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A MULTI-STAGE RISK-ADJUSTED CONTROL CHART FOR MONITORING AND EARLY-WARNINGOF PRODUCTS SOLD WITH TWO-DIMENSIONAL WARRANTY

KARTA KONTROLNA DO WIELOETAPOWEGO MONITOROWANIA PRODUKTÓW SPRZEDAWANYCH Z GWARANCJĄ DWUWYMIAROWĄ, Z KOREKTĄ RYZYKA I WCZESNE OSTRZEGANIE O WADACH PRODUKCYJNYCH NA PODSTAWIE DANYCH Z REKLAMACJI

Warranty claims data contain valuable information about the quality and reliability of products. The monitoring and early-warning of warranty claims data are of great significance to the manufacturer by identifying and solving the emerging quality or reliability problem as soon as possible. However, though it has been used widely in the automobile industry, there are no studies that have been carried out on the monitoring and early-warning of claims data for products sold with two-dimensional warranty. In order to fill this gap, fitting the two-dimensional warranty claims data with accelerated failure model (AFT), a multi-stage risk-adjusted control chart is proposed by this paper, for which a reasonable product sales tracking time and a monitoring time are suggested to reduce the influence of sales delay and fluctuating claim rates. Comparing with traditional Cumulative Sum Control Chart (CUSUM), the applicability and availability of the proposed model are demonstrated in the final.

Keywords: *two-dimensional product warranty, claims data, monitoring and early-warning, multi-stage control chart, accelerated failure model, risk adjustment.*

Roszczenia gwarancyjne stanowią cenne źródło informacji na temat jakości i niezawodności produktów. Monitorowanie danych dotyczących roszczeń gwarancyjnych i wczesne ostrzeganie w oparciu o te dane ma wielkie znaczenie dla producenta, ponieważ pozwala rozpoznawać i rozwiązywać pojawiające się problemy związane z niezawodnością w jak najkrótszym czasie. Chociaż ten rodzaj monitorowania i wczesnego ostrzegania jest szeroko stosowany w przemyśle motoryzacyjnym, nie przeprowadzono dotąd żadnych badań na temat tych procesów w odniesieniu do produktów sprzedawanych z gwarancją dwuwymiarową. W celu wypełnienia tej luki, dane o reklamacjach składanych na podstawie gwarancji dwuwymiarowych dopasowano modelem uszkodzeń przyspieszonych (accelerated failure model, AFT), a następnie przedstawiono koncepcję karty kontrolnej monitorowania wieloetapowego z korektą ryzyka, dla której zaproponowano odpowiedni czas śledzenia sprzedaży produktu i czas monitorowania, mając na uwadze zmniejszenie wpływu opóźnień w sprzedaży i wahań liczby roszczeń zgłaszanych z tytułu gwarancji. Możliwości zastosowania i dostępność proponowanego modelu porównano z tradycyjną kartą sum skumulowanych.

Słowa kluczowe: *dwuwymiarowa gwarancja na produkt, dane o roszczeniach z tytułu gwarancji, monitorowanie i wczesne ostrzeganie, karta kontrolna procesu wieloetapowego, model przyspieszonego uszkodzenia, korekta ryzyka.*

1. Introduction

In today's competitive market, offering attractive warranty service has been used by many manufacturers to capture more market share and customers' satisfaction. Warranty is a contractual agreement between the manufacturer and the customer, which specifies the manufacturer's obligation in the event that the product is unable to perform satisfactorily when properly used [1]. Based on the number of variables used to define warranty coverage, warranty policies can be broadly divided into two categories: one-dimensional warranty and two-dimensional warranty [2]. One-dimensional warranty is usually defined on the basis of age or usage, while two-dimensional warranty takes age and usage or the potential interaction between them into consideration. In practice, two-dimensional warranty has been widely applied in automobile industry. For example, a new automobile is usually sold with a two-dimensional warranty, offering free repair for 3 years or 60000 km, whichever occurs first.

With the intensification of market competition, the percentage of warranty cost to the profit of manufacturers is gradually increasing. For the US automotive industry, manufacturers spend roughly \$10 billion–\$13 billion per year on warranty claims, consuming roughly 1%–5.2% of the product profit [3]. Among various factors affecting warranty cost, product reliability and quality control have attracted significant attention from practitioners and academics. So far, reliability design, quality control management, and many other reliability technologies have been widely used in the design and development stages of products, which have resulted in great contributions to improve product reliability and quality [4]. However, many reliability or quality issues still occur during the product warranty period, such as unknown failure modes, unanticipated changes in operating environments, unknown changes in raw materials, etc., which may result in huge warranty costs. As warranty data contain valuable information about the quality and reliability of products, the use of appropriate statistical detection rules in a warranty database has the potential to identify or warn serious reliability problems long before they would otherwise be discovered [5]. Detecting these problems, a month or even a week earlier, can effectively reduce tangible or intangible costs for the manufacturers, especially for products sold with two-dimensional warranty, such as automobile industry, the warranty cost of which is relatively expensive.

The research about warranty data analysis have been classified into five different directions by Wu [5], including identifying early warnings of abnormalities in their products, providing useful information for product modification and improvement; estimating and explaining the costs of warranty claims; predicting failure claims and warranty costs, and estimating product reliability [4-7]. With the aim to provide early indications of unexpected quality or reliability problems for the manufacturer, the monitoring and early-warning of warranty claims data have become the most important aspects for the research about warranty data.

In general, the model for monitoring and early-warning of warranty claims data can be divided into two stages: first, it is to analyze and fit warranty claims data with parameter model or non-parameter model; next, the control chart for monitoring warranty claims data is proposed to balance Type-1and Type-2 error in the monitoring models [8].As far as we know, most of the monitoring and early warning research have only considered the product sold with one-dimensional warranty, for which the impact of correlation time on the claim rate is conspicuous. Therefore, in the first stage of modeling, claims data are always hierarchically aggregated according to product production time, product sales time and product usage time [3,4,9], the reason/objective for doing this is to analyze and model the influence of product correlation time on product claim rate clearly,then the detecting and tracing analysis for abnormal conditions can become more easily. However, for product sold with two-dimensional warranty, the warranty claim rate is assumed to be affected by the time and usage of a product simultaneously, so fitting warranty claims data only with correlation time cannot meet the need for monitoring of products sold with two-dimensional warranty, which deserves further research.

In the second stage, for product sold with one-dimensional warranty, a variety of statistical techniques, theories and methods, such as Shewhart control chart, Cumulative Sum Control Chart (CUSUM), change-point theory, artificial intelligence algorithms, and many other tools have been proposed for monitoring and early warning of warranty claims data [3,4,9,10,15]. For example, based on change-point theory, a monitoring model is provided by Karim [9] to detect whether the product reliability has been changed and record the time and the form it happened. Classifying and fitting warranty claims data by production time, sales time and usage time, a multi-stage control chart is proposed by Wu and Meek [4] to monitor claims data month by month, which has laid the foundation for the research of warranty claims data monitoring. To improve the monitoring capability of the above models, the traditional Cumulative Sum Control Chart (CUSUM) is suggested by Lawless [10] to monitor warranty claims data subsequently. At present, monitoring warranty claims data according to the correlation time independently can lead to many control charts working at the same time, which may result in poor robustness and high false alarm rates. Furthermore, when the quantity of products monitored at different stages or types is small, the over classification of warranty claims data may result in many inefficient monitors, which have also restricted the applicability of the control chart.

No studies have been focused on the monitoring and early-warning of products sold with two-dimensional warranty ,however, the monitoring of one-dimensional warranty claims data with different covariates in the previous studies have built the foundation for it. For instance, considering supply chain quality information as key covariates, a multi-stage monitoring model with cox proportional hazard (PH) model is proposed by Zhou [3]to monitor the warranty claims data with a relatively high claim rate. Taking into account some qualitative factors, such as automobile type, warranty service area, seasonal factors and so on, a warning threshold for the monitoring of warranty claims data is obtained with Artificial Neural Networks (ANNs) and analytic hierarchy process (AHP) by Lee [10]. Subsequently, this model has been extended by Na [11] to monitor warranty claims data with both qualitative and quantitative influencing factors. For product under uncertain service environment, a fuzzy feedback control method is proposed by Lee [13] to adjust the warning threshold according to the environmental implication.

The pertinent literatures have been reviewed above. For monitoring and early-warning of products sold with two-dimensional warranty, the influence of product usage rate on warranty claims data has to be considered. This is the main difference between monitoring one-dimensional and two-dimensional warranty claims data. In this study, on the condition of fitting two-dimensional warranty claims data with Accelerated failure time model (AFT), a multi-stage risk-adjusted control chart is provided for the monitoring and early-warning of two-dimensional warranty claims data. Firstly, considering the information can be used at different stages, risk adjustment has been carried out to allocate the false rate probability. Secondly, to reduce the effect of sales delay and fluctuating claim rates on the monitoring capability, a reasonable limit of product sales tracking time and monitoring time is suggested in this study. Finally, comparing with traditional CUSUM control chart, the availability and applicability of the proposed control chart are demonstrated by a case study.

2. Modeling two-dimensional warranty claims data

2.1. Introduction of warranty data

Warranty data are composed of claims data and supplement data [5]. Claims data are the data collected during the servicing of claims under warranty, and supplement data are additional data (such as production and marketing related, warranty cost, items with no claims, etc.) that are needed for effective warranty management. Both claims data and supplement data are usually collected according to the fail-

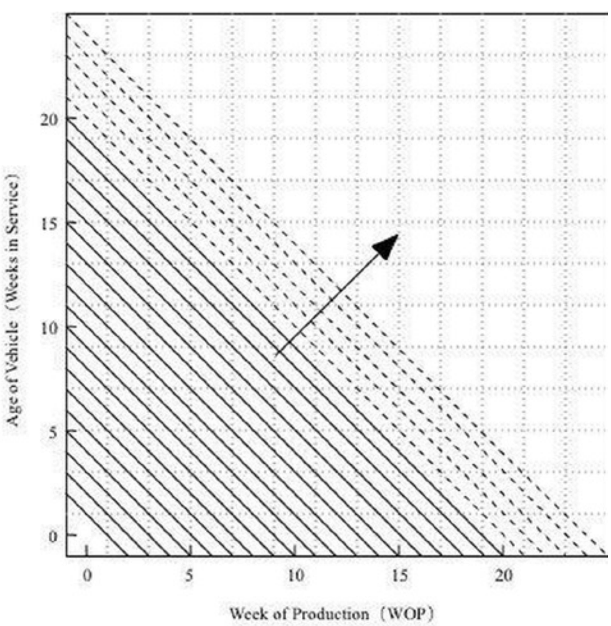

Fig .1.Warranty claim quantity for different monitoring stages ure mode of products, which can provide valuable information about product quality and field reliability.

As we all know, the production and sales time of products are continuous for the manufacturer, and it is necessary to record and monitor the warranty claims data at different stages continuously, such as weekly or monthly, so as to detect the abnormal conditions of claims data as soon as possible. For a specified monitoring period or stage, the monitoring index of warranty claims data can be the whole claim quantity of products, the whole warranty cost of products, or the claim quantity for a specified failure mode, and so on. All of them can provide useful information to the manufacturer. In this paper, we choose the claim quantity for a specified failure mode as the monitoring index, which can also be extended by other options. As shown in Fig 1, the claim quantity of products that can be collected and monitored at different stages is changing gradually.

2.2. Warranty claims data analysis

For products sold with two-dimensional warranty, the claim rates of them are also affected by product correlation time, such as production time, sales time and service time. So, it is still necessary to classify warranty claims data according to correlation time. Considering one week as the smallest unit of time, it is supposed that products are produced and sold at the first week or stage, as shown in Fig 2. N_i is the quantity of products produced in stage *i*; N_{ij} is the quantity of products produced in stage *i* and sold in stage *j*; R_{ijk} is the quantity of products produced in stage *i*, sold in stage *j*, and claimed in stage *k*, where *j* and *k* represent the relative sales stage and service stage, respectively. For example, R_{111} represents the claim quantity of products that occurred during the first stage.

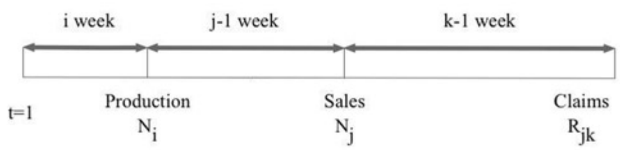

Fig. 2. Production, sale, and services schedule

In addition to the correlation time, the usage rate is also a main factor influencing the claim rate ofa product sold with two-dimensional warranty. In order to model the claim rate of a product in terms of its age and usage, three different approaches have been proposed for modelling claims data under two-dimensional warranty, including marginal approach, bivariate failure distribution approach and composite scale approach [14]. Readers can refer to [21] for a brief review of the recent publications on these approaches. For the current study, Accelerated failure time (AFT) and proportional hazard (PH) models, as well-known variations of the marginal approaches, are widely used to model the effect of customer usage rate on the claim rate of products [25]. In most recent researches, Iskandar and Jack [22], Jack et al. [23], and Baik and Murthy [24] have used AFT model to investigate the effect of usage rate on the claim rate of products, which would also be followed in this study.

Assume X_d with cumulative distribution function $F_d\left(x:\alpha_0,\beta\right)$, is the time to the first failure of the product under the nominal usage rate r_0, α_0 and β are the scale parameter and shape parameter of the distribution model of X_d, respectively.In the marginal approach, it is assumed that the usage rate of a given customer over the warranty period is constant but it varies randomly from customer to customer. Suppose r is the usage rate (random variable), and $g(r)$, $G(r)$ are its probability density function (PDF) and cumulative distribution function(CDF) respectively. According to the AFT model, for a specified usage rate r, the cumulative distribution function of the time to the first claim of the product X_r,will be:

$$\mathrm{F}\left(\mathrm{x}:\alpha\left(\mathrm{r}\right),\beta\right)=F_D\left(x:\alpha_0\left(\frac{r_0}{r}\right)^{\gamma},\beta\right) \tag{1}$$

Where γ≥1 is the accelerating failure factor. The nominal usage rate r_0 is also defined as experimental utilization rate in the model of AFT[24], which is often set as an average usage level in practice, then:

$$r_0=\int_0^{+\infty} rg\left(r\right)dr \tag{2}$$

Suppose $z_D\left(x\right)$ and $z_r\left(x\right)$ are the product hazard rate functions for the nominal usage rate r_0, and a random usage rate r, respectively, they can be calculated by:

$$z_D\left(x\right)=\frac{dF\left(\mathrm{x}:\alpha,\beta\right)/dx}{1-F\left(\mathrm{x}:\alpha,\beta\right)}$$
$$z_r\left(x\right)=\frac{dF\left(\mathrm{x}:\alpha\left(\mathrm{r}\right),\beta\right)/dx}{1-F\left(\mathrm{x}:\alpha\left(\mathrm{r}\right),\beta\right)} \tag{3}$$

3. The multi-stage risk-adjusted control chart

In this section, a multi-stage risk-adjusted control chart is provided to monitor the two-dimensional warranty claims data. Firstly, the proposed multi-stage control chart means monitoring the warranty claims data at different stages independently and continuously, such as weekly or monthly. The actual claim quantity, the theoretical distribution function of warranty claims data at different stages are analyzed elaborately. Secondly, considering the information available at different monitoring stages, risk adjustment has been considered to allocate the overall false rate of the multi-stage control chart, which can balance quick detection and the overall probability of detection for potential reliability problem. Then, the monitoring thresholds of the proposed control chart are calculated subsequently. Finally, in order to reduce the influence of sales delay and fluctuating claim rates on the monitoring capability of the control chart, reasonable product sales tracking time and monitoring time are suggested in this study.

3.1. Monitoring index

As mentioned above, the production and sales of products are continuous to the manufacturer. Monitoring warranty claims data independently according to different production time, sales time and service time can result in many issues, such as high false alarm rate, poor robustness, inefficient monitor, and so on. In this study, for a specified monitoring stage, the claim quantity of products with different production and sales time are monitored as an overall monitoring index, which can avoid the problems mentioned above largely. Suppose M is the prespecified number of stages for monitoring, taking monitoring stage l as an example, the actual warranty claim quantity of products monitored at stage l is:

$$\mathrm{o}_l=\sum_{i+j+k-2=l} R_{ijk},\forall i+\left(j-1\right)+\left(k-1\right)\le M \tag{4}$$

As the monitoring and early warning of warranty claims often start at the sales time of products, and stop long before the deadline of warranty time limit, there is no need to consider the extreme case that the product under monitoring will exceed the warranty period because of product usage.

Subsequently, the theoretical distribution function of warranty claims data at different stages need to be estimated. In order to do this, the maintenance strategies applied for product claims during product warranty period have to be considered. According to degree of restorability of the product, Pham and Wang [17] classified the maintenance actions during warranty into three main categories, including minimal repair, imperfect repair and perfect repair. Minimal repair means restoring the product or system to its as-good-as old operation condition [18-20]. For a complex system as automobile, the claim of it may be caused by failures of one or few components, and simply repair or replacing these items would not significantly improve or reduce the system reliability. Therefore, the application of minimal repair for products such as automobile is an appropriate assumption. At present, minimal repair strategy has been widely used in the automotive industry, electronic products, and other complex systems.

Assuming minimal repairs with negligible repair times applied for warranty claims, then, the claim quantity up to the time t will constitute a non-homogenous Poisson process (NHPP), the intensity function of which is the same as the hazard function of the time to the first failure of the product. Thus, as given in eq. (3), for a given probability density function (PDF) of product usage rate, $g(r)$,the expected claim quantity at different service stage are:

$$\gamma_k^0 = \int_{r_{min}}^{r_{max}} g(r) \int_{k-1}^{k} z_r(x) dx dr, k = 1,2\ldots \quad (5)$$

Where r_{min} and r_{max} are the minimum and maximum usage rates for customer respectively. Then, the theoretical claim quantity at different monitoring stages will also follow Non-Homogeneous Poisson Process (NHPP) with different hazard rate functions. According to this, we can calculate the quantity of theoretical product warranty claims e_l in monitoring stage l , which is:

$$e_l = \sum_{i+j+k-2=l} N_{ij} * \gamma_k^0,\ \forall i + (j-1) + (k-1) \le M \quad (6)$$

As any failures occurring during the warranty period will result in one claim and maintenance with minimal repair, the claim quantity at different monitoring stages will follow NHPP independently. Therefore, the method proposed in this study can monitor the claim quantity of products at different monitoring stages independently.

3.2. Risk adjustment for the multi-stage control chart

For the multi-stage control chart proposed in this study, the monitoring at different stages are working independently. However, as the valuable information collected and monitored at different stages are not the same, it is unreasonable to allocate the false alarm probability and power of detection identically. To deal with this problem, risk adjustment has been considered, for which the overall false alarm rate of the multi-stage control chart is allocated according to the available information at different monitoring stages.

Define α_l as the false alarm rate at the l monitoring stage, $l \in [1, M]$. As the monitoring of claims data at different stages is independent, the overall false alarm rate for the multi-stage control chart up to stage M is given by:

$$\alpha = 1 - \prod_{l=1}^{M} (1 - \alpha_l) \quad (7)$$

For a given overall false alarm rate α, increasing M, the prespecified number of monitoring stages, will lead to the decrease of the false alarm rate allocated at different monitoring stages. But meanwhile, the early-warning ability, which means the detection ability for abnormal conditions of the control chart, has been increased. Therefore, M should be set properly to balance the false alarm rate and the early-warning ability.

The available information at different stages has an important influence on the balance between the false alarm rate and the early-warning ability. In this paper, the expected warranty claims e_l , is used to represent the information available at stage l, then the false alarm rate is allocated proportionally to the expected claims at different stages, that is:

$$\alpha_l = \eta * \sum_{i+j+k-2=l} N_{ij} * \gamma_k^0 = \eta * e_l \quad (8)$$

η is a constant to ensure that $1 - \prod_{L=1}^{M} (1 - \alpha_L) = \alpha$, which can be calculated by interpolation method. Risk adjustment can not only effectively reduce the high false alarm rate caused by the limited information in the early period of the monitoring, but also enhance the robustness of the proposed monitoring model. According to the allocation of false alarm probability, the monitoring threshold C_l of the monitoring stage l can be calculated by distribution parameter $\lambda = e_l$ and the false alarm rate α_l, which is given by:

$$\mathrm{P}\left(O_l > C_l | O_l \sim \mathrm{P}(e_l)\right) = \alpha_l \quad (9)$$

In other words, the monitoring threshold C_l is the α_l upper quartile for the NHPP distribution, with parameter $\lambda = e_l$ .Because of the independence of the monitoring at different stages, the overall false alarm probability for the proposed model up to stage l is given by:

$$\alpha_{2l} = 1 - \prod_{k=1}^{l} (1 - p(o_l > c_l)) \quad (10)$$

It should be noted that α_l is the false alarm rate at stage l, while α_{2l} is the overall false alarm rate up to stage l, which may be easily confused. The response time of abnormal conditions of the proposed control chart cannot be accurately predicted. In this study, $T= l$ is define as the early warning response time, up to when the overall alarm probability for the abnormal condition is just over or equal to 50%. Comparing the value of T, the monitoring capability of different control charts can be analyzed and compared.

3.3. Product sales tracking time and monitoring time

The production and sales of products are continuous to the manufacturer, and most of them have a time delay, which could be several months or even up to years, from production to sales. For example, the average sales delay of automobile is about 25 weeks, which implies that the automobiles will be completely sold out in about 25 weeks after their production. For the monitoring model proposed by this study, it is suggested that products of different production and sales time monitored as a whole index at different monitoring stages. If the reliability of products starts to exhibit abnormalities after some certain production stages, owing to sales delay, it is not obvious in the early monitoring stages because of the small proportion of these products, as shown in Fig 1. Then, the initial reliability variance will difficult to be detected as soon as possible. To deal with this problem, a reasonable limit of product sales tracking time in the proposed model is necessary in this study.

In addition, the products may have the same claim rate at different stages during the product warranty, then the theoretical claim quantity at different monitoring stages is only dependent on the quantity of products monitored, which is quite simple and will not be expanded in this study. More commonly, for product such as automobile, its claim rates at different stages are not the same, which is often fitted with Weibull distribution or lognormal distribution. For this circumstance, a long monitoring time of the products will result in poor detection power of abnormal conditions for the newly listed products, which is caused by the relatively small proportion of them under monitoring. Therefore, it is also necessary to limit the monitoring time for the products according to its claim rate function.

Suppose A and B to be the limit of product sales tracking time and monitoring time respectively. Taking monitoring stage l as an example,the actual claim quantity and expected claim quantity will be adjusted to:

$$o_l = \sum_{\substack{i+j+k-2=l \\ j\le A, k\le B}} R_{ijk}$$

$$e_l = \sum_{\substack{i+j+k-2=l, \\ j\le A, k\le B}} N_{ij} * \gamma_k^0$$

$$\forall i + (j-1) + (k-1) \le M \tag{11}$$

A limit of both product sales tracking time and monitoring time willnot only improve the early-warning ability of the proposed model, but also facilitate the process of data collection and modeling effectively.

4. Case study

In order to illustrate and test our proposed monitoring model for products sold with two-dimensional warranty, a case study is carried out in this section by using the warranty data of an automobile manufacturing enterprise in China. Firstly, the main parameters of the proposed model are set or estimated from the enterprise in subsection 4.1. Secondly, through analyzing the response time of the proposed model, the validity and applicability of the proposed monitoring model have been demonstrated. Thirdly, comparing the response time for abnormal conditions, it has been verified that a reasonable combination of A and B can effectively improve the early-warning ability of the proposed control chart. Fourthly, comparing with CUSUM control chart, the monitoring capability of the proposed model has been proven in subsection 4.4. Finally, an actual failure mode has been monitored by both CUSUM control chart and the proposed control chart in this study.

4.1. Parameter estimation

Parameter estimation for Accelerated failure time (AFT) and probability density function (PDF) of product usage rate have been elaborately studied in literature [7, 19], which isn't described in detail in this study. Considering one week as the smallest unit of time for the independent stage, the prespecified number of stages for monitoring is M=40, product sales tracking time is $A = 20$, and monitoring time is $B = 20$ respectively. The overall false alarm rate up to stage M is 0.05, and the sales of automobiles within 20 weeks is uniform distribution over $[4500,5500]$.Besides, the usage rate r is also uniform distribution over $[0.5,3.5]$. For the nominal usage rate $r_0 = 2$, the time to the first failure of product is Weibull distribution with dimension parameter $\alpha_0 > 0$ and shape parameter $\beta > 0$, then the cumulative distribution function (CDF)of the time to the first failure is:

$$F_d(x:\alpha_0,\beta) = 1 - e^{-(x/\alpha_0)^\beta} \tag{12}$$

With the accelerating factor γ=1.2, which is calculated through AFT. The hazard function of the time to the first failure for product with nominal usage rate r_0 and actual usage rate r are calculated respectively, which are:

$$z_D(x) = \beta(\frac{1}{\alpha_0})^\beta t^{\beta-1}$$

$$z_r(x) = \left(\frac{r}{r_0}\right)^{\gamma\beta} z_D(x) \tag{13}$$

4.2. Model applicability analysis

For Weibull distribution with shape parameter β, when β > 1, the hazard rate function is increasing with time, while when 0<β> 1, the hazard rate function is decreasing [18]. In other words, when $\beta \ne 1$, the product claims data in the whole warranty obey non-homogeneous Poisson distribution (NHPP). In order to test the monitoring capability of the proposed model for different hazard rate function, two different hazard functions, which are α_0=200, β=2, and α_0=12500, β=0.5, are chosen to analyze the response time of the proposed control chart. For the first instance, the allocation of false alarm probability and the

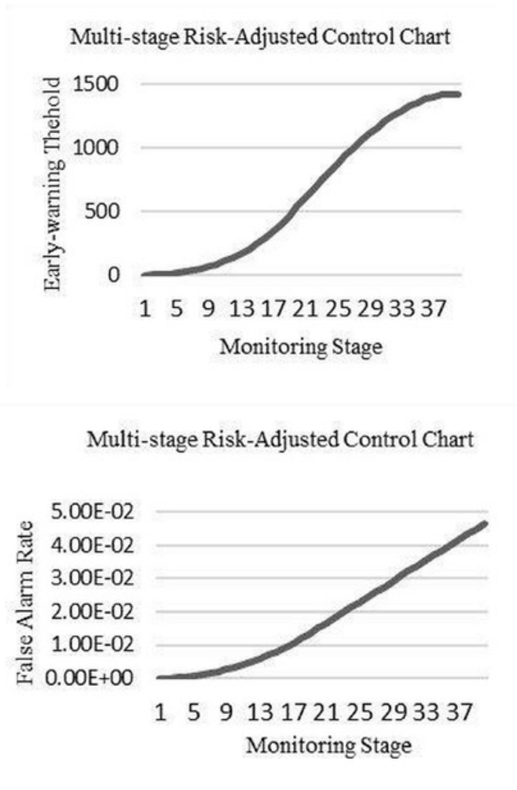

Fig. 3. Early-warning threshold and false alarm rate of each monitoring phase

Table. 1. Early-warning ability of two different distributions

stage	α 0=200, β=2			α 0=12500, β=0.5		
	1.2	1.3	1.4	1.2	1.3	1.4
10	0.003232	0.003236	0.003241	0.005497	0.006575	0.008152
12	0.004999	0.00511	0.005227	0.030431	0.07723	0.173059
14	0.008096	0.008862	0.00976	0.179185	**0.515428**	**0.852389**
16	0.01332	0.016624	0.021111	**0.587684**	0.967233	0.99982
18	0.0249	0.038204	0.059221	0.939524	0.99997	1
20	0.048138	0.091951	0.168141	0.998978	1	1
22	0.099147	0.227474	0.436915	0.999999	1	1
24	0.20822	**0.503929**	**0.81835**	1	1	1
26	0.420305	0.844438	0.990402	1	1	1
28	**0.720033**	0.989995	0.999988	1	1	1
30	0.944992	0.999972	1	1	1	1
32	0.997092	1	1	1	1	1
34	0.999975	1	1	1	1	1
36	1	1	1	1	1	1
38	1	1	1	1	1	1
40	1	1	1	1	1	1

threshold for different monitoring stages are calculated with $z_r(x)$ and α_l by eq. (7) and eq. (8), as shown in Fig. 3.

Assuming that the warranty claim rates of products produced after 10 weeks has become 1.2-1.4 times of the original standard. The response time for these abnormal conditions of the proposed model is analyzed in table 1

As shown in table 1, the theoretical alarm probabilities for abnormal conditions at monitoring stages from 10 to 40 have been calculated. It is obvious that the distribution of warranty claim rate has an important influence on the response time of the proposed model, which should to be considered in the design of the proposed model. Through timely and effectively detecting the claim rate change occurring during monitoring stages, the experimental results show that the proposed control chart can be applied to the monitoring and early-warning of product claim rate with different distribution function.

4.3. Parameter sensitivity analysis

In order to test the influence of product sales tracking time A and monitoring time B on the monitoring capability of the proposed model, various parameter combinations of A and have been considered for monitoring the first example mentioned above, the monitoring results are shown in table 2.

Table 2. Results of Early-warning Ability under different A and B

Monitoring Stage	α 0=200, β=2				α 0=12500, β=0.5			
	B=20		B=10		B=20		B=10	
	A=10	A=20	A=10	A=20	A=10	A=20	A=10	A=20
10	0.005896	0.003232	0.005896	0.003232	0.008942	0.005497	0.008942	0.005415
12	0.008655	0.004999	0.008986	0.004896	0.043345	0.030431	0.043449	0.029906
14	0.012558	0.008096	0.012866	0.0079	0.249646	0.179185	0.261572	0.187031
16	0.018755	0.01332	0.020244	0.014252	**0.745238**	0.587684	0.786577	0.618208
18	0.030785	0.0249	0.039151	0.030662	0.990549	0.939524	0.996431	0.958025
20	0.055415	0.048138	0.101427	0.074181	0.999992	0.998978	1	0.999678
22	0.11061	0.099147	0.248367	0.166329	1	0.999999	1	1
24	0.23438	0.20822	0.477935	0.312786	1	1	1	1
26	0.475417	0.420305	**0.70817**	**0.505569**	1	1	1	1
28	**0.781122**	**0.720033**	0.860335	0.709367	1	1	1	1
30	0.967884	0.944992	0.934663	0.878262	1	1	1	1
32	0.99892	0.997092	0.969434	0.966794	1	1	1	1
34	0.999992	0.999975	0.985701	0.994366	1	1	1	1
36	1	1	0.993311	0.999376	1	1	1	1
38	1	1	0.996871	0.999947	1	1	1	1
40	1	1	0.998536	0.999996	1	1	1	1

As can be seen in table 2, for abnormal conditions occurred after 10th stage, the alarm probabilities of the 10th–30th monitoring stages have been calculated. The experimental results show that different combinations of A and B can effectively influence the alarmability of the proposed model. Firstly, the small value of can reduce the risk of a weak early-warning ability caused by sales delay,but it also need sufficient data to improve the confidence level of the proposed model. Therefore, we set the time when sales percentages of products up to 80% as the value ofin this study, which not only ensures that there is enough warranty data, but also greatly reduces the deficiency of the weak warning ability caused by sales delay. Secondly, the value of has the same influence on the early-warning ability of the proposed model, the choice of which depends on the hazard rate function of the product during the warranty period. When the product warranty claim rates decrease throughout the warranty period, it can effectively improve the early-warning ability to shorten the monitoring time. On the contrary, when the product claim rates increase throughout the warranty period, reasonably prolonging the monitoring time is necessary to ensure that the warranty claims data are large enough for monitoring. Therefore, the value of should be adjusted according to the hazard rate function of the time to the first failure of the product. On the whole, a reasonable combination of product sales tracking time and monitoring timecan effectively reduce the risk of weak warning ability caused by sales delay or fluctuating claim rates.

Table 3. Results of Early-warning ability under different Control Charts

Monitoring Stage	Multi-stage Risk Adjustment Control Chart			CUSUM Control Chart		
	1.2	1.3	1.4	1.2	1.3	1.4
10	0.003232	0.003236	0.003241	0.046123	0.046151	0.04618
12	0.004999	0.00511	0.005227	0.051691	0.051992	0.052308
14	0.008096	0.008862	0.00976	0.054124	0.054926	0.05584
16	0.01332	0.016624	0.021111	0.055111	0.056457	0.058189
18	0.0249	0.038204	0.059221	0.055499	0.057335	0.060088
20	0.048138	0.091951	0.168141	0.055636	0.057853	0.061846
22	0.099147	0.227474	0.436915	0.0557	0.058322	0.064544
24	0.20822	**0.503929**	0.81835	0.055747	0.059054	0.071637
26	0.420305	0.844438	0.990402	0.055812	0.061231	0.10218
28	0.720033	0.989995	0.999988	0.055984	0.071743	0.251154
30	0.944992	0.999972	1	0.056638	0.126209	**0.671404**
32	0.997092	1	1	0.058899	0.303362	0.969075
34	0.999975	1	1	0.0665	**0.637179**	0.99981
36	1	1	1	0.088608	0.916635	1
38	1	1	1	0.147474	0.995197	1
40	1	1	1	0.280899	0.999966	1

4.4. Early-Warning Ability Analysis

Traditional CUSUM control chart has been used in the monitoring and early warning of products sold with one-dimensional warranty, which can also be adjusted to monitor products sold with two-dimensional warranty. In order to test the early-warning ability of the proposed model in this study, a comparison of response timefor the above-mentioned control charts has been carried out in this subsection.

As seen in table 3, the first instance in subsection 4.2, has been chosen as an example in this analysis. The hazard rate function of the time to the first failure is Weibull distribution with parameters, and it is known that the warranty claim rate of product produced after 10 weeks has become 1.2-1.4 times of the original standard. On the condition that the overall false alarm rate up to 40 weeks is 0.05, the alarm probabilities of the above two control charts at different monitoring stages have been calculated. It is obvious to find that comparing to traditional CUSUM control chart, the alarm probabilities for abnormal conditions of the proposed model are greater at the following monitoring stages, which have reached to 50% as soon as possible. The result means that the response time of the proposed model for abnormal conditions is shorter than traditional CUSUM control chart. However, we have only analyzed the response time for relatively small change of warranty claims data in table 3, extensive experimental results show both traditional CUSUM control chart and the proposed control chart in this study have a shorter response time for huge change of claim rates, which isn't discussed in detail in this study.

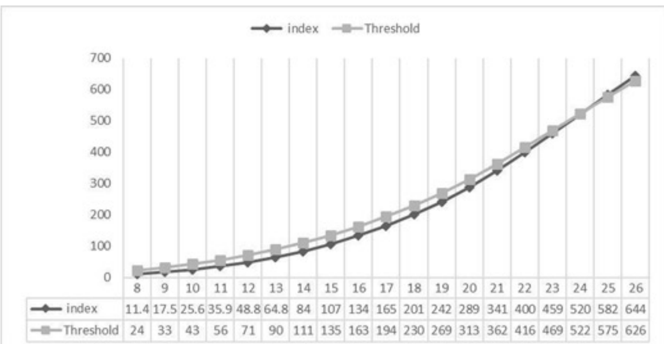

Fig . 4. Multi-stage risk-adjusted control chart

4.5. Example analysis

In this subsection, the proposed control chart and CUSUM control chart are both used to monitor the actual claims data for a common failure mode in 2014.For the normal usage rate , the hazard rate function of the time to the first failure is fitted by Weibull distribution with ,and the accelerating factor of AFT is $\gamma = 1.2$, The prespecified monitoring stages, the product sales tracking time and all other parameters of the proposed model are the same as studied in subsection 4.1.It is known that the claim rate of products has been increased about 1.2 times of the original standard from week 8, which is caused by the change of raw material. The real monitoring performance of the proposed model and CUSUM control chart are shown in Fig 4 and Fig 5.

As seen in Fig. 4 and Fig. 5, both the proposed control chart and CUSUM control chart have been applied for monitoring actual warranty claims data. It is obvious to find that the monitoring thresholds of the proposed control chart are changing stage by stage, while the monitoring thresholds of CUSUM control chart are constant among all the monitoring stages.

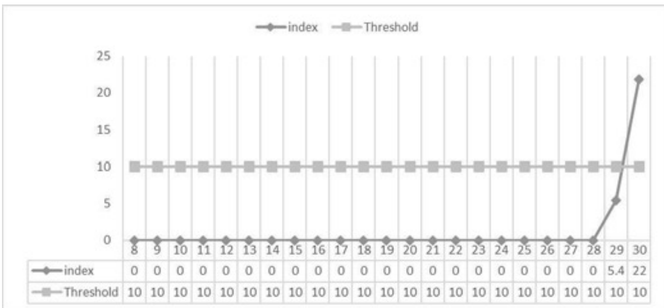

Fig . 5. Traditional CUSUM control chart

For the abnormal conditions of claim rates occurring from week 8, the proposed control chart needs 16 weeks to detect the change and raise the alarm, while the traditional CUSUM control chart needs23 weeks. The multi-stage risk-adjusted control chart proposed by this study, which has a relatively shorter response time for the abnormal change of claim rate, can effectively apply for the monitoring and early-warning of two-dimensional warranty claims data.

5. Summary

Early detection of reliability problems can aid manufacturers to reduce the associated warranty costs and improve customer satisfaction and brand image. For products sold with two-dimensional warranty, a multi-stage risk-adjusted control chart is provided by this study, which can detect and warn the quality or reliability issues of products as soon as possible. However, in consideration of the fact that the maintenance strategies have a significance influence on the claim rate distribution during the warranty period, only minimal repair strategy, for which the monitoring of different stages is independent from each other, has been considered in this study. It is, therefore, a great challenge to design a control chart for the monitoring of time-related claims data, which will be faced by the application of other maintenance strategies.

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