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## ORIGINAL PAPER

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# VOLUMES DETERMINATION IN TERMS OF VARIOUS DATA DENSITY AND SURFACE DIVERSITY

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## ABSTRACT

<b>Purpose</b>	The aim of this paper is to assess the impact of input data density and diversity on surfaces obtained using the terrestrial laser scanning (TLS) method for creating digital elevation model (DEM). For this we can use several approaches, while we have chosen an intermediary parameter – volume calculation, which is in practice the most frequently requested requirement from surveyors.
<b>Methods</b>	Precise terrestrial measurement and terrestrial laser scanning were used to ensure that detailed knowledge about the surface and volumes of two piles of earth and a stone pit in comparison with theoretical defined surfaces was obtained.
<b>Results</b>	Mathematically defined surfaces generally have smooth shapes, and thus the effect of different density on the input data is less apparent in the final comparison of volumes. In our case the results for most of the different interpolation methods and the different density of the input data was less than 0.5%. From the experimental measurements of the two earth bodies and the quarry, which have an irregular shape with unsmooth surfaces, we can only test the relative precision of the calculated volumes to the data with the highest density. Experimental measurements in the area of the quarry, where the scanned surface was uneven and considerably different in height, confirmed the assumption that a vastly irregular surface should exhibit more significant variations than a smooth surface, but for the nearest neighbour method relative errors under 1% were achieved.
<b>Practical implications</b>	According to the results from the analysis above, the lower density of input data we have, the lower the precision of calculating volumes we can assume, but it is interesting that we did not achieved significantly worse results with strongly irregular surfaces compared to a less irregular surface.
<b>Originality/value</b>	The input values for the analysis of theoretically defined surfaces were obtained by the calculation of integral calculus and earth-moving bodies and quarry from an experimental measurement terrestrial laser scanning method and were used in Slovakia for the first time.

## Keywords

*theoretical surface, terrestrial laser scanning, volume calculation, density and diversity of surface*

## 1. INTRODUCTION

The computer environment together with software enables the creation of digital models of a selection of different types of interpolation methods. The choice and the employment specific interpolation method influence several factors that are linked with the character of the modelled area or the specific exploitation of the executed model.

The accuracy of the digital elevation model (DEM) is decisive in its use, which is in addition to terrain roughness, affected by the interpolation function, interpolation method and the attributes of the input data (Li, 1992; Li, Zhu, & Gold 2004; Aguilar, Agüera, Aguilar, & Carvajal, 2005; Chaplot et al., 2006). The most commonly used methods are interpolation and

approximation, eg. Kriging, the inverse distance method, the nearest neighbour method and the splines method.

An important foundation for creating DEM is having a finite number of points on the surface. For the collection of spatial data, surfaces or objects from smaller scale surveying methods are usually used, such as conventional methods using universal measuring stations and methods based on GNSS and terrestrial laser scanning (TLS). TLS methods provide high accuracy input data, but on the other hand, if we get denser data collection, they are time consuming (Gašinec, Gašincová, Černota, & Staňková, 2012)

Theoretical assessment can be made on figures whose surface is generated on the basis of known functions of planar coordinates and the volume can be calculated using integral

calculus (Yanalak & Baykal, 2003; Easa, 1998; Chen & Lin, 1991), or on the figures, where the exact volume is known (Yilmaz, 2009). Secondly, comparison of results obtained from measurements of different input data densities or by comparison with the results of various methods can be determined by their relative accuracy (Křemen, Pospíšil, & Koska, 2009). In this paper the theoretical surfaces and the surfaces measured by terrestrial laser scanning have been analysed.

**2. ANALYSIS ON THE THEORETICAL SURFACES**

For this purpose, we defined a rectangular area with the dimensions 20 × 30 m and its origin is given the planar coordinates  $x = 0$  m and  $y = 0$  m (Fig. 1). Subsequently, within this area and on its borders, we created sets of approximately regularly distributed points with a density from 20 mm up to 1 m (Fig. 1). In order to create the theoretical surfaces over the defined area, we used two functions for all sets of points:

- theoretical surface A:  $z(x, y) = \sqrt{\frac{x \cdot y}{100}}$ ,
  - theoretical surface B:  $z(x, y) = 7 - \sqrt[3]{(x-10)^2 + (y-15)^2}$ ,
- where  $x$  and  $y$  are the planar coordinates of the points.

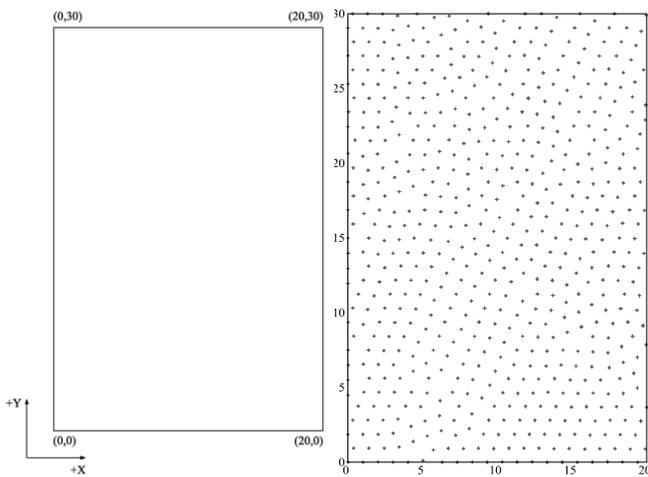


Fig. 1. Defined area (left) and a set of points with a density of 1 m (right)

Based on these sets of the spatial coordinates of points, we created the grid DEMs using the nearest-neighbour, the inverse distance squared weighted and the Kriging method from Surfer software. The grid size was adjusted according to the density of the source data (McCullagh, 1998). Graphical representations of the contoured maps and DEMs of both theoretical surfaces are shown in figures 2 and 3.

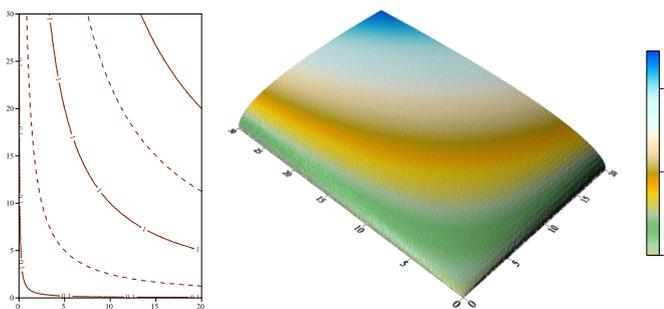


Fig. 2. The contoured map and the DEM visualization of the theoretical surface A

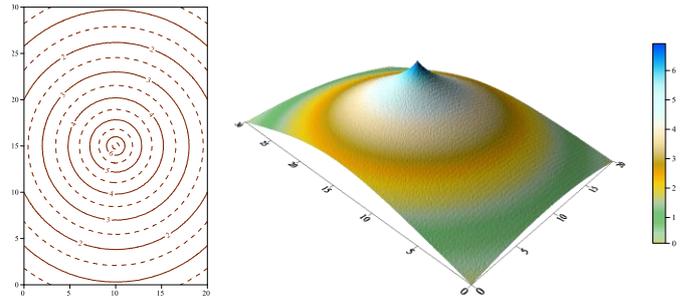


Fig. 3. The contoured map and the DEM visualization of the theoretical surface B

Calculation of the volumes from the generated DEMs was also carried out using the Surfer program and the cross-section method. As a reference plane we chose a horizontal plane with a height of 0 m. The correct values of the volumes were determined using the integral calculus:

$$V = \int_{x_{\min}}^{x_{\max}} \int_{y_{\min}}^{y_{\max}} z(x, y) dy dx \tag{1}$$

The correct volume between the horizontal plane with a height of 0 m and theoretical surface A is 653.19 m<sup>3</sup> and with theoretical surface B it is 1530.20 m<sup>3</sup>. To evaluate the effect of the density, relative errors were calculated using the following formula:

$$r = \frac{V - V_{DEM}}{V} \cdot 100 \tag{2}$$

where  $r$  is the relative error in [%],  $V$  – the volume calculated using the integral calculus,  $V_{DEM}$  – the volume determined using the cross-sections from the DEM. The results obtained for both theoretical surfaces are shown in tables 1 and 2.

Table 1. Relative errors (%) for theoretical surface A

Density [mm]	Nearest neighbour	Inverse distance	Kriging
20	–	–	–
50	0.00	0.01	0.00
100	0.01	0.02	0.01
200	0.04	0.07	0.04
300	0.05	0.12	0.07
400	0.10	0.20	0.10
500	0.21	0.26	0.14
750	0.19	0.49	0.29
1000	0.11	0.76	0.41

Table 2. Relative errors (%) for theoretical surface B

Density [mm]	Nearest Neighbour	Inverse distance	Kriging
20	–	–	–
50	0.00	0.00	0.00
100	0.00	0.00	0.00
200	–0.01	0.01	0.01
300	0.01	0.02	0.01
400	–0.01	0.05	0.02
500	0.11	0.07	0.03
750	0.12	0.11	0.07
1000	0.25	0.35	0.13

From the above results it can be observed that the regular surfaces have high accuracy when calculating volumes, we have therefore focused on a further experiment, surfaces with an irregular shape.

**3. RESULTS AND DISCUSSION**

For practical measurement and subsequent analysis, two piles of earth in the shape of an irregular truncated cone were

chosen. The height of both piles of earth was approximately 3.5 m and the diameter of the smaller pile was approximately 15 m and 30 m for the larger pile. The Trimble GX scanner with the Trimble accessories was used for scanning (Fig. 4). This system allows for scanning of up to 350 m with a speed up to 5000 points per second.



Fig. 4. The Trimble GX scanner (left) and the Trimble planar target (right)

**3.1. Analysis on experimentally measured surfaces**

We defined the resolution of scanning 20 mm at 50 m and the orientation of the scanner was provided by measuring the Trimble planar targets (Fig. 4). Scanning of the larger pile of earth (PE-B) was performed using 9 scanner stations (5001–5009) and to more detailed coverage of the upper part of the pile, 4 stations (5006–5009) were placed directly on its upper surface. The smaller pile of earth (PE-S) was scanned together using 7 stations (5001–5003, 5007, 5010–5012), and 2 of these stations were also on its upper surface (Fig. 4).

Data from TLS was loaded into the Trimble RealWorks software for processing. Because the scanner captures everything within its field of view, the point clouds were filtered from the unwanted ambient vegetation and objects. Thus the prepared point clouds were resampled into the datasets of spatial coordinates of points with densities from 20 mm to 1 m. The other steps of the DEM construction were the same as those used for theoretical surfaces (Fig. 5 and 6). The volumes were computed using the Surfer software and the cross-section method with respect to a horizontal plane with a height of 136,60 m for PE-S and of 136,30 m for PE-B.

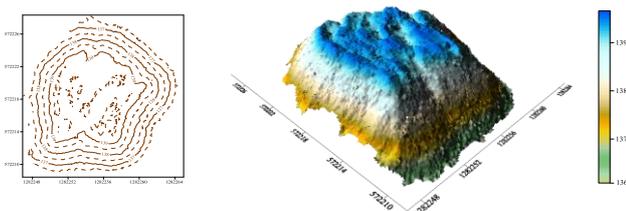


Fig. 5. The contoured map and the DEM visualization of PE-S

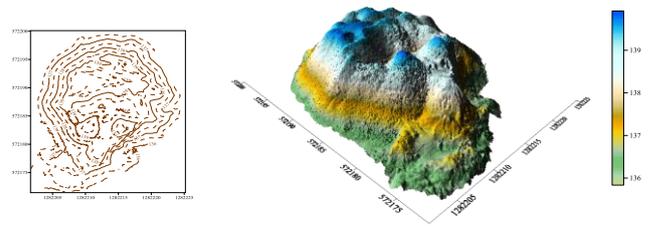


Fig. 6. The contoured map and the DEM visualization of PE-B

Because the exact volume of both piles was unknown, relative errors were calculated with respect to the 20 mm DEM volume. The results obtained for both piles are shown in tables 3 and 4.

Table 3. Relative errors (%) for PE-S

Density [mm]	Nearest neighbour	Inverse distance	Kriging
20	–	–	–
50	0.04	0.05	0.04
100	0.09	0.11	0.07
200	0.18	0.26	0.14
300	0.18	0.41	0.21
400	0.24	0.48	0.17
500	0.41	0.70	0.34
750	0.62	1.32	0.47
1000	0.58	1.65	0.10

Table 4. Relative errors (%) for PE-B

Density [mm]	Nearest neighbour	Inverse distance	Kriging
20	–	–	–
50	0.02	0.02	0.03
100	0.07	0.06	0.07
200	0.26	0.22	0.20
300	0.15	0.26	0.23
400	0.22	0.42	0.30
500	0.39	0.46	0.34
750	0.56	0.73	0.49
1000	1.16	1.54	0.73

**3.2. Analysis of data density in a stone pit**

For the second experiment a former stone pit was chosen. The stone pit is located in the Klubina cadastre unit in northern Slovakia. The stone pit was used for sandstone mining and has been closed for almost 25 years. The width of the stone pit is approximately 280 m with a height difference of 80 m (Fig. 7).



Fig. 7. Stone Pit

The stone pit measured consists of a terraced hillside; weathering together with water and wind erosion has caused disruption at particular height levels. These factors mean that measurements by means of conventional methods using

a total station or GNSS would be difficult and insufficient in terms of the accuracy and detail of any 3D model created. In such cases terrestrial laser scanning represents a suitable method in terms of safety, point density, speed and accuracy.

Scanning was carried out from 5 stations (3 stations in front and 2 stations on the side of the stone pit). Unwanted vegetation from the acquired point cloud was removed using TRW tools. For this purpose the Sampling tool was used to divide the whole area into several smaller regions in which it was possible to accurately remove unwanted vegetation. After removing the noise, these regions were merged into one single point cloud which served as the basis for creating the stone pit 3D model (Fig. 8).

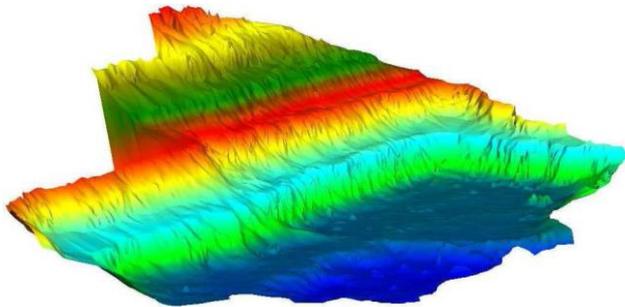


Fig. 8. Visualization of the stone pit DEM

The point cloud was resampled into datasets with spatial coordinates with a point density of 100 mm to 2 m. In the subsequent steps of 3D model creation the same approach as in the previously described experiment was used. The volumes were computed using Surfer software and the cross-section method with respect to the horizontal plane with a height of 460.00 m. Relative errors were calculated with respect to the volume obtained from the 100 mm density dataset. Results are given in table 5.

Table 5. Relative errors (%) for the Stone Pit (bottom plane 460 m above sea level)

Density [mm]	Nearest neighbour	Inverse distance	Kriging
100	–	–	–
200	0.0	0.1	0.2
300	0.0	0.1	0.4
400	0.1	0.2	0.5
500	0.3	0.4	0.5
750	0.1	0.4	1.0
1000	0.3	0.8	1.1
1500	<b>0.9</b>	<b>1.6</b>	0.9
2000	<b>0.9</b>	<b>1.9</b>	<b>1.5</b>

The relative errors (Table 5) for the nearest neighbour method did not reach a value of 1%. Therefore, we decided to verify this fact by analysis of another part of the stone pit with a base heights of 480 m and 500 m. The relative errors of volumes were only calculated for densities from 500 mm to 2 m with respect to the volume obtained from the 100 mm density dataset. Results are given in tables 6 and 7.

Table 6. Relative errors (%) for the Stone Pit (bottom plane 480 m above sea level)

Density [mm]	Nearest neighbour	Inverse distance	Kriging
100	–	–	–
500	0.4	0.7	1.1
750	0.1	0.7	2.0
1000	0.5	1.3	2.4
1500	1.1	2.5	2.8
2000	1.1	3.2	3.9

Table 7. Relative errors (%) for the Stone Pit (bottom plane 500 m above sea level)

Density [mm]	Nearest neighbour	Inverse distance	Kriging
100	–	–	–
500	0.5	1.1	2.3
750	0.1	1.2	3.9
1000	0.6	1.9	4.7
1500	1.2	3.4	5.8
2000	1.1	4.2	8.2

#### 4. CONCLUSIONS

According to the analysis results, the mathematically definable surfaces have a generally smooth shape, and thus the effect of a different density of the input data is less apparent in the final comparison of the volumes. In our case the results for most of the different interpolation methods and the different density of the input data varied by up to 0.5%.

From the experimental measurements of the two earth bodies, which have the irregular shape of a truncated cone with unsmooth surfaces, we can only test the relative precision of the calculated volumes to the data with the highest density (20 mm). The largest relative deviations for the larger natural body showed that when using the smallest density of the test data they were equal to 1000 mm, for the smaller natural body it was for the data with density of 750 mm and 1000 mm. Exceeding 1% took place primarily when using 1000 mm density of input data.

The second experimental measurements in the area of the quarry where the scanned surface was uneven and considerably different in height, confirmed the assumption that strongly irregular surface should exhibit more significant variations than a smooth surface. Here we tested the relative volume errors between data with a density of 100 mm to the data with a density of up to 20 000 mm. At a density of 1000 mm, the relative error when using the method of the nearest neighbour and the method of inverse distance was below 1% (0.3%, 0.8%), with the method of kriging just above 1% (1.1%). The largest relative errors were achieved when using input data at a density of 2000 mm, and despite such a low density, we calculated a value of 0.9% using the method of nearest neighbour. Using the method of inverse distance and kriging the values were respectively 1.9% and 1.5%. Following further calculations of the relative volume error for density from 500 mm to 2000 mm for the new DEM defining the bottom edge at 480 meters and 500 meters above sea level we verified the authenticity of the results obtained earlier. Nearly identical results were obtained using the nearest neighbour method, but when using the method of inverse distance and kriging there was a deterioration of relative errors of volumes determination.

From the aforementioned facts it can be stated that the lower density of input data we have, the lower the precision of calculating the volumes we can assume, but it is interesting that we did not achieve significantly worse results with strongly irregular surfaces at an elevation of 460 m n. m. compared to less irregular surfaces.

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