Control and Cybernetics

vol. **45** (2016) No. 2

Analysis of strategies in a monetary-fiscal game. The case of Poland*

by

Lech Kruś and Irena Woroniecka-Leciejewicz

Systems Research Institute, Polish Academy of Sciences,
Newelska 6, 01-447 Warsaw, Poland
Lech.Krus@ibspan.waw.pl
Irena.Woroniecka@ibspan.waw.pl

Abstract: A monetary-fiscal game describing the interactions of the fiscal and monetary authorities is formulated and analyzed. A macroeconomic model for the Polish economy has been formulated on the basis of the concept of New Neoclassical Synthesis and respectively extended so as to accommodate the effects of fiscal policy. Several variants of the model have been estimated using statistical data for the Polish economy. It is assumed in the game that each party (monetary and fiscal) tries to achieve its own goal: the fiscal authority – the assumed GDP growth, and the monetary authority – an inflation level. The best response strategies of the authorities and the Nash equilibria are calculated and analyzed in two cases, namely when the decisions are made simultaneously and sequentially. The simulation results obtained indicate that when the authorities try to achieve independently their goals, in a general case the Nash equilibrium is not Pareto optimal. The best response strategies may lead to conflict escalation and to results which are not beneficial for both parties.

Keywords: monetary-fiscal game, macroeconomic modeling, Nash equilibria, Pareto optimality

1. Introduction

The paper deals with the choice of policy mix in the context of mutual decision conditioning between the fiscal authority (the government) and the monetary authority (the central bank). Mathematical modeling, game theory and multicriteraia optimization methods are applied in the respective analysis. The policy mix means in this context a combination of monetary and fiscal policies with the given levels of restrictiveness/expansiveness of each of them.

*Paper presented in its original version at the BOS2016 conference of the Polish Operational and Systems Research Society, Warsaw, October 2016. Accepted for publication in November 2016.

There exists a relatively rich bibliography dealing with interactions of fiscal and monetary policies. Blinder (1983), and after him Bennett & Loayza (2001), considered a simplified monetary-fiscal game with the fiscal and monetary authorities as players, having respectively two fiscal and two monetary strategies: the restrictive and expansive ones. The authors mentioned showed that independent actions of the authorities may lead to the Nash equilibrium (Nash, 1951), which is not Pareto optimal. They presented a similar interpretation relating to the prisoner's-dilemma problem and similar arguments for coordination of the policies. Nordhaus (1994) analyzed the problem of independence versus coordination of fiscal and monetary policies also using a monetary-fiscal game. This game was based on a simple hypothetical macroeconomic model with utility functions of the government and of the central bank, dependent on their policy instruments. He presented an extended discussion, relating to the Nash equilibria, Pareto optimality of payoffs, possible conflicts of interests of the authorities, and suggestions for policy coordination. The Nordhaus game model constituted a starting point for further research. In the monograph of Marszałek (2009, pp. 131-132) a list and a characterization of selected game models, describing relations between the government and the central bank is presented. Dixit & Lambertini (2001), when considering a monetary-fiscal game, underlined the importance of players' credibility and fiscal discipline for the results of the game. Lambertini & Rovelli (2003) continued the above research by comparing the Nash and Stackelberg games. Many authors discussed and explained the fact that solutions in the models of non-cooperative monetary-fiscal games, referred to above, are not optimal and lead to a suboptimal policy mix. There are also some papers, discussing policy mix problems, formulating arguments for policy coordination and presenting interesting results with the use of statistical data for Poland: Darnault & Kutos (2005), Stawska (2014). Libich, Nguyen & Stehlik (2014) presented analysis and comparison of selected countries in the so called *monetary vs fiscal leadership space*. Poland is located in the central part of the space. This means that in the case of Poland the fiscal authority does not dominate over the monetary authority nor vice versa. Let us also note that in the case of Poland there are no publications dealing with interactions of the fiscal and monetary policies, analyzed with the use of computational game models. The research presented in this paper tries to fill this gap.

The paper presents current results of the research carried on within the game theory, macroeconomic modeling and optimization methods applied in the analysis of the policy mix problem. We try to analyze the efficiency of decisions made by the authorities, considering the Nash equilibria and Pareto optimality of their decisions. We try also to answer the following questions: how priorities of the monetary and fiscal authorities relate to the choice of the authorities' strategies; when and under what conditions the independent choice of strategies by the monetary and fiscal policies leads to the decisions which are economically effective and when a coordination of the decisions is required.

A non-cooperative game called monetary-fiscal game is formulated and analyzed, in which the fiscal and monetary authorities play the roles of players.

Payoffs table		**Central bank – the monetary policy** ← restrictive			expansive →
		Monetary strategy M_1 (interest rate r_1)	Monetary strategy M_2 (interest rate r_2)	…	Monetary strategy M_n (interest rate r_n)
Government – fiscal policy ← expansive restrictive →	Fiscal strategy F_1 (budgetary deficit b_1)	p_{11} y_{11}	p_{12} y_{12}	… …	p_{1n} y_{1n}
	Fiscal strategy F_2 (budgetary deficit b_2)	p_{21} y_{21}	p_{22} y_{22}	… …	p_{2n} y_{2n}
	…			…	
	Fiscal strategy F_m (budgetary deficit b_m)	p_{m1} y_{m1}	p_{m2} y_{m2}	… …	p_{mn} y_{mn}

Figure 1. The monetary-fiscal game – the table of outcomes (Source of all tables and figures: own elaboration of the authors)

Strategies of the monetary authority relate to the monetary policies having different restrictiveness/expansiveness levels. Similarly, strategies of the fiscal authority mean the budget policies featuring different restrictiveness/expansiveness levels. The level of restrictiveness of each policy is defined by the value of the respective policy instrument, that is – the real interest rate in the case of monetary policy and the budget deficit in relation to the GDP in the case of fiscal policy. Each authority tries to achieve its respective economic target: a desired value of the GDP dynamics in the case of the fiscal authority, and a desired value of inflation rate in the case of the monetary authority. It is assumed that the authorities make decisions independently.

Outcomes of the game in the discrete form are presented in Table 1. Payoffs in the table are denoted in the following manner: y_{ij}– payoff of the fiscal authorities (GDP growth rate) in the case where the government applies the fiscal strategy $F_i$and the central bank applies the monetary strategy M_j; p_{ij}– payoff of the monetary authorities (inflation) for the same pair of policies. The symbol $b_i$denotes the budgetary deficit in relation to GDP, corresponding to the i-th fiscal strategy, while $r_j$denotes the real interest rate, ascribed to the j-th monetary strategy. It is assumed that the fiscal and monetary authorities take decisions independently, and the Nash equilibrium state in such a game is identified with the choice of a given combination of the budgetary and monetary policies.

A macroeconomic model for the Polish economy has been formulated on the basis of the concept of New Neoclassical Synthesis. It includes four fundamental equations, referring to the output gap, inflation, expected inflation, and the

Taylor rule of the interest rate. It allows for analyzing the economic situation over time. The model takes into account the interest rate effect on the economy. The classical form of the model has been extended to include also the effects of the influence of fiscal policy on inflation. The model has been estimated using quarterly time series of data for Poland from the period 2000-2014.

A computer-based system calculating the results of the game has been constructed using the above model. A sequence of simulations was performed, in which payoffs of the game were derived for alternative monetary and fiscal policies. This paper presents a continuation of the research described in the previous papers of the authors: Woroniecka-Leciejewicz (2010, 2015), and Kruś & Woroniecka-Leciejewicz (2015, 2017).

The paper is organized as follows. The next Section 2 presents the mathematical formulation of the game. The proposed macroeconomic NNS-MFG model is described in Section 3. Section 4 presents the results of model estimation and examples of simulation runs. Analysis of the proposed monetary-fiscal game is shown and discussed in Section 5, which is followed by Conclusions and References. This paper differs from Kruś & Woroniecka-Leciejewicz (2017) in that it presents three different variants of the macroeconomic model with respective estimation results, new simulation results of the game for one stage as well as for sequential decisions made by the monetary and fiscal authorities, and the respective discussion of the results.

2. The monetary-fiscal game - mathematical formulation

Relations between the fiscal and the monetary authorities can be described in terms of a non-cooperative game. It is a single stage, non-zero sum, perfect information game played by the central bank and the government. Each player takes decision independently, while considering the possible reaction of the counter-player. The game is defined in the strategic form as follows:

1. There are two players i =1, 2: the fiscal authority (the government) and the monetary authority (the central bank).
2. For each player a set Ω^i of pure strategies is defined. The strategies of the fiscal authority are those of the budgetary policy – from the extremely restrictive one to the extremely expansive one. The measure, denoted by b, of the degree of restrictiveness/expansiveness of the fiscal policy is constituted here by the level of budget deficit in relation to GDP. The strategies of the monetary authority range also from extremely restrictive to extremely expansive. The measure of the degree of restrictiveness/expansiveness is equivalent simply to the value of the real interest rate and is denoted by r. Let Ωdenote the Cartesian product of the sets of strategies, $\Omega = \Omega^1 \times \Omega^2$.
3. For each player $i = 1, 2$, a function h^i: $\Omega \rightarrow \boldsymbol{R}$ is given, defining the outcome for the player i for the given strategies, applied by both players. The outcome for the fiscal authority is measured by the GDP growth rate, denoted by y, where $y = h^1(b, r)$. In the case of the monetary authority

it is the value of inflation rate, denoted by p, where $p = h^2(b, r)$. The functions $h^i, i = 1, 2$, are defined by the model relations.

4. For each player i =1, 2, a preference relation is given in the set of attainable outcomes. It is assumed here that each authority tries to achieve a given goal: the fiscal authority – a desired value of GDP growth, the monetary authority – a desired value of inflation.

In this paper, two types of the non-cooperative game are analyzed: the first, when the decisions of players are made simultaneously, and the second, when the decisions are made sequentially.

3. The macroeconomic NNS-MFG model

The macroeconomic NNS-MFG model is proposed according to the basic concepts of the New Neoclasscal Synthesis'model (Galí, 2010), and it includes the fundamental equations describing: demand gap, inflation, and the Taylor rule. The last one is formulated in order to describe the interest rate decisions of the central bank. The equations allow for simulating the results of the monetary policy. Additionally, the NNS-MFG model has been extended and takes into account budget expenditures, so as to simulate the results of the fiscal policy. The model includes also the equation of expected inflation. Some delayed variables are introduced into the model, because it is used in recursive calculations to simulate the results of the considered game. Several variants of the model equations are presented below.

3.1. The NNS-MFG model – variants 1 and 2

Equation of the output gap, referring to the dynamic, inter-period version of the IS (investment-savings) curve, describes an aggregated demand as the result of the optimal decisions made by a representative consumer. It has the following form:

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2(r_t - \pi_t^e - r_t^n) + \alpha_3 g_t \tag{1}$$

where

$$x_t = y_t - y_t^n, g_t = G_t - G_t^n, \tag{2}$$

or:

$$x_t = \ln y_t - \ln y_t^n, g_t = \ln G_t - \ln G_t^n \tag{3}$$

The output gap x_t is defined as the difference between the current real production y_t and its natural level y_t^nin the equilibrium state with the perfectly elastic prices. Production is measured by the real Gross Domestic Product (GDP). The natural levels of the product and of the natural interest rate r_t^n, have been calculated using the Hodrick-Prescott filter. The current value of the production gap depends on its delayed value and on the interest rate gap, where the interest rate gap is defined as the difference between the real interest rate and its natural level r_t^n. The real interest rate is calculated as the difference: the nominal interest rate r_t(WIBOR 1M) minus the expected inflation π_t^e. The proposed model takes, additionally, into account the effects of the fiscal policy – the influence of the real budget expenditures G_t on inflation in the gap category, i.e. as the deviation from its natural value G_t^n. The values of G_t^n have been also derived using the Hodrick-Prescott filter. The production gap and the budget expenditure gap are defined in two versions, namely (a) as the absolute deviation from the natural value, (b) as the difference of logarithms.

The inflation equation is known as the New Keynesian version of the Phillips curve. It presents a function of the aggregated supply, based on price decisions of firms in conditions of imperfect competition. Inflation depends on the expected inflation π_t^e and on the output gap x_t. The equation has the form:

$$\pi_t = \beta_0 + \beta_1 \pi_{t-1}^e + \beta_2 x_t. \tag{4}$$

Expected inflation is explained by its delayed value and by inflation in the current quarter. The respective equation has the form:

$$\pi_t^e = \delta_0 + \delta_1 \pi_{t-1}^e + \delta_2 \pi_t. \tag{5}$$

The equation of the interest rate (Taylor rule) describes the rule, defining the nominal interest rate setting by the central bank. The central bank derives the nominal interest rate in reaction to deviation of inflation from the target π_t^* and to the current economic situation, measured by the production gap. In this model it is the inflation target assumed by the National Bank of Poland in the Monetary Policy Guidelines. The equation describes the reaction of the central bank according to the Taylor rule and it has the form:

$$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 (\pi_{t-1} - \pi_{t-1}^*) + \phi_3 x_{t-1} \quad . \tag{6}$$

3.2. The NNS-MFG model – variant 3

Equation of the output gap:

$$\frac{y_t}{y_t^n} = \alpha_0 \left(\frac{y_{t-1}{}^n}{y_{t-1}}\right)^{\alpha_1} \left(\frac{r_t^r}{r_t^n}\right)^{\alpha_2} \left(\frac{G_t}{G_t^n}\right)^{\alpha_3}, \tag{7}$$

where the real interest rate equals the minimal rate minus the expected inflation:

$$r_t^r = r_t - \pi_t^e. \tag{8}$$

The output gap equation in logarithmic form:

$$\ln y_t - \ln y_t^n = \ln \alpha_0 + \alpha_1 \left(\ln y_{t-1} - \ln y_{t-1}^n\right) + \alpha_2 \left(\ln r_t^r - \ln r_t^n\right) + \alpha_3 \left(\ln G_t - \ln G_t^n\right) \tag{9}$$

Inflation equation:

$$\pi_t = \alpha_0 \left(\pi^e_{t-1}\right)^{\alpha_1} \left(\frac{y_t}{y^n_t}\right)^{\alpha_2} \tag{10}$$

and in logarithms:

$$\ln \pi_t = \ln \alpha_0 + \alpha_1 \ln \pi^e_{t-1} + \alpha_2 \left(\ln y_t - \ln y^n_t\right). \tag{11}$$

Equation of the expected inflation:

$$\pi^e_t = \alpha_0 \left(\pi^e_{t-1}\right)^{\alpha_1} \left(\pi_t\right)^{\alpha_2}, \tag{12}$$

and in logarithms:

$$\ln \pi^e_t = \ln \alpha_0 + \alpha_1 \ln \pi^e_{t-1} + \alpha_2 \ln \pi_t. \tag{13}$$

Equation of the interest rate (Taylor rule):

$$r_t = \alpha_0 (r_{t-1})^{\alpha_1} \left(\frac{\pi_{t-1}}{\pi^*_{t-1}}\right)^{\alpha_2} \left(\frac{y_t}{y^n_t}\right)^{\alpha_3}, \tag{14}$$

and in logarithms:

$$\ln r_t = \ln \alpha_0 + \alpha_1 \ln r_{t-1} + \alpha_2 \left(\ln \pi_{t-1} - \ln \pi^*_{t-1}\right) + \alpha_3 \left(\ln y_t - \ln y^n_t\right). \tag{15}$$

4. Model estimation

Three variants of the macroeconomic model have been estimated: variant 1 – the linear model, equations (1), (2), (4), (5), (6); variant 2 – model with equations (1), (3), (4), (5), (6), variant 3 – model with equations (9) through (13). The model variant, described by the equations (9) through (15), was also estimated, but the obtained goodness of fit was not acceptable. Finally, in the third variant, the Taylor rule has been omitted. All variants of the NNS-MFG model were estimated as systems of simultaneous equations using the Three-Stage Least Squares Method (3SLS) from the econometric GRETL package. As indicated before, the time series for the Polish economy from the period of years 2000-2014 (quarterly data) have been used. The statistical data have been collected from the following sources: the Central Statistical Office of Poland, the National Bank of Poland (NBP), and the Ipsos group. The names and the descriptions of the variables used for estimation are presented in Table 1.

4.1. Variant 1

The detailed estimation results for the NNS-MFG model are presented in Table 2 (GRETL package, system of equations, 3SLS). The estimation results show an acceptable goodness of fit. All the variables are statistically significant. The R-squared values exceed 90% in the case of equations 3 and 4. Worse

Table 1. The variables used in the model estimation

Variable	Description
output_gap	The output gap is defined as the difference between the real GDP and the natural level of output presented by the Central Statistical Office in time series according to the principles of the "European System of National and Regional Accounts" (ESA); GDP in constant prices. The natural level is calculated using the Hodrick–Prescott filter.
output_gap_dl	The difference of logarithms: of the output gap and its natural level
inflation	Inflation calculated on the basis of the consumer price index
l_inflation	The logarithm of inflation
expected_infl	Expected inflation measured as the average inflation level expected in the next year (NBP, Ipsos data)
WIBOR	The interest rate WIBOR 1M, nominal, at the beginning of each period (data from Money.pl (http://www.money.pl/))
WIBOR_gap	The interest rate gap measured on the basis of WIBOR 1M
WIBOR_gap_dl	The difference of logarithms: of the WIBOR gap and its natural level
expend_gap	The gap of the expenditure of the public sector
expend_gap_dl	The difference of logarithms: of the expenditure of the public sector and its natural level
infl_target_dif	The difference between inflation and the inflation target (the inflation target: Monetary Policy Guidelines data)
variable_1	The variable one period delayed

estimation results have been obtained in the case of equations 1 and 2, with R-squared values of 63% and 76%, respectively. However, also in this case all the variables are statistically significant. The estimation inaccuracy is caused by model simplifications. The model describes the influence of the economic policies only. It does not describe any influence of exogenous factors in the explicit form. Figure 2 presents the matching of the endogenous variables: the theoretical values calculated by the estimated model vr. the empirical values, calculated by the estimated model, are compared with the empirical values.

4.2. Variant 2

The detailed estimation results for the NNS-MFG model, relative to its variant 2, are presented in Table 3 (GRETL package, system of equations, 3SLS).

Table 2. Model (variant1) - equation system, Three-Stage Least Squares – GRETL outputs

Equation 1: Estimation 3SLS, observations 2001:1-2014:4 (N = 56)					
Dependent variable (Y): output_gap					
	coefficient	std. error	z	p-value	
const	0.0272869	0.119475	0.2284	0.8193	
output_gap_1	0.693819	0.0840444	8.255	1.51e-016	***
WIBOR_gap	-0.424272	0.124742	-3.401	0.0007	***
expend_gap	0.137646	0.0620940	2.217	0.0266	***
Mean dependent var - 0.113802 S.D. dependent var 1.541290 Sum squared resid 46.33381 S.E. of regression 0.909610 R-squared 0.647118 Adjusted R-squared 0.626760					
Equation 2: Estimation 3SLS, observations 2001:1-2014:4 (N = 56) Dependent variable (Y): inflation					
	coefficient	std. error	z	p-value	
const	0.554993	0.189279	2.932	0.0034	***
expected_infl_1	0.753164	0.0577744	13.04	7.61e-039	***
output_gap	0.338427	0.0911186	3.714	0.0002	***
Mean dependent var 2.606757 S.D. dependent var 1.678949 Sum squared resid 36.97738 S.E. of regression 0.812595 R-squared 0.765484 Adjusted R-squared 0.756635					
Equation 3: Estimation 3SLS, observations 2001:1-2014:4 (N = 56) Dependent variable (Y): expected_infl					
	coefficient	std. error	z	p-value	
const	-0.196344	0.102801	-1.910	0.0561	*
expected_infl_1	0.143050	0.0587171	2.436	0.0148	**
inflation	0.915195	0.0741551	12.34	5.41e-035	***
Mean dependent var 2.586357 S.D. dependent var 1.736927 Sum squared resid 11.16849 S.E. of regression 0.446584 R-squared 0.936581 Adjusted R-squared 0.934188					
Equation 4: Estimation 3SLS, observations 2001:1-2014:4 (N = 56) Dependent variable (Y): WIBOR					
	coefficient	std. error	z	p-value	
const	0.265344	0.0946242	2.804	0.0050	***
WIBOR_1	0.922672	0.0131957	69.92	0.0000	***
infl_target_dif	0.196669	0.0290075	6.780	1.20e- 011	***
output_gap_1	0.229699	0.0345724	6.644	3.05e- 011	***
Mean dependent var 5.858571 S.D. dependent var 3.643124 Sum squared resid 8.191705 S.E. of regression 0.382466 R-squared 0.988783 Adjusted R-squared 0.988136					

The estimation results, similarly as those for the variant 1, show an acceptable goodness of fit. All the variables turn out to be statistically significant. Consequently, Fig. 3 shows the matching of the endogenous variables: the theoretical values, calculated with the estimated model, are compared in this figure to the empirical values.

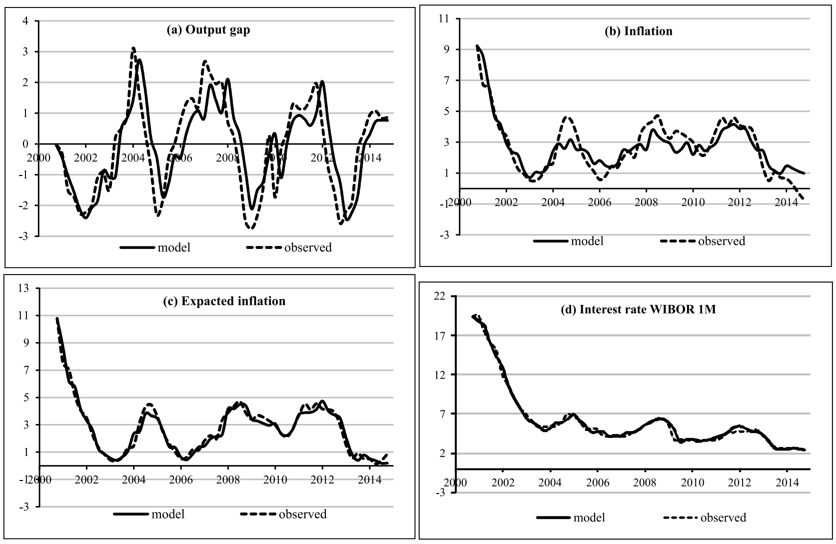

Figure 2. Estimated (3SLS, model: variant 1) and observed values of the variables: (a) output gap, (b) inflation, (c) expected inflation, (d) interest rate WIBOR 1M

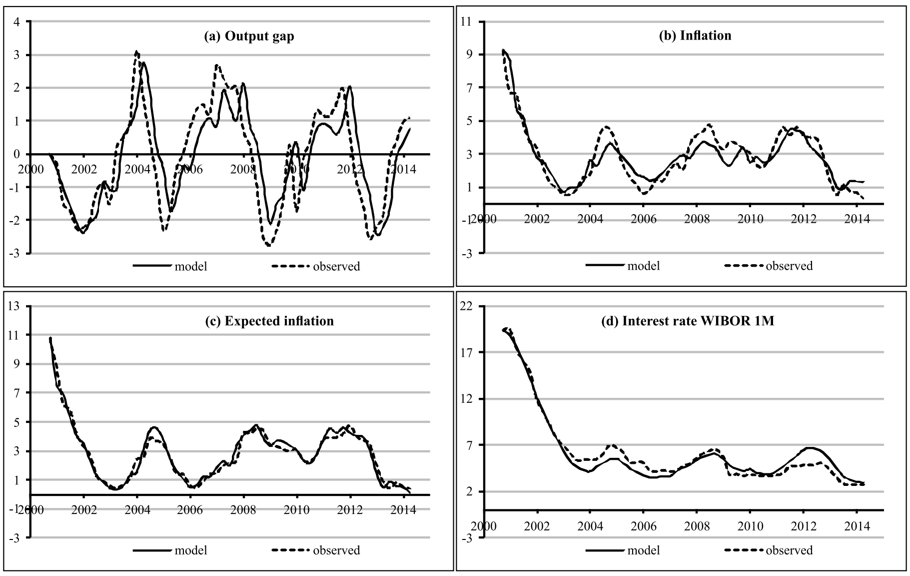

Figure 3. Estimated (3SLS model: variant 2) and observed values of the variables: (a) output gap, (b) inflation, (c) expected inflation, (d) interest rate WIBOR 1M

Table 3. Model (variant 2) – equation system, Three-Stage Least Squares – GRETL outputs

Equation 1: Estimation 3SLS, observations 2001:1-2014:4 (N = 56) Dependent variable (Y): output_gap_dl					
	coefficient	std. error	z	p-value	
const	0.000249665	0.00115176	0.2168	0.8284	
output_gap_dl_1	0.692150	0.0839817	8.242	1.70e- 016	***
WIBOR_gap	- 0.00411401	0.00120112	- 3.425	0.0006	***
expend_gap_dl	0.139269	0.0622089	2.239	0.0252	**
Mean dependent var - 0.001242 S.D. dependent var 0.014871 Sum squared resid 0.004294 S.E. of regression 0.008757 R-squared 0.648625 Adjusted R-squared 0.626760					
Equation 2: Estimation 3SLS, observations 2001:1-2014:4 (N = 56) Dependent variable (Y): inflation					
	coefficient	std. error	z	p-value	
const	0.558213	0.188761	2.957	0.0031	***
expected_infl_1	0.753829	0.0577956	13.04	6.97e-039	***
output_gap_dl	35.0927	9.44080	3.717	0.0002	***
Mean dependent var 2.606757 S.D. dependent var 1.678949 Sum squared resid 36.92949 S.E. of regression 0.812068 R-squared 0.765759 Adjusted R-squared 0.756920					
Equation 3: Estimation 3SLS, observations 2001:1-2014:4 (N = 56) Dependent variable (Y): expected_infl					
	coefficient	std. error	z	p-value	
const	- 0.195819	0.102815	- 1.905	0.0568	*
expected_infl_1	0.142757	0.0587765	2.429	0.0151	**
inflation	0.915305	0.0742366	12.33	6.28e- 035	***
Mean dependent var 2.586357 S.D. dependent var 1.736927 Sum squared resid 11.17172 S.E. of regression 0.446649 R-squared 0.936540 Adjusted R-squared 0.934145					
Equation 4: Estimation 3SLS, observations 2001:1-2014:4 (N = 56) Dependent variable (Y): WIBOR					
	coefficient	std. error	z	p-value	
const	0.267267	0.0943623	2.832	0.0046	***
WIBOR_1	0.922911	0.0131780	70.03	0.0000	***
infl_target_dif	0.196216	0.0289553	6.777	1.23e-011	***
output_gap_dl_1	23.8733	3.58004	6.668	2.59e-011	***
Mean dependent var 5.858571 S.D. dependent var 3.643124 Sum squared resid 8.158654 S.E. of regression 0.381694 R-squared 0.988828 Adjusted R-squared 0.988184					

4.3. Variant 3

The detailed estimation of results for the NNS-MFG model (variant 3) are presented in the subsequent Table 4 (GRETL package, system of equations, 3 SLS).

The obtained goodness of fit, however, is not so good as for the previous two variants. Figure 4 shows the corresponding matching of the endogenous variables: the theoretical values, calculated with the estimated model, are compared there again with the empirical values.

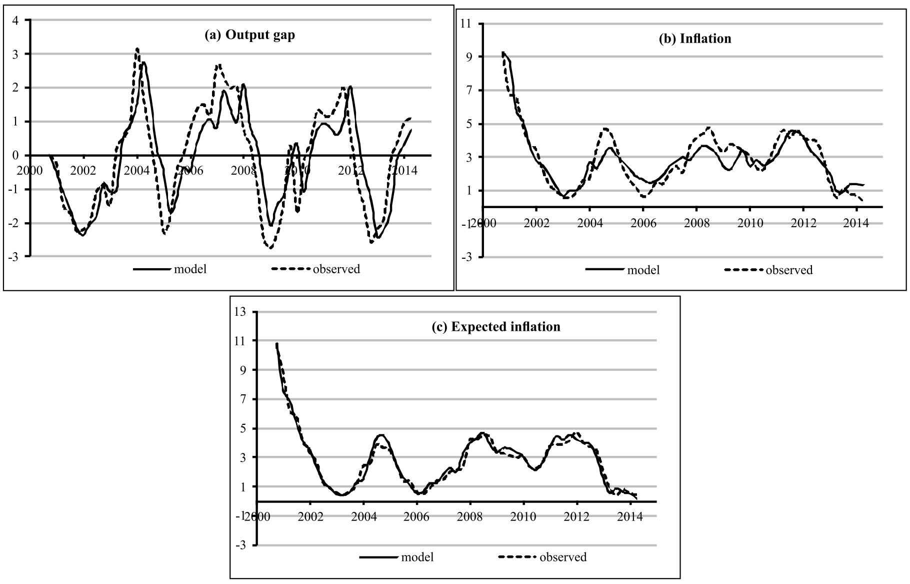

Figure 4. Estimated (3SLS model: variant 3) and observed values of the variables: (a) output gap, (b) inflation, (c) expected inflation

5. Analysis of the monetary-fiscal game

Computer simulations of the game have been carried out using the NNS-MFG model presented in Section 3 for the estimation results shown in Section 4. Payoffs were calculated for different variants of the model and different assumptions regarding the policy-mix. Selected results for the variant 2 of the model are presented.

The initial state of the economy is represented by the model variables on the basis of the empirical data from the last quarter of the year 2000. Model variables are calculated using the NNS-MSG model since the first quarter of 2001, while the nominal interest rate is calculated by the Taylor rule and the public expenditure gap according to the statistical data on the real public expenditure. In a selected period (in the presented results: 8 quarters since the 1^{st} quarter of 2008) an impulse of changing the policy mix is introduced. The instruments of the policy mix, i.e. the real interest rate and the budget deficit in relation to GDP are assumed on a definite constant level for this period of time. After this period of time the real interest rate is derived according to the Taylor rule

Table 4. Model (variant 3) - equation system, Three-Stage Least Squares – GRETL outputs

Equation 1: Estimation 3SLS, observations 2001:1-2014:2 (N=54) Dependent variable (Y): output_gap_dl					
	coefficient	std. error	z	p-value	
const	-0.00032626	0.00125476	-0.26	0.7949	
output_gap__dl _1	0.776879	0.0847354	9.168	4.81E-20	***
WIBOR_gap	-0.00079699	0.00132606	-0.601	0.5478	
expend_gap__dl	0.0807724	0.065497	1.233	0.2175	
Mean dependent var −0.001593 S.D. dependent var 0.015032 Sum squared resid 0.004248 S.E. of regression 0.008869 R-squared 0.645308 Adjusted R-squared 0.624026					
Equation 2: Estimation 3SLS, observations 2001:1-2014:2 (N = 54) Dependent variable (Y): l_inflation					
	coefficient	std. error	z	p-value	
const	0.104439	0.0783159	1.334	0.1823	
l_expected_infl_1	0.887585	0.0770965	11.51	1.14E-30	***
output_gap__dl	22.1922	5.00912	4.43	0.00000941	***
Mean dependent var 0.767283 S.D. dependent var 0.77022 Sum squared resid 8.679739 S.E. of regression 0.400919 R-squared 0.725533 Adjusted R-squared 0.714770					
Equation 3: Estimation 3SLS, observations 2001:1-2014:2 (N = 54) Dependent variable (Y): l_expected_infl					
	coefficient	std. error	z	p-value	
const	-0.0901053	0.0436126	-2.066	0.0388	**
l_expected_infl_1	0.207927	0.0942799	2.205	0.0274	**
l_inflation	0.848633	0.110557	7.676	1.64E-14	***
Mean dependent var 0.724598 S.D. dependent var 0.806199 Sum squared resid 2.441407 S.E. of regression 0.212629 R-squared 0.929671 Adjusted R-squared 0.926913					

and the budget deficit to GDP ratio - according to the ex post data. Effects of the changed policy mix are measured by the average annual production growth and by average annual inflation in the period of 8 quarters since the changes of the policy mix have been introduced.

The sets of admissible values of the policy instruments have been assumed in the form of intervals. The interest rate has been changed in the interval [6%, -1%] and the budget deficit in relation to GDP – in the interval [-1%, 6%]. Selected results are presented and discussed in the further course of this section.

Figure 5 presents a part of the payoff matrix. A low level of inflation can be achieved when a restrictive monetary policy and a restrictive fiscal policy are applied simultaneously. More expansive monetary and fiscal policies lead to an increase of inflation and of the economic growth. On the contrary, more restric-

Figure 5. Table of payoffs

interest rate

budget deficit	5.0%	4.6%	4.2%	3.8%	3.4%	3.00%	2.6%	2.2%
-1.0%	-1.45% / -0.40%	-1.10% / -0.02%	-0.76% / 0.38%	-0.41% / 0.77%	-0.06% / 1.16%	0.28% / 1.56%	0.63% / 1.96%	0.97% / 2.35%
-0.6%	-1.23% / -0.20%	-0.88% / 0.19%	-0.54% / 0.59%	-0.19% / 0.98%	0.15% / 1.37%	0.50% / 1.77%	0.85% / 2.17%	1.19% / 2.57%
-0.2%	-1.01% / 0.01%	-0.67% / 0.40%	-0.32% / 0.79%	0.02% / 1.19%	0.37% / 1.58%	0.72% / 1.98%	1.06% / 2.38%	1.41% / 2.78%
0.2%	-0.80% / 0.21%	-0.45% / 0.61%	-0.11% / 1.00%	0.24% / 1.39%	0.59% / 1.79%	0.93% / 2.19%	1.28% / 2.59%	1.62% / 2.99%
0.6%	-0.59% / 0.42%	-0.24% / 0.81%	0.11% / 1.20%	0.45% / 1.60%	0.80% / 2.00%	1.14% / 2.40%	1.49% / 2.80%	1.84% / 3.20%
1.0%	-0.38% / 0.62%	-0.03% / 1.01%	0.32% / 1.41%	0.66% / 1.80%	1.01% / 2.20%	1.35% / 2.60%	1.70% / 3.00%	2.05% / 3.41%
1.4%	-0.17% / 0.82%	0.18% / 1.21%	0.52% / 1.61%	0.87% / 2.01%	1.22% / 2.41%	1.56% / 2.81%	1.91% / 3.21%	2.25% / 3.61%
1.8%	0.04% / 1.02%	0.38% / 1.41%	0.73% / 1.81%	1.08% / 2.21%	1.42% / 2.61%	1.77% / 3.01%	2.11% / 3.41%	2.46% / 3.82%
2.2%	0.24% / 1.22%	0.59% / 1.61%	0.93% / 2.01%	1.28% / 2.41%	1.63% / 2.81%	1.97% / 3.21%	2.32% / 3.62%	2.66% / 4.02%
2.6%	0.44% / 1.41%	0.79% / 1.81%	1.14% / 2.21%	1.48% / 2.61%	1.83% / 3.01%	2.17% / 3.41%	2.52% / 3.82%	2.87% / 4.22%

tive monetary and restrictive fiscal policies lead to a decrease of the economic growth.

We assume that the monetary and fiscal authorities try to achieve some given targets of their policies. Let the monetary authority assume the inflation goal at the level p^g, and let the fiscal authority try to achieve the GDP growth rate at the level y^g.Let Ω denote the set of admissible pairs (b, r) of strategies. The best response strategies of the authorities can be obtained as solutions of the optimization problems:

Min $|h^1(b, r) - y^g|$with respect to $b \in \Omega^1$ solved for all $r \in \Omega^2$, in the case of the fiscal authority

and

Min $|h^2(b, r) - p^g|$with respect to $r \in \Omega^2$, solved for all $b \in \Omega^1$, in the case of the monetary authority, respectively.

Examples of the best response strategies derived for different targets of the authorities are presented in the following figures. Thus, Fig. 6 shows the best response strategies of the monetary authority for two different targets: inflation = 2% and = 2.5%, and the best response strategy of the fiscal authority for the target: of GDP growth = 3.75%. Then, Fig. 7 shows the best response strategies of the fiscal authority for three targets of GDP growth = 3.5%, 3.75%, and 4%, and of the monetary authority for the target of inflation rate = 2.5%. The Nash equilibria, which are Pareto optimal in the assumed interval of the policy instruments, are shown.

It can be observed how the level of restrictiveness/expansiveness of the monetary policy for the best response strategy depends upon the level of restrictiveness/expansiveness of the fiscal policy. A more expansive fiscal policy leads to a more restrictive monetary policy, taken by the central bank, trying to limit inflation, which tends to exceed the inflation target. If the budget deficit is

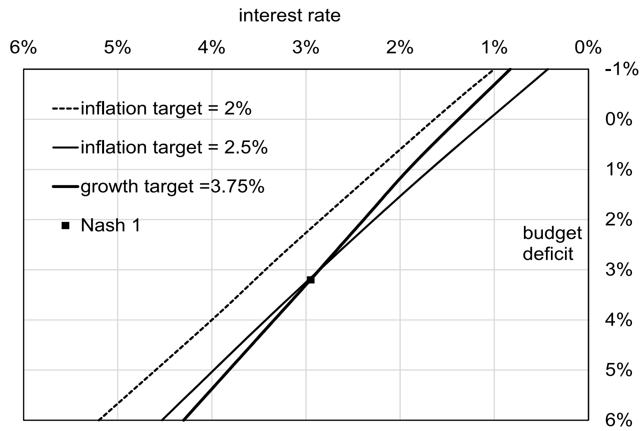

Figure 6. The best response strategies for two different monetary targets and the fiscal target: GDP growth=3.75%

higher, then the required inflation is obtained for respectively higher interest rates. Analogously, if the government carries out a more restrictive budget policy, then the central bank will apply a less restrictive (more expansive) monetary policy with relatively lower interest rates.

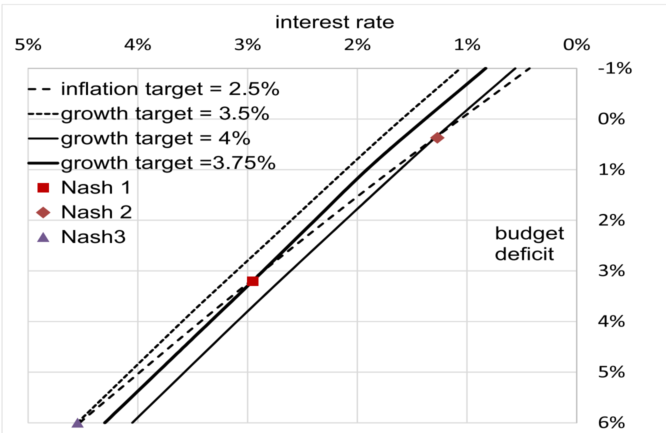

Figure 7. The best response strategies for different fiscal targets and the monetary target: inflation = 2.5%

On the other hand, a more restrictive monetary policy causes, in reaction, a more expansive budget policy. If the interest rate is higher, then the required growth rate can be achieved by applying a more expansive fiscal policy, supporting a higher growth rate. This means that the government should assume a relatively bigger budget deficit. On the contrary, the government should implement a more restrictive fiscal policy, limiting the budget deficit, in reaction to a more expansive monetary policy.

The simulation results show how the changes in the targets of fiscal and monetary policies influence the best response strategies and the Nash equilibrium state, i.e. the choices of the respective policy mixes. A more ambitious target of fiscal policy with a high required economic growth causes that the best response budget strategies become more expansive ones, and vice versa in the opposite case. The lower inflation targets, assumed by the monetary authority, cause that the best response strategies of the central bank tend towards more restrictive monetary policies. The changes of the targets, assumed by the fiscal and monetary authorities, result in respective positioning of the Nash equilibrium.

The Nash equilibrium (point indicated as Nash 1) in Fig. 4 corresponds to a state, which can be compared to the real state of the economy in Poland in the analysed period of 8 quarters between 2008:1 and 2009:4. Upon comparing the obtained results (growth rate = 3.3% and inflation =3.8%) with the historical data for that period one can state that the calculated equilibrium state indicates the possibility of applying policies giving better economic effects, namely: higher growth rate = 3.75% and lower annual inflation = 2.5%, than in the past. More expansive monetary and fiscal policies could be applied with the real interest rate = 2.95%, lower than the historical one, equal to 3.4% on the average in the period 2008 - 2009, and with the budget deficit to GDP ratio = 3.3%, higher that the historical one, equal to 2.7% on the average in this period. It is an open question why the authorities did not implement these feasible better policies. The decision making process was in this time rather difficult, when negative effects of economic recession were observed at the end of 2008 and in 2009. A more general analysis of the alternative policies mix as compared to the historical ones, implemented in Poland, is presented in a separate paper (Kruś & Woroniecka-Leciejewicz, 2016).

Figure 8 illustrates the case when the fiscal and monetary authorities assume too ambitious targets as regards the given economic state. An example of a dynamic game is analyzed. Let us assume that decisions of the authorities are made sequentially and that the fiscal authority makes the first decision, starting from a given state (status quo) of the economy. Each authority tries to achieve the assumed target – the monetary authority assumes rather restrictive inflation target (2%) and the fiscal authority would like to achieve a relatively high growth (4%). The best response strategies do not cross in the assumed intervals of the policy instruments. The decisions are represented by arrows. The theoretical Nash equilibrium (the point indicated as Nash 4 in this figure) is located at the most expansive admissible fiscal policy and a respective monetary policy. It is not Pareto optimal. Another example (of the apparently not-so-demanding targets) is presented in Fig. 9 for the fiscal target of GDP growth =3% and the inflation target = 2.5%. A sequence of decisions is presented with the fiscal authority making the first decision. The theoretical Nash equilibrium (the point Nash 5 in the figure) is in this case situated at the most restrictive admissible fiscal policy and the respective monetary policy. It can be shown that in these two cases if the first decision is made by the monetary authority, the same results of the game will be achieved. The results show that in such cases a coordination of the fiscal and monetary policies is desired.

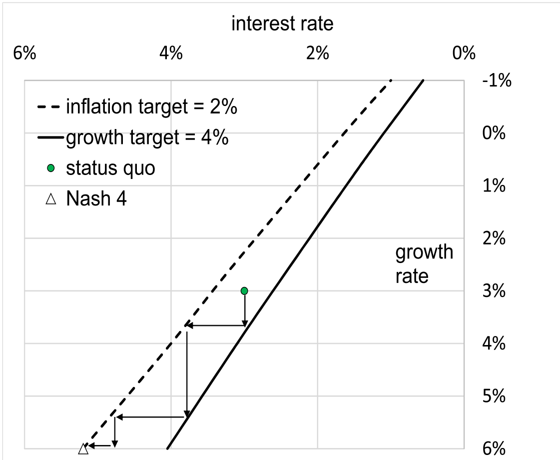

Figure 8. The best response strategies for the targets: GDP growth= 4%, inflation= 2%

The results here presented indicate a truly interesting issue of the existence and determination of the borders of possibility to reach the Pareto-optimal Nash points. The authorities, when planning their policies, assume the intervals of respective instruments: the monetary authority – an interval of admissible values of real interest rates, and the fiscal authority – an interval of admissible budget deficit in relation to GDP. The authorities determine also the targets for their policies. The computer-based system developed calculates the best response strategies of the authorities for the assumed targets, and derives the Nash equilibria, allowing also for the verification of the Pareto-optimality of these equilibria.

When analyzing the numerical results obtained for the case of the one-stage game, as well as for the case of a sequential game, we observe two qualitatively different strategic situations. In the first situation, the best response strategies of the players cross within the interval of the admissible values of instruments. The crossing point defines the Nash equilibrium, which is then also the Pareto optimal solution. The second situation is observed when the monetary and fiscal authorities determine conflicting targets, for example – the targets that are “too ambitious” (see Fig. 9), when the best response strategies do not cross within the interval of admissible strategies. In this latter case the realization of the best response strategies leads to the selection of extremely restrictive/expansive strategies from the interval of admissible values of instruments. This leads to the non-effective decision in Pareto sense, deteriorating the outcomes as com-

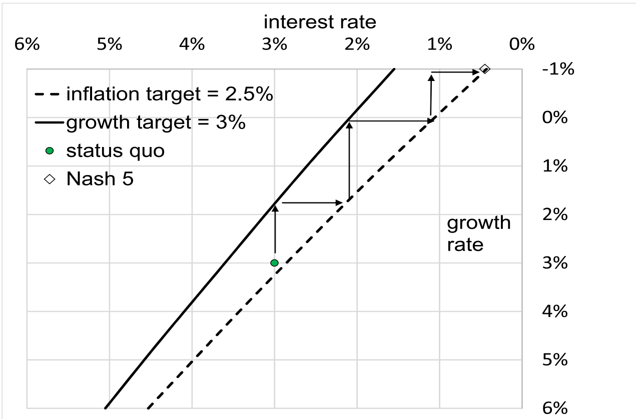

Figure 9. The best response strategies for the targets: GDP growth= 3%, inflation= 2.5%

pared to the status quo. Finally, we can observe the escalation of the conflict and the increase of risk that the economic and strategic stability may get lost. A small correction to the target, made by the monetary or fiscal authority may entail a great leap in the policy mix and imply sudden unpredictable economic effects. The here discussed first or second situation appears depending upon the macroeconomic circumstances and upon the admissible values of the instruments, but first of all – as the effect of the targets, assumed by the monetary and/or fiscal authorities. The results obtained from the analysis imply the necessity of coordination of the policies when such conflicting targets do actually appear.

6. Conclusions

This paper presents some selected results from the research dealing with mutual interactions of the monetary and fiscal policies. The results have been obtained using game theory and optimization methods. A dynamic macroeconomic model, called NNS-MFG, has been formulated and estimated using the statistical data for Poland. A noncooperative monetary-fiscal game has been formulated, in which payoffs of players – namely of the monetary and fiscal authorities – are calculated using model equations.

The macroeconomic NNS-MFG model describes the influence exerted by the instruments of the monetary and fiscal policies on the state of the economy, i.e. influence of the real interest rate and of the budget deficit in relation to GDP on the growth rate and inflation. The model is based on the concept of the New Neoclassical Synthesis model. Several variants of the model developed, with equations describing production gap, inflation, expected inflation, and the Taylor rule are proposed.

The basic NNS model describes a transmission of the monetary policy impulses. In comparison with the basic NNS model, this particular model has been extended to describe the influence of the fiscal policy. It takes into account the budget expenditure gap. The model parameters have been estimated using the quarterly time series for 2000-2014. The Three-Stage Least Squares method from the GRETL package has been used for the model, treated as a system of simultaneous linear equations. The estimated model has been implemented in the form of a recursive algorithm in a computer-based system. The system calculates the payoffs of players and other variables of the model, as depending on the strategies implemented by the players. The system derives also the best response strategies, which depend on the targets assumed by the monetary and fiscal authorities, as well as the Nash equilibria and the Pareto optimal outcomes.

A number of simulations have been performed and the obtained results have been analyzed. The system derives detailed quantitative results, but also some qualitative conclusions can be formulated. There are some values of the targets assumed by the monetary and fiscal authority, for which the best response strategies cross in the interval of the assumed admissible values of the instruments. The crossing point relates to the Nash equilibrium and the equilibrium is then Pareto optimal. However, for some values of the targets, the Nash equilibrium can be non Pareto optimal. For example, in the case of too ambitious targets of the authorities, the Nash equilibrium shifts towards the most restrictive monetary policy and the most expansive fiscal policy, which leads to non-effective, non Pareto optimal solutions. The results show that in such cases a coordination of the monetary and fiscal policies is required.

The system may support the search for the Pareto-optimal consensus of the authorities in the policy mix problem. It can be checked when the targets assumed by the fiscal and monetary policies lead to the Pareto-optimal Nash equilibrium and the equilibrium can be derived. On the other hand, one can check when the priorities of the monetary and fiscal authorities lead to non-effective Nash equilibria and when coordination of the monetary and fiscal policies is required. Further discussion of the here mentioned more general issue of the borders of possibility to reach the Pareto-optimal Nash equilibria is planned for the future work, together with construction of an algorithm for calculation of the respective borders.

The constructed and used macroeconomic model is relatively simple, but the proposed approach can be applied also for more extended versions of this or similar model. Such an extended nonlinear model describing the influence of the policy mix instruments on the economy in a more adequate way is also planned. Further research would include also the analysis of the problem using dynamic game concepts, when a sequence of decisions made by the authorities is considered. Another direction would deal with development of multicriteria optimization tools, supporting the analysis and consensus seeking. The methods of multicriteria bargaining support proposed by Kruś (2011, 2014) can be applied in construction of such optimization tools.

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