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Zbigniew KAMIŃSKI

MATHEMATICAL MODELLING OF THE TRAILER BRAKE CONTROL VALVE FOR SIMULATION OF THE AIR BRAKE SYSTEM OF FARM TRACTORS EQUIPPED WITH HYDRAULICALLY ACTUATED BRAKES

MODELOWANIE MATEMATYCZNE ZAWORU STERUJĄCEGO HAMULCAMI PRZYCZEPY DO SYMULACJI POWIETRZNEJ INSTALACJI HAMULCOWEJ CIĄGNIKÓW ROLNICZYCH Z HAMULCAMI URUCHAMIANYMI HYDRAULICZNIE*

Agricultural tractors are equipped with air braking systems to control and operate the braking systems of towed agricultural vehicles. This paper presents a mathematical model of a hydraulically actuated trailer brake control valve. The results of the statistical Kolmogorov-Smirnov test confirmed the consistence between the experimental and simulated pressure transients during testing the response time of a farm tractor's control circuit. The computer model developed in Matlab-Simulink can be used as a tool to analyze transient processes by using simulation methods in the process of designing the air braking systems of farm tractors.

Keywords: *farm tractor, air braking system, hydraulically actuated trailer brake control valve, mathematical modelling, simulation.*

Ciągniki rolnicze są wyposażone w powietrzne instalacje hamulcowe do sterowania i napędu układów hamulcowych pojazdów ciągnionych. W niniejszej pracy przedstawiono model matematyczny uruchamianego hydraulicznie zaworu sterującego hamulcami przyczepy. Wyniki testu statystycznego Kołmogorowa-Smirnowa oceny zgodności doświadczalnych i symulowanych przebiegów czasowych ciśnienia podczas badania czasu reakcji obwodu sterującego ciągnika Pronar 1531A potwierdziły adekwatność opracowanego w Matlabie-Simulinku modelu komputerowego. Model komputerowy może być wykorzystany jako podsystem do analizy metodami symulacyjnymi procesów przejściowych w procesie projektowania powietrznych układów hamulcowych ciągników rolniczych.

Słowa kluczowe: *ciągnik rolniczy, powietrzny układ hamulcowy, zawór sterujący hamulcami przyczepy uruchamiany hydraulicznie, modelowanie matematyczne, symulacja.*

1. Introduction

Currently, in farm tractors moving at speeds of up to 50–60 $km \cdot h^{-1}$, hydraulically actuated wet multi-disc brakes [24] are commonly used. In low- and medium-power tractors, simple and inexpensive hydraulic brake systems with no power assistance are preferred. These manually operated systems work at a low operating pressure and a small required brake cylinder volume [14]. However, in more powerful tractors, hydraulic systems powered by the tractor's hydraulic system are mainly used [15, 18]. These hydraulic brake systems allow the supply of the flow of fluid from the tractor hydraulic pump to the service brake cylinders, ranging from small to large in volume, at a greater pressure in a relatively short time. Apart from the service and parking brake systems, today's farm tractors are fitted with air brake systems to actuate the air braking systems of assembled machinery or trailers. Often, so called combined systems [29] are used to operate with both the single- and dual-line air braking systems of towed vehicles.

A typical air braking system of an agricultural tractor consists of two major parts: an energy supply unit and a control device. The role of the energy supply unit is to purify and compress the air and to maintain the adequate pressure in the tractor and trailer reservoirs so that the required trailer braking performance is ensured. The control device permits a smooth gradual braking process for the tractor-trailer combinations. The co-operation of the tractor hydraulic braking system and the trailer air braking system is provided by a hydraulically actuated trailer brake control valve (Fig. 1).

The braking systems of agricultural vehicles driven on public roads must meet several specific requirements [3, 4] for braking performance, response time during rapid braking (up to 0.6 s) and braking compatibility with combination vehicles [27].

The behaviour of the braking system of a tractor and a trailer, including the unsteady states, may be predicted already at an early stage of design using simulation methods. The main benefit of the model-based design (MBS) is that it increases the speed and efficiency in testing new solutions, provides the possibility of confronting them with the applicable requirements, and detects any errors resulting from malfunctioning or mistakenly taken assumptions earlier than in the case of constructing physical prototypes [25].

In mathematical modelling of multi-circuit air braking systems, pneumatic elements of a discrete nature (such as line filters, fasteners) and even elements of parameters distributed in a continuous manner (e.g. pipes) are usually substituted with idealized elements in the form of a lumped volume and resistance [9, 13, 19]. This yields a mathematical model as a system of ordinary differential equations, which are able to be solved within a majority of software designed for numerical simulation, including object-oriented programs [12, 30]. The main difficulty in modelling the air braking system of farm tractors is the lack of appropriate mathematical models for some components and devices, particularly the trailer brake control valves. Most of the brake valve models known from the literature concern typical air brake valves used in commercial vehicles [6, 22, 28] and trailers [8, 10, 11, 21, 23].

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

This paper presents mathematical modelling of a trailer brake control valve hydraulically actuated by brake fluid pressure from the service hydraulic brake system of a farm tractor. The mathematical and computer model of this valve considers the heat exchange and the friction and inertia forces of movable elements, which are generally omitted in a modelling process [6, 22, 28]. The computer model implemented in Matlab-Simulink is used to test the response time of the control device of a typical tractor's air braking system. For modelling the energy supply unit, the relationship described in [12] is used. The developed model can be used in a farm tractor design process for the dynamic calculation of the tractor's air braking system.

2. Experimental setup for testing of the tractor pneumatic braking system

A simplified schematic diagram of the combined pneumatic braking system of a Pronar 1523A agricultural tractor [26] equipped with a hydraulic brake drive is shown in Fig. 1. The energy supply unit includes filter 1, air compressor 2, unload valve (governor) 3, and compressed air reservoir 4. Compressed air is also fed to the control device, which includes proportional brake valve 11 and inverse brake valve 7. The trailer control valve 11 is connected, via ports 41 and 42, with the hydraulic system of the tractor service brakes. When the brake pedal is depressed, the hydraulic control pressure will act on valve 11, causing this valve to open thus increasing the air pressure in the line with coupling head 10 that controls the dual-line trailer braking system. The trailer braking system supply line is connected to coupling head 9. For controlling the single-line trailer braking system, inversion valve 7 is used, which, upon the pressure increase in its control port 4 causes a pressure drop in the supply and control line with coupling head 8.

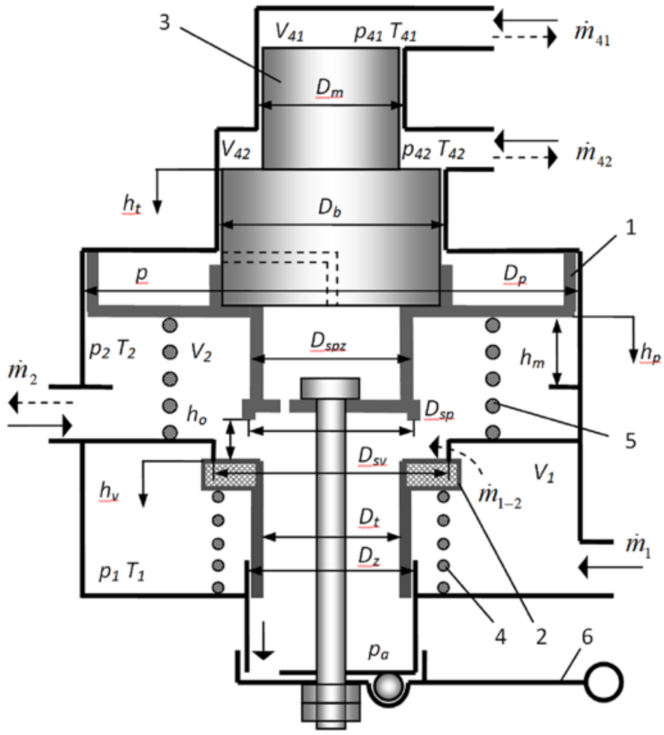

Fig. 2. A hydraulically actuated trailer brake control valve: 1 – pneumatic control piston, 2 – inlet head, 3 – hydraulic graduating piston, 4 – head spring, 5 – piston return spring, 6 – hand brake mechanism [13]

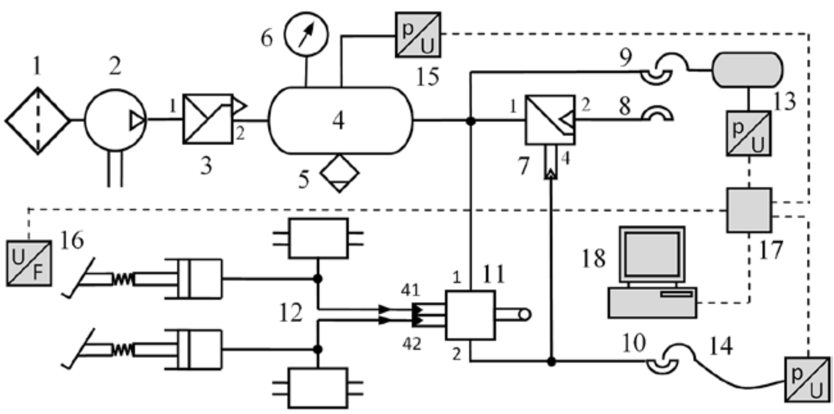

Fig.1. Schematic diagram of a Pronar 1523A farm tractor's combined air braking system coupled with the experimental control line response time testing setup: 1 – filter, 2 – FOS Polmo Łódź 601.23.931 compressor, 3 – Visteon 51 10 011 unloader valve, 4 – 20 dm³ air reservoir, 5 – drain valve, 6 – single pressure gauge, 7 – Visteon 45 10 016 inversion trailer control valve, 8 – "single-line" coupling head (black), 9 – "supply" coupling head (red), 10 – "brake" coupling head, 11 – Haldex 329 020 311 trailer brake control valve , 12 – tractor service brake hydraulic system, 13 – 385 cm³ air vessel, 14 – 2.5 m-long 13 mm-internal diameter pipe, 15 – pressure transducer, 16 – pedal force transducer, 17 – input/output adapter, 18 – computer with a measuring card

The experimental setup for testing the response time of the tractor dual-line pneumatic system control device is distinguished in Fig. 1 with the grey background. Pressure in selected locations of the air brake system was measured with Danfoss type MBS 32 industrial pressure transducer 10 (with a pressure range of 0÷10 bar; an output range of 0÷10 V; and an accuracy class of 0.3%). CL 23-type extensometric brake pedal force sensor 16 complete with a CL10D type ZEPWN industrial amplifier (with a measuring range of 0÷1 kN; an output signal range of 0÷10 V; and an accuracy class of 0.1%) was used for measuring the force on the brake pedal. The transducers were voltage supplied from input/output adapter 17. The output voltage signals from the force and pressure transducers were acquired from the input/output adapter using a (12 bit resolution) Senga MC1212 measuring card and then directly converted into force and pressure data with the use of integrated software installed on computer 18 designed for data collection during the run of the tests.

3. Modelling of the hydraulically actuated trailer brake control valve

In the control circuit of the Pronar 1523A tractor's air system, a Haldex 329 020 311 brake valve [5] was used as trailer control valve 11 (Fig. 1), which has the same design as a Wabco 470 015 brake valve [29]. A schematic diagram of the structure of a typical hydraulically actuated valve is given in Fig. 2.

When the brake pedal is depressed, the pressure rises both in the hydraulic wheel brake cylinders of the tractor and in the hydraulic chambers V_{41} and V_{42} of the control valve coupled with the tractor's twin (parallel) master cylinder. Under the influence of the brake fluid pressure, graduating piston 3 and the piston 1 move downwards, causing inlet head 2 to open. The compressed air from the inlet chamber V_1 flows into a variable-capacity outlet chamber V_2 as a mass flux $\dot{m}_{1-2}$. The flux $\dot{m}_2$ flowing out from the outlet chamber V_2 is routed to the control circuit of the trailer braking system, resulting in the activation of the trailer brakes. The air mass flux $\dot{m}_1$ flows into a fixed-volume inlet chamber V_1 from the supply circuit.

During the brake release caused by the decrease in hydraulic control pressure, piston 1 raises again by the action of spring 5. Poppet valve 2 closes (thus cutting off the inlet chamber from the outlet chamber) and the passage between the seat of piston 1 and head 2 to the atmosphere opens. The outlet chamber V_2 is vented. The compressed air from the control line returns to the outlet chamber as the mass flux $\dot{m}_2$ (flow reversal), and then flows out to the atmosphere as the mass flux $\dot{m}_{2-3}$. The venting of the outlet chamber and the control line causes a drop in the trailer brake force. The follow-up control (tracking) is accomplished by the negative feedback as the air pressure p_2 acts on control piston 1.

When creating the current mathematical model of the trailer control valve, a number of simplifying assumptions were made [11, 13] including the following:

- Compressed air was regarded as a thermodynamically ideal gas (i.e. the one obeying the Clapeyron law), while being viscous and compressible.
- The valve adjusting element, irrespective of its design, was regarded as a local resistance (nozzle), whose effective flow field (conductance) depended on the head lift.
- The airflow through the adjusting element was considered one-dimensional.
- The air properties were assumed to be uniform both in the individual valve chambers and in the entire cross-section of flow through the local resistance.
- In the valve opening phase, the force interaction between the head 2 and control piston 1 and the clearance between this piston and hydraulic piston 3 were omitted, which means that all the elements moved together as a single mass (h_t=h_p).
- The influence of the housing on the control piston in its end position was neglected; the stopping of the piston was accomplished via the acceleration control logic (hard stopping).
- There is always atmospheric pressure in the chambers above piston 1 and under piston 3
- The heat exchange between the system's air and the environment took place by natural convection at a constant wall temperature which was equal to ambient temperature.
- The air and brake fluid leaks from the valve chambers were neglected.
- Variation in brake fluid temperature was neglected.

In accordance with the law of conservation of matter, the brake fluid mass changes in the hydraulic control chambers of the volumes V_{41} and V_{42}, respectively, and the air mass changes in the inlet chamber of the volume V_1 and the outlet chamber of the volume V_2 (Fig. 2) are described by the equations:

$$\frac{dm_{V41}}{dt} = \pm\dot{m}_{41} \qquad \frac{dm_{V42}}{dt} = \pm\dot{m}_{42} \tag{1}$$

$$\frac{dm_{V1}}{dt} = \dot{m}_1 - \dot{m}_{1-2} \qquad \frac{dm_{V2}}{dt} = \dot{m}_{1-2} \mp \dot{m}_2 - \dot{m}_{2-3} \tag{2}$$

where: $\dot{m}_i$ – the mass flux [kg·s^{-1}] flowing into (the sign +) or from (the sign –) a given chamber; the respective flux subscripts conform to the convention on denoting the braking valve chamber ports (the small numerals at the valve ports in Fig. 1).

The air mass flux through the local pneumatic resistances is described by the Saint-Venant–Wantzel relationship [2] in the following generalized form:

$$\dot{m} = (\mu A_m)\frac{p_m}{\sqrt{RT_m}}\Psi_{max}\Psi(\sigma) \tag{3}$$

where: (μA_m) – conductance, i.e. the product of the discharge coefficient μ and the flow cross-section area A_m [m^2], p_m – pressure [Pa] upstream of the resistance, T_m – air temperature [K] upstream of the resistance, R – gas constant for air, being equal to 288 [J·kg^{-1}K^{-1}], Ψ_{max} – maximum value of the Saint-Venant–Wantzel function for the critical product σ^* of the pressure upstream and downstream of the resistance, respectively, which is given by:

$$\Psi_{max} = \Psi(\sigma^*) = \sqrt{\kappa\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}} = 0.68473 \tag{4}$$

where : κ – adiabatic exponent, for air being equal to κ=1.4.

Instead of the dimensionless two-range Saint-Venant flow function $\Psi(\sigma)$, as given by:

$$\Psi(\sigma) = \begin{cases} 1 & \text{for} \quad \sigma \le \sigma_* \\ \dfrac{1}{\Psi_{max}}\sqrt{\dfrac{2\kappa}{\kappa-1}\left(\sigma^{\frac{2}{\kappa}} - \sigma^{\frac{\kappa+1}{\kappa}}\right)} & \text{for} \quad \sigma_* < \sigma \le 1 \end{cases}$$

a single-range hyperbolic function [19, 20] with a constant parameter value of b=1.13 typical of pneumatic air braking systems was used, as being more convenient for numerical computation and still sufficiently accurate:

$$\Psi(\sigma) = b\frac{1-\sigma}{b-\sigma} \tag{5}$$

Using equations (3) and (5), the equations of mass fluxes flowing through the braking valve are obtained as:

$$\dot{m}_{1-2} = \mu_{12}A_{12}\frac{p_1}{\sqrt{RT_1}}\Psi_{max}\Psi\left(\frac{p_2}{p_1}\right) \tag{6}$$

$$\dot{m}_{2-3} = \mu_{23}A_{23}\frac{p_2}{\sqrt{RT_2}}\Psi_{max}\Psi\left(\frac{p_a}{p_2}\right) \tag{7}$$

The flow cross-section areas A_{12} (during braking) and A_{23} (during releasing) are dependent on the piston travel h_p and the distance h_v of head 2 from the valve seat and are given by:

$$A_{12} = \begin{cases} 0 & \text{if} \quad h_v \le h_{vo} \\ \pi D_{sv}(h_v - h_{vo}) & \text{if} \quad h_{vo} < h_v \le h_{vm} \\ \dfrac{\pi\left(D_{svw}^2 - D_{spz}^2\right)}{4} & \text{if} \quad h_v > h_{vm} \end{cases} \tag{8}$$

$$A_{23} = \begin{cases} 0 & \text{if} \quad h_p \ge h_o - h_{po} \\ \pi D_{sp}(h_o - h_{po} - h_p) & \text{if} \quad h_{pm} \le h_p < h_o - h_{po} \\ \dfrac{\pi\left(D_t^2 - d_s^2\right)}{4} & \text{if} \quad h_p < h_{pm} \end{cases} \tag{9}$$

where: D_{sv}, D_{svw} – stationary seat average diameter [m] and inner diameter [m], respectively; D_{spz} – moveable seat outer diameter [m] (in the piston), h_{vo}, h_{vm} – head 2 position [m] corresponding to the beginning of opening (allowing for plate seal deformation) and position [m] corresponding to the attaining of the maximum flow field value, respectively, D_{sp} – the average diameter [m] of the moveable seat, D_z, D_t – the outer and the inner diameter [m], respectively, of the head sleeve, d_s – the screw diameter [m] of the parking brakes mechanism, h_{po}, h_{pm} – piston position [mm] corresponding to the beginning of opening the passage to the atmosphere (allowing for plate seal deformation) and the position [mm] in which the flow field attains the maximum value, respectively.

The following relationship exists between the displacement h_p of control piston 1 and the displacement h_v of head 2:

$$h_v = \begin{cases} 0 & h_p \le h_o \\ h_p - h_o & h_p > h_o \end{cases} \tag{10}$$

where: h_o – the maximum distance (clearance) [m] between head 2 and piston 1 in the upper extreme position.

With the assumed simplifications, the mechanical elements of the brake valve can be considered as a dynamical system with one degree of freedom with variable mass, as described by the following equation of motion:

$$m_z \frac{d^2 h_p}{dt^2} = F_H + F_{p2a} + F_{sp} + F_v + F_{fp} \tag{11}$$

where: m_z – reduced mass [kg] of the elements moving together with piston 1, F_H – pressure force [N] of the brake fluid acting on piston 3, F_{p2a} – pressure force [N] acting on piston 1, F_{sp} – force of piston return spring 5 [N], F_{fp} – force [N] of piston friction against the housing, F_v – force [N] of the pressure of head 2 acting on piston 1.

The reduced mass of the elements moving together with piston 1 is:

$$m_z = \begin{cases} m_\rho + m_h + m_p + m_{sp}/3 & h_p \le h_o \\ m_\rho + m_h + m_p + m_v + \left(m_{sp} + m_{sv}\right)/3 & h_p > h_o \end{cases} \tag{12}$$

where: m_h – mass [kg] of graduating piston 3, m_p – mass [kg] of control piston 1, m_v – mass [kg] of head 2 with the sleeve guide, m_{sv} – mass [kg] of spring 4 clamping head 2, m_{sp} – mass [kg] of return spring 5, m_ρ – mass [kg] of the brake fluid.

The forces of pressure acting on the pistons are:

$$F_H = \frac{\pi D_m^2}{4} p_{41} + \frac{\pi\left(D_b^2 - D_m^2\right)}{4} p_{42} \tag{13}$$

$$F_{p2a} = \frac{\pi D_p^2}{4} p_a - \frac{\pi D_{sp}^2}{4} p_a - \frac{\pi\left(D_p^2 - D_{sp}^2\right)}{4} p_2 = -\frac{\pi\left(D_p^2 - D_{sp}^2\right)}{4}\left(p_2 - p_a\right) \tag{14}$$

where: p_{41}, p_{42} – pressures [Pa] of the brake fluid in the control hydraulic chambers, p_a – atmospheric pressure [Pa], p_2 – air pressure [Pa] in chamber V_2, D_m, D_b – diameters [m] of graduating piston 3 [m], D_p – diameter [m] of piston 1.

The force of the pressure of return spring 5 acting on piston 1 is calculated from the relationship:

$$F_{sp} = -\left(F_{spo} + c_{sp} h_p\right) \tag{15}$$

where: F_{spo} – preset force [N] of spring 5 for h_p=0 [N], c_{sp} – stiffness [N·m^{-1}] of spring 5.

Assuming that the poppet valve has an unladen design ($D_{sp} \approx D_z$), the force of head pressure acting on the piston is described by the relationship:

$$F_v = \begin{cases} 0 & h_p \le h_o \\ -\left[F_{svo} + c_{sv}\left(h_p - h_o\right)\right] - \left(p_1 - p_2\right)\dfrac{\pi\left(D_{sv}^2 - D_{sp}^2\right)}{4} - \operatorname{sgn}\left(\dfrac{dh_p}{dt}\right)\left[F_{cv} + k_v\left(\dfrac{dh_p}{dt}\right)\right] & h_p > h_o \end{cases} \tag{16}$$

where: F_{cv} – head guide kinematic friction force [N], c_{sv} – stiffness [N·m^{-1}] of spring 4, k_v – viscous friction coefficient [N·s·m^{-1}].

The force of static and kinetic friction of the pistons against the housing is described using the Karnopp model [1] according to:

$$F_{fp}\left(\frac{dh_p}{dt}, F_e\right) = -\begin{cases} \operatorname{sgn}\left(F_e\right)\cdot\min\left(F_{sp}, \left|F_e\right|\right) & \text{if } \left|\dfrac{dh_p}{dt}\right| < \Delta v \\ \operatorname{sgn}\left(\dfrac{dh_p}{dt}\right)\cdot\left[F_{cp} + k_v\cdot\left(\left|\dfrac{dh_p}{dt}\right| - \Delta v\right)\right] & \text{if } \left|\dfrac{dh_p}{dt}\right| \ge \Delta v \end{cases} \tag{17}$$

The total kinetic friction force F_{sp} and the kinetic friction force F_{cp} of piston 1, as well as the kinetic friction force F_{cv} of the guide of head 2, are described by the experimental relationships:

$$\begin{aligned} F_{sp} &= \pi D_p\left(f_s + k_s\left|p_2 - p_a\right|\right) + \pi D_m\left(f_s + k_s\left|p_{41} - p_{42}\right|\right) + \pi D_b\left(f_s + k_s p_{42}\right) \\ F_{cp} &= \pi D_p\left(f_c + k_c\left|p_2 - p_a\right|\right) + \pi D_m\left(f_c + k_c\left|p_{41} - p_{42}\right|\right) + \pi D_b\left(f_c + k_c p_{42}\right) \\ F_{cv} &= \pi D_z\left(f_c + k_c\left|p_1 - p_a\right|\right) \end{aligned} \tag{18}$$

where: F_e – total external force [N], f_s, f_c – static friction force and kinetic friction force [N·m^{-1}], respectively, per unit piston perimeter, independent of the differential pressure across the piston, k_s, k_c – proportionality factors [m].

Based on the energy conservation law for open systems, excluding the kinetic and potential energy, the following equations for the change in the internal energy of the air in the control volumes V_1 and V_2 of the chambers are obtained:

$$\frac{dU_1}{dt} = \dot{Q}_1 + \dot{H}_1 - \dot{H}_{1-2} \tag{19}$$

$$\frac{dU_2}{dt} = \dot{Q}_2 + \dot{W} + \dot{H}_{1-2} \mp \dot{H}_2 - \dot{H}_{2-3} \tag{20}$$

where: U_i – internal energy [J] of the air in a given chamber, $\dot{Q}_i$ – rate of heat flow [W] exchanged with the environment, $\dot{H}_i$ – rate of enthalpy flow [W] reaching (+) or leaving (−) the chamber, $\dot{W}_i$ – rate of external work [W] done by the air in the chamber:

$$\begin{aligned} U_i &= m_V \cdot c_v T_i & \dot{H}_i &= \dot{m}_i \cdot c_p T_m \\ \dot{W}_i &= -p_i \cdot \dot{V}_i & \dot{Q}_i &= \alpha_i \cdot A_i\left(T_w - T_i\right) \end{aligned} \tag{21}$$

where: c_v, c_p – specific heat capacity of the gas at constant volume and constant pressure: c_v=717 [J·kg^{-1}K^{-1}], c_p=1005 [J·kg^{-1}K^{-1}], T_i – absolute temperature [K] of air in the i-th chamber, T_m – flow temperature [K] (for the flow leaving the chamber $T_m=T_i$), α_i – heat transfer coefficient [W·m^{-2}K^{-1}], p_i – absolute pressure in the i-th chamber; A_i – heat transfer area [m^2], T_w – valve wall temperature [K] equal to ambient temperature T_a.

After the differentiation of the internal energy and using Equation (2) and the Clapeyron equation in the differential form [16]:

$$R\left(\frac{dm_{Vi}}{dt}T_i + m_{Vi}\frac{dT_i}{dt}\right) = p_i\frac{dV_i}{dt} + V_i\frac{dp_i}{dt} \tag{22}$$

the following differential equations for the variations of air pressure and temperature in individual valve chambers are obtained:

$$\frac{dp_1}{dt} = \frac{1}{V_1}(\kappa-1)\left(\dot{Q}_1 + \dot{H}_1 - \dot{H}_{1-2}\right) \qquad \dot{Q}_1 = \alpha_1 A_1\left(T_w - T_1\right) \tag{23}$$

$$\frac{dT_1}{dt} = \frac{T_1}{p_1V_1}\left[V_1\frac{dp}{dt} - RT_1\left(\dot{m}_1 - \dot{m}_{1-2}\right)\right] \tag{24}$$

$$\frac{dp_2}{dt} = \frac{1}{V_2}\left[(\kappa-1)\left(\dot{Q}_2 + \dot{H}_{1-2} \mp \dot{H}_2 - \dot{H}_{2-3}\right) - \kappa\cdot p_2\frac{dV_2}{dt}\right] \tag{25}$$

$$\frac{dT_2}{dt} = \frac{T_2}{p_2V_2}\left[p_2\frac{dV_2}{dt} + V_2\frac{dp_2}{dt} - RT_2\left(\dot{m}_{1-2} \mp \dot{m}_2 - \dot{m}_{2-3}\right)\right] \tag{26}$$

Using the relationship between the mass flow and the volume flow of the brake fluid ($\dot{m}_V = \dot{m}/\rho$) and after differentiation of mass in relationships (1):

$$m = d\rho\cdot V + dV\cdot\rho$$

the following equations for the volume flux are obtained by taking into account the fluid bulk modulus and housing compliance [7]:

$$\pm\dot{m}_{41V} = \frac{\pi D_m^2}{4}\frac{dh_p}{dt} + \frac{V_{41}}{B_z}\cdot\frac{dp_{41}}{dt} \tag{27}$$

$$\pm\dot{m}_{42V} = \frac{\pi\left(D_b^2 - D_m^2\right)}{4}\frac{dh_p}{dt} + \frac{V_{42}}{B_z}\cdot\frac{dp_{42}}{dt} \tag{28}$$

where: B_z – effective bulk modulus [Pa].

4. Example tractor air braking system simulation

For the validation of the computer model of the trailer control brake valve, as well as the models of other components, the experimental results of the response time testing of the Pronar 1523A farm tractor's dual line air braking system (Fig. 1) were utilized. The response time of the tractor's dual-line pneumatic system control circuit was determined according to the method specified in Regulation 13 of the ECE [4] based on the variations in the brake pedal force and pressure as measured at the end of a 2.5 m-long 13 mm-internal diameter line (an imitation of the trailer control line) connected to the brake coupling.

The computer models of most components of the energy supply unit and the control device were created in the form of *S*-function type graphical sub-systems written in the m-files of the Matlab-Simulink program, based on own algorithms and procedures [9, 12]. For modelling the pipes of the hydraulic brake system, the transmission line method, as described by Krus [17], was applied.

The actual time variations of the pedal force F_p (input signal) and the actual pressure variations (output signals) recorded during the experimental tests at the points in the system, as indicated in Fig.1, including pressure p_{te} in air reservoir 4, pressure p_{se} in vessel 13 and pressure p_{ce} at the end of line 14 connected to brake coupling head 10, were input to the computer model in the form of *From File*-type source blocs.

Sample simulation (the solid lines) and experimental (the dotted lines) test results obtained during the testing of the response time of the Pronar 1523A air braking system's control device are presented in Fig. 3. From the modelling viewpoint, i.e. the calculation of the

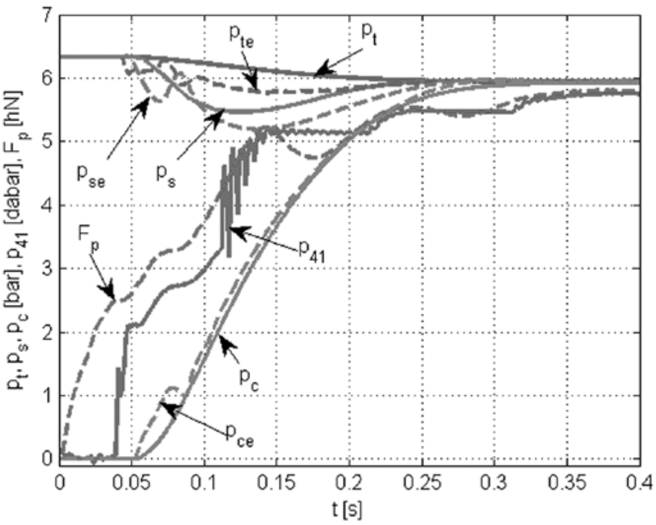

Fig. 3. Results of the experimental and simulation tests of transient processes in the farm tractor's air braking system: F_p – actual brake pedal force, p_{41} – model pressure in the hydraulic control chamber of the valve, p_t,p_{te} – model and experimental pressure in the air reservoir, p_s,p_{se} – model and experimental pressures in the vessel at the end of the supply line, p_c, p_{ce} – model and experimental pressures at the end of the control line

response time from the theoretical pressure p_c, it can be assumed that the developed model is sufficiently accurate. This is evidenced by the obtained values of the statistical indicators of agreement [31] between the time function graphs of the experimental pressure p_{ce} and the simulated pressure p_c at the end of the control line (the coefficient of determination R^2 >98% and the mean absolute percentage error *MAPE*<5%) and, above all, a small shift in time (less than 0.01 s) between both graphs at the pressure equal to 75% of the asymptotic pressure. The adequacy of the computer model was also confirmed by the results of the Kolmogorov-Smirnov test at a significance level of 5%. The obtained results of the validation of the tractor braking system computer model are also indirectly indicative of the suitability of the mathematical and computer trailer control brake valve model for simulation of the transient processes occurring in the braking systems of vehicles.

5. Summary

In the mathematical modelling of the trailer brake control valve, the actual course of the phenomena accompanying the operation of braking valves, usually omitted at the stage of creating a physical or a computer model, has been taken into account to a considerable extent. The differential equations of the variations in air pressure and temperature in individual valve chambers consider the heat exchange with the environment, while the differential equation of the piston motion takes into account the static-kinetic friction forces and the inertial forces. The discrete brake model equations can be used to formulate the mathematical and computer models of other braking valves of a similar design.

The obtained results of the Kolmogorov-Smirnov test, as well as the values of the statistical indicators R^2, *MAPE* assessing the consistence between the experimental and the simulation pressure transients at the end of the control line during testing of the control circuit response time have confirmed the adequacy of the Pronar 1523A tractor's air braking system computer model implemented in Matlab-Simulink.

The developed computer model can be used to study transient processes in the air braking system of agricultural tractors equipped with hydraulic brakes and to predict the dynamic properties of the air braking system of tractor-trailer units (speed and the synchrony of action) using simulation methods.

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