

DESIGN OF A LINEAR QUADRATIC REGULATOR BASED ON GENETIC MODEL REFERENCE ADAPTIVE CONTROL

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Abstract:

The conventional control system is a controller that controls or regulates the dynamics of any other process. From time to time, a conventional control system may not behave appropriately online; this is because of many factors like a variation in the dynamics of the process itself, unexpected changes in the environment, or even undefined parameters of the system model. To overcome this problem, we have designed and implemented an adaptive controller. This paper discusses the design of a controller for a ball and beam system with Genetic Model Reference Adaptive Control (GM-RAC) for an adaptive mechanism with the MIT rule. Parameter adjustment (selection) should occur using optimization methods to obtain an optimal performance, so the genetic algorithm (GA) will be used as an optimization method to obtain the optimum values for these parameters. The Linear Quadratic Regulator (LQR) controller will be used as it is one of the most popular controllers. The performance of the proposed controller with the ball and beam system will be carried out with MATLAB Simulink in order to evaluate its effectiveness. The results show satisfactory performance where the position of the ball tracks the desired model reference.

Keywords: *model reference adaptive control, gradient approach, Linear Quadratic Regulator, genetic algorithm*

1. Introduction

Adaptive control is a method of control that uses a controller with adaptable parameters that change with respect to the variation in system response. This method has advantages over conventional control, where it has been used for better performance and accuracy of advanced control systems design, and for systems with uncertain or unknown parameter variations and environmental changes. These characteristics made adaptive control find numerous applications in control problems where it has the ability to be automatically compensated for changes in the plant dynamics [1]. Model Reference Adaptive Control (MRAC) is considered one of the most popular types of adaptive controllers for its straight adaptive strategy with adjustable parameters [2].

This adaptive effect will be given through a reference model, where the error between the real plant (system) and the reference model will be used to modify its parameters to make the plant output follow the reference model response [3]. As a result, MRAC will force the real plant to track the reference system, which has been chosen precisely. In the area of self-tuning controllers, MRAC is considered very popular. It is a robust control that can deal with disturbances and rapid changes in the parameters despite not needing a priori information about the bounds of the uncertainties or the time-varying parameters [4]. An example of a system requiring adaptive control is an aircraft, which when flying will reduce its mass at a slow rate as it consumes fuel. In this case, the controller needs to adapt itself continuously.

As for the controller to be used inside the MRAC, the Linear Quadratic Regulator (LQR) was chosen, as it is one of the most utilized techniques for the feedback control design [5]. Optimal feedback LQR is one of the tools that might be implemented for stability improvement of the system performance, where a set of optimal feedback gains may be found by using minimization of a quadratic index [6].

The challenge of using the LQR application is the adjustment process that is used to find the elements of both weighting matrices Q and R. Therefore, for the LQR that will be used, the genetic algorithm (GA) optimization method will be used for Q and R adjustment. The GA has advantages over other optimization methods due to its ability to deal with complex problems and different optimization. For instance, it can deal with the linear or nonlinear, or with a system with random noise [7-10]. The MRAC that uses the GA to optimize its parameters and mechanism is called Genetic Model Reference Adaptive Control (GMRAC).

The proposed GMRAC will be applied on a ball and beam system whose open loop is inherently unstable. This system has some uncertainty about its model due to the many assumptions considered when deriving the model. Also, this system is linked directly to real control problems in settings such as in an airplane: for instance, issues have arisen in horizontal stabilizing during landing, turbulent airflow, and automatic ball balancers in optic disk drivers [11]. Another problem of this system is nonlinearity where the open loop transfer function is nonlinear; to overcome

this problem, linearization with the modern state space method will be used around the horizontal region [12]. Although the model of the system has been linearized, it still represents typical systems in real life (e.g., horizontal stabilization of airplanes during landing) [12-14].

This paper deals with the design of the adaptive controller with a model reference scheme using the MIT rule. The principle of this work is to adjust the controller parameters in order to make the output of the plant (process) follow the output of the reference model for the same input.

2. Mathematical Model of Ball and Beam System

The ball-beam system involves a beam whose position can be adjusted by using an electrical motor and a ball that rolls on the top of the beam. This system has two degrees of freedom: one is for the rolling up and down of the ball, while the second one is from the beam rotating around its central axis. For this system, the torque generated by the motor will be used to control the ball position on the beam.

The mathematical model of the ball and beam system has been explained in detail by many researchers, depending on the mathematical equations that drive the model of the system [12, 15-20]. Figure 1 shows the sketch of the system that this mathematical model is drawn from, including the torque balance of the beam as well as the force balance.

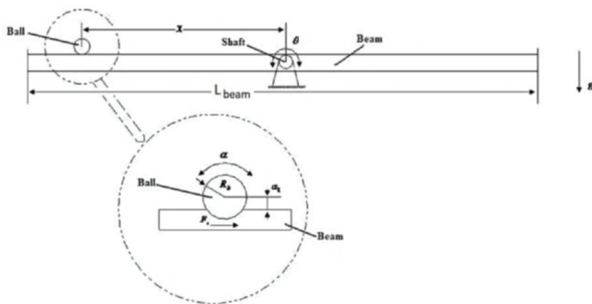


Fig. 1. Sketch map of the system [11]

Many analyses must be completed to derive the mathematical model of the system. Firstly, the analysis of the balance force (F_b) depending on Newton's law, and the torque balance of the motor (T_{motor}), must be completed. Next, the equation of the used DC motor must be derived. All equations analyzed will be represented in state space, which can be used with state space control methods.

The mathematical model may be expressed in state space form. The parameters that need to be controlled are the beam tilt angle (θ), the rate of change in θ, the ball position (x), and the rate of change in x.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ x \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{g}{1 + \frac{2}{5} \left(\frac{R_b}{a_1}\right)^2} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{M_{ball}}{J_{bm}} & 0 & 0 & -\frac{KK_e}{J_{bm}} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K}{RJ_{bm}} \end{bmatrix} V \quad (1)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}, \quad (2)$$

where x is the position of the ball (m) and v is the control voltage [12]. Table 1 shows the parameter values that have been used.

Tab. 1. Parameters of the ball-beam system

Parameters	Value
Mass of ball (M _{ball})	0.0327 kg
Electric resistor (R)	4.7 ohms
Electromotive force constant (K)	4.91 Nm/A
Ball radius (R _b)	0.01 m
Moment of inertia (J _{bm})	0.062 kg/m ²

It is crucial to notice that the system model depending on the physical and electrical laws in Equations 1 and 2 is depending on some assumptions. For instance, there is no slip between the ball and beam and the gearbox of the motor does not have backlash.

The final model after the assumption and simplifications will be represented in Equations 3 and 4.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ x \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 3.7731 & 0 \\ 0 & 0 & 0 & 1 \\ -5.17 & 0 & 0 & -105.1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 16.85 \end{bmatrix} \quad (3)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad (4)$$

where the input is the voltage (v) and the outputs are θ and x.

3. MRAC Methodology

Adaptive controllers generally consist of two loops: the outer loop (normal feedback loop), and the inner loop (parameter adjustment loop). The traditional MRAC strategy is used to adjust the controller parameters so that the response of the actual plant follows the response of the reference model, where both have the same reference input.

Whitaker proposed a MRAC in 1958 whose block diagram is illustrated below (Fig. 2) [21].

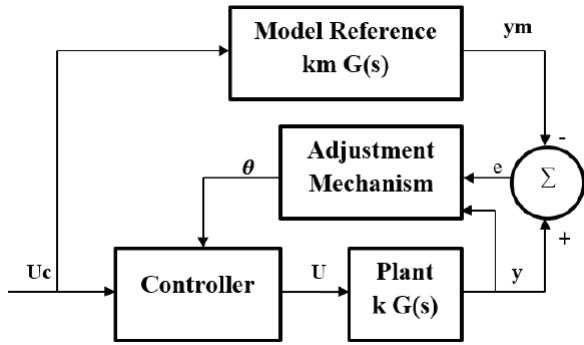


Fig. 2. Basic block diagram of a model-reference adaptive control (MRAC) system

The reference model $G_m(s)$ is used to create an optimal response of the adaptive system to the reference input $U_c(s)$.

The adjustable parameters are implemented to describe the controller while the values of θ depend on the adaptation gain. The most important block in the system is the “adjustment mechanism,” which is considered the heart of the MRAC, and its determination is crucial. For this work, the MIT rule has been chosen as the parameter adjustment mechanism, which is originally used in MRAC.

For perfect tracking between the output of the plant (y) and the output of the reference model (y_m), the squared model cost function must be minimized, so that the error function can be minimized.

Using Equation 5, the error between y and y_m can be determined.

$$E(s) = y(s) - y_m(s) \tag{5}$$

According to the MIT rule, the cost function is defined as:

$$J(\theta) = \frac{e^2(\theta)}{2} \tag{6}$$

where θ , the controller parameter vector, is an adjustable parameter used to minimize J to zero. The parameter adjustment mechanism showed in Equation 3 is called the MIT rule.

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta} \tag{7}$$

where the $\partial e / \partial \theta$ component is the derivative of the sensitivity of the error while γ is the adaptation gain. Both indicate the error changing with respect to θ . The selection of γ is crucial to reduce the error

and there are many methods to select its value. In this paper, we have used the GA adjustment mechanism.

As the MIT rule is a gradient scheme which aims to minimize the squared model error e^2 [22], the change in the parameter is in the negative gradient of J .

If the process is linear with the transfer function while the $k.G(s)$ equation is unknown, the underlying design provides a system with the transfer function $k_m.G(s)$, where the value of K_m is known [2, 23]. From Equation 5:

$$E(s) = KG(s)U(s) - k_m G(s)U_c(s) \tag{8}$$

Defining a control law:

$$U(s) = \theta U_c(s) \tag{9}$$

Substituting Equation 9 in Equation 8:

$$E(s) = kG(s)\theta U_c(s) - k_m G(s)U_c(s) \tag{10}$$

Taking the partial derivative:

$$\frac{\partial E(s)}{\partial \theta} = kG(s)U_c(s) \tag{11}$$

From Figure 1:

$$y_m = k_m G(s)U_c(s) \text{ so, } G(s)U_c(s) = \frac{y_m}{k_m}$$

substitute it in (11).

$$\frac{\partial E(s)}{\partial \theta} = \frac{K}{k_m} y_m(s) \tag{12}$$

Substituting Equation 12 in Equation 7,

$$\frac{d\theta}{dt} = -\gamma e \frac{k}{k_m} y_m(s) = -\gamma e y_m \tag{13}$$

Equation 13, which shows the law for adjusting the parameter θ , can be represented in Figure 3 as such:

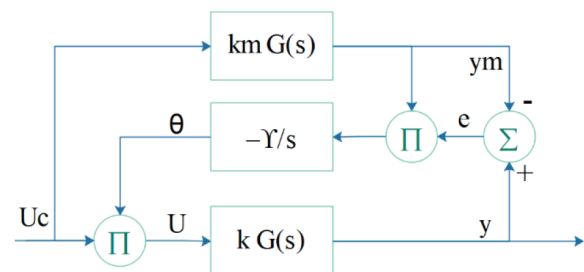


Fig. 3. MIT rule for adjusting feed forward gain

A reference model, whose pole positions determine the stability of the whole system, has been selected. For the output y_m , which is the desired position of the ball on beam x , the input of the reference model is $U_c(s)$, which is the voltage to the motor that rotates

the beam forward and backward. Reference model parameters have been selected so that the poles of the transfer function at x_1 and x_2 are placed on the left half of the s -plane. For the selected system, the ball-beam system, the most important specifications that need to be considered are the overshoot and settling time. This is so that the ball will reach its desired position in a specific time (settling time) while not going far from the desired position (overshoot). For this research, the required $\% OS \leq 10\%$, while $T_s \leq 3$ sec. To achieve these values, the transfer function with poles $x_1, 2 = -2.5 \pm i1.3229$ will be chosen for the reference model.

The transfer function $G_m(s)$ of the reference model is defined as:

$$G_m(s) = \frac{y_m}{U_c} = \frac{k_m}{(s+x_1)(s+x_2)} = \frac{8}{s^2+5s+8} \quad (14)$$

4. MRAC with LQR Controller and GA Optimization Method

As the adjustment mechanism requires a method to choose θ values, the GA optimization method will be used with the MRAC. This combination is called the Genetic Model Reference Adaptive Control (GMRAC). Figure 4 shows the schematic representation of GMRAC (Fig. 4).

The error between the outputs of the reference model and the plant is used to drive the linear quadratic regulator (LQR) controller parameters. The reference model is designed based on both control specifications and the position controller. This appropriate selection of a reference model leads to the stabilization of the entire system. To design the genetic adaptive controller, the behavior of the ball and beam system with the output of the reference model will be used.

The genetic algorithm (GA) can be applied to tune the weight matrices Q and R of the LQR controller gains, which are unknown and approximated to reference values per requirement, to ensure an optimal control performance at nominal operating conditions. By using the approximation and adaptation of the reference model, the error derivatives will be calculated based on the GA [24].

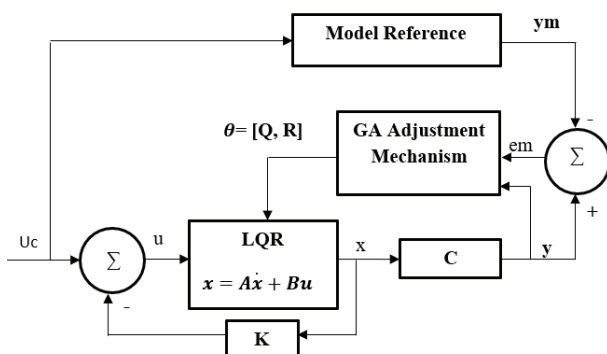


Fig. 4. GA-tuned LQR controller based on model-reference approach

For the LQR controller, the cost function used to find values of Q, R , which are the control input (θ), is represented in Equation 15. The goal is to reduce its value to the minimum.

$$J = \int_0^{\infty} (x^T Q x + U_c^T R U_c) dt \quad (15)$$

where R can be defined as the control-weighting matrix and Q is the state-weighting matrix. They are usually square and symmetric, and their choosing will be used to penalize the control signal and state variables respectively. Choosing a larger R means keeping the control input $u(t)$ smaller to keep J small, while choosing a larger Q means keeping the state variables $x(t)$ smaller.

The other element that needs to be found is the P matrix, which represents the solution of the Algebraic Riccati equation, and it is given in Equation 16.

$$PA + A^T B + Q - PBR^{-1}P^T P = 0 \quad (16)$$

To find the K matrix, Equation 17 can be used.

$$K = R^{-1}B^T P \quad (17)$$

The optimal control signal (u) can be found using Equation 18.

$$u = -K X(t) \quad (18)$$

where $K = [k_1 \ k_2 \ \dots \ k_n]$, $X = [x_1 \ x_2 \ \dots \ x_n]^T$

for our system, $n=4$, where the system has four state variables. The closed-loop system that has the optimal Eigen values is given by:

$$\dot{x} = A_c x = (A - B K) x \quad (19)$$

The genetic algorithm is a random search method that copies the process of natural evolution. The GA begins with no awareness of the accurate solution while relying on the response from its environment and evolution operators to find the best solution. The application of the basic operations permits the creation of new individuals, which have the opportunity to be better than their parents.

The process above will keep repeating until it reaches individuals that represent the optimal solution. The architecture of the GA is shown in Figure 5 (Fig. 5) [25, 26, 27].

The tuning procedure using the GA starts with the definition of the chromosome representation (θ) where $\theta = [q_{11}, q_{22}, q_{33}, q_{44}, R]$. As illustrated in Figure 6, the chromosome is defined by five values that correspond to the five gains to be adjusted in order to achieve satisfactory behavior. [28]

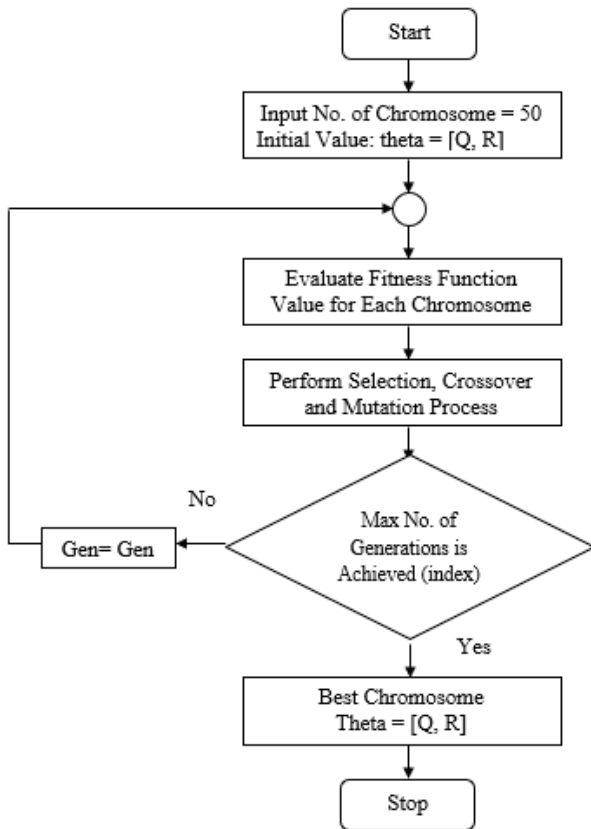


Fig. 5. Simulation flow chart for the computation of GA-LQR controller parameters

q11	q22	q33	q44	R
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Fig. 6. Chromosome Definition $\theta = [Q, R]$

Choosing the suitable GA tuning strategy to pick the target work is considered to be the most essential step, and it has been utilized to assess the fitness value of every chromosome.

The objective function, J , is the sum of the square error between the tracking errors of ball-beam and the reference model along the same trajectory. It is crucial to use the squared error in the objective function in order to have more accurate results for smaller values of error.

$$J = \sum (y - y_m)^2 / 2$$

where y is the system response, while y_m is the model response.

The genetic algorithm parameters chosen for the purpose of tuning are shown in Table 2. They were chosen depending on the system specifications, where these parameters are different from one system to another.

Tab. 2. Parameters of GA.

GA property	Value/Method
Population Size	50
Maximum Number Of Generations	100

(continued)

Tab. 2. Continued.

GA property	Value/Method
Fitness Function	$J(\theta) = \frac{e^2(\theta)}{2}$
Selection Method	Normalized Geometric Selection
Probability Of Selection	0.05
Crossover Method	Scattering
Mutation Method	Uniform Mutation
Mutation Probability	0.01

5. MRAC Simulation and Results

The simulation of the GMRAC with the system has been carried out with MATLAB and the Simulink in order to examine its effectiveness. MATLAB has m-files that can be used to build the controller and the optimization method; in the same time, Simulink can be used to show the results and analyze them. The first step of the simulation is running the GA program. For the GA, the convergence curve for each gain is called a particle. These particles, $q11, q22, q33, q44$ and R , are plotted in Figure 7 with population size 50 to give an initial idea how the GA converged to its final value.

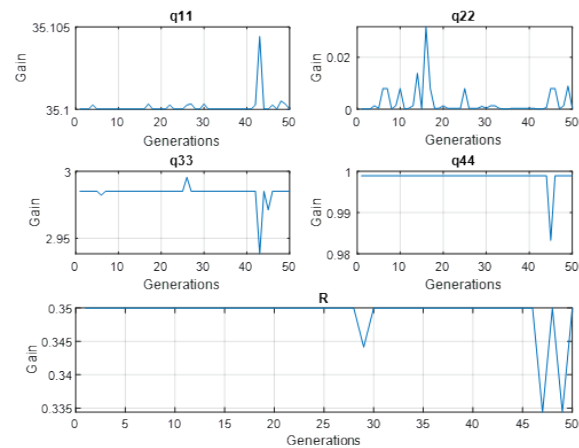


Fig. 7. Convergence curve for Q and R matrices of LQR Controller with population size 50 and reference model Gm

The control weight matrix R and the state matrix Q obtained below are usually square and symmetric.

$$Q = \begin{bmatrix} 35.1 & 0 & 0 & 0 \\ 0 & 0.000191313 & 0 & 0 \\ 0 & 0 & 0.99505 & 0 \\ 0 & 0 & 0 & 0.998911 \end{bmatrix}$$

$$R = 0.349986$$

After finding the Q and R -values, the closed loop poles of the system and controller are:

$$P_{cl} = [-108.87; -1.81; -0.9 \pm i1.55]$$

In addition, the solution of the algebraic Riccati equation matrix P can be found to be:

$$P = \begin{bmatrix} 39.4176 & 22.1122 & 22.7911 & 0.2022 \\ 22.1122 & 18.7683 & 26.1233 & 0.2340 \\ 22.7911 & 26.1233 & 54.8376 & 0.4954 \\ 0.2022 & 0.2340 & 0.4954 & 0.0091 \end{bmatrix}$$

Finally, the feedback gain matrix K can be found using Equation 17. After running the simulation for the whole system, the response control law u is shown in Figure 8.

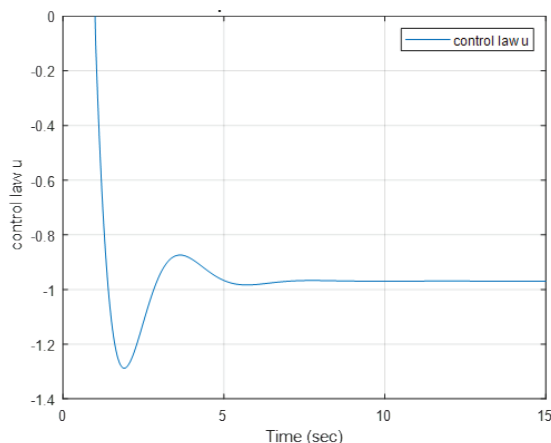


Fig. 8. Response of control law u

The response of the LQR controller tuned by the GA, according to the fitness function with reference model (G_m), are illustrated in Figure 9.

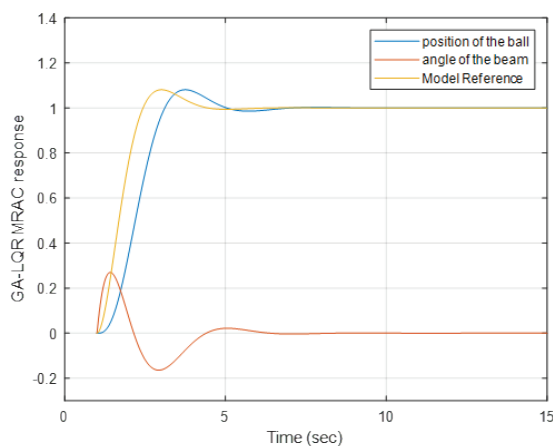


Fig. 9. Response of ball-beam using MRGAC-LQR controller for reference model G_m

6. Conclusion

In this paper, the model of the ball-beam system has been presented and discussed in detail, and a linearized model with the modern state-space method has been used around the horizontal region. A Model Reference Adaptive Control (MRAC) using the MIT rule with the LQR controller was designed to control the position of a ball over a beam. As an optimization

method, the genetic algorithm (GA) has been used for parameter tuning of the LQR controller.

A test of these results has been performed on SIMULINK, and the results show satisfactory performance. Adaptation of LQR based on MRGAC techniques improves the performance of the system, thus bringing up quick tracking and steady state control (% OS \leq 10%, while $T_s \leq$ 3 sec).

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