions

In this paper an IM scaling procedure for the machine's 2D FE solutions is proposed. Besides the geometrical scaling of the IM, the scaling laws for variations in the stator and rotor resistance, for the maximum speed and for the number of turns of the stator winding are presented. The reference FE solutions of the IM are scaled in the  $I_1$ - $f_2$ -plane and transformed to the T- $n$ -map, by considering boundary conditions, such as maximum current, and taking into account an operation

strategy. The results of the scaling process show a very good agreement with the FE results of the scaled IM. The scaling procedure is tested in the optimization of a traction drive. The results of the optimization show that the proposed scaling process is suitable for the use in numerical optimization processes.

The rotor resistance scaling can be used to analyze the machine's behavior for different rotor bar materials, their quality differences or different rotor temperatures. It can also be used in combination with a thermal model of the IM to calculate the IM's thermal behavior in different operation conditions. The proposed scaling methodology is a rapid and very accurate tool to scale entire operation maps of IMs. In further work, the scaling laws for variations in the electrical steel due to temperature differences or the quality of the material will be studied. Furthermore, the limits of this method will be analyzed and the scaling will be validated for different machine configurations and designs, such as closed rotor slots. A validation with experimental results will be conducted. The focus of further publications can also be the optimization process itself.

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