Available online at www.ilcpa.pl

International Letters of Chemistry, Physics and Astronomy

20(1) (2014) 10-20 ISSN 2299-3843

Introduce of Generalized Uncertainty Principle

S. Davood Sadatian

Department of Physics, Faculty of Basic Sciences, University of Neyshabur,
P. O. Box 91136-899, Neyshabur, Iran

E-mailm address: sd-sadatian@um.ac.ir

ABSTRACT

We study and explain the uncertainty principle. We've discussed how to reform the uncertainty principle. In this regards, we have used the mechanisms of noncommutative algebra for obtain a generalized uncertainty principle. Following, Due to modified relationship uncertainty, we consider some application of these relation.

***Keywords*:** Quantum Gravity; Uncertainty principle; Symmetry Breaking; Poincare alghebra

1. CLASSICAL UNCERTAINTY RELATIONS

We consider a wave as $y = y_1 \sin kx$. This wave continues from $x = -\infty$ to $x = +\infty$, consistently. The position of this wave can not be determined precisely, but its wavelength is precisely determined and we get: $\lambda = \frac{2\pi}{k}$ [1] .

If we want to use the wave in order to show a particle, its position must precisely be determined. In fact the wave must be localized. Or could be limited in analmost short area of the space .Now if we add another wave with a different wavelength to the initial wave , these two waves act together and we get to the conjunction of waves.

In this case another wave would be produced, that it would continues consistently again from $x = -\infty$ to $x = +\infty$, but we can determine its position more with details , because in some positions wave's wavelength is different. In fact beat occurs because of conjunction of waves .

Consequently the possibility of storm increases for some numbers of x.

So we would have more information about wave's position, but as a result of adding two wavelengths, the initial wavelength is not accurately defined.

Now if we add more waves with different wavelengths and accurate amplitudes and phases, so we'll have a wave which practically has no amplitude outside of an almost narrow area of the space. According to this purpose we have added many waves with different wavelengths, k_i , so the wave would be a demonstration of the average of wave numbers (or wave lengths) shownas Δk . When we have only a single wave, $\Delta k = 0$, and Δx is unspecified, by increasing Δk , Δx decreases; it means the wave becomes more limited. So there is a

contrary relation between Δk and Δx ; a relation such as

$$\Delta x\,\Delta k \geq 1 \qquad 1\text{-}1$$

It means Δx times Δk is of the order of one. So the position of any kind of waves can only be determined by decreasing the accuracy of measuring its wavelength.The 1-1 relation is the first classical uncertainty relation for classical waves. Yang's experiment is one of reversal examplesof the ability to measure the place and wavelength in the same time. In a way that weather we can find out which gap the wave has crossed through, or to measure crossed photons. Now if we want to use eq.(1-1) uncertainty relation for dobro wave; the fundamental relation of dobroi is as : $p = \frac{h}{\lambda}$, using $\hbar = \frac{h}{2\pi}$ and $p = \frac{hk}{2\pi}$ we get :

$$\Delta k = \frac{\Delta p}{\hbar} \qquad 2\text{-}1$$

So using eq. (1-1) uncertainty relation we get; $\Delta x\,\Delta p \geq \hbar$ 3-1

Eq. (3-1) is Heisenberg uncertainty relation.

Indeed it's the mathematical present of Heisenberg uncertainty principle which states that; it's impossible to determine the position and momentum of a particle,simultaneously.

The Heisenberg Energy _ time uncertainty principle is considered as:

$$\Delta E\,\Delta t \geq \hbar \qquad 4\text{-}1$$

and states that it's impossible to determine the energy and the coordinate of the time of a particle, simultaneously.

2. UNCERTAINTY PRINCIPLES

There are observables that have compatible Eigen states, (i.e. Hermitian operators, A and B with [*A*,*B*] = 0 commutator.). These are commuting observables because of their commuting relation, for example if we consider A and B (Hermitian operators) with [*A*,*B*] = 0, they have compatible Eigen states and so a determinate state of A might also be a determinate state of B. But does this always come true?

We know that operators such as x, p, H do not commute, so we can conclude that measuring one thing doesn't always denote to the measuring of any thing else. Although sometimes it happens. So we can determinate the uncertainty principles in such measurementexperimentally, that these principles would be built in mathematical formulation. We only can measure transition from one state into another. The different states of a system cannot be measured even though they ever existed.

2. 1. Uncertainty Relations

We want to relate variances to the commutativity relations between two operators. Weconsider the operator ΔQ . $\Delta Q \equiv Q - \langle Q \rangle$. Then the variance of Q would be as $\langle (\Delta Q)^2 \rangle$ defined below [2]:

$$\left\langle (\Delta Q)^2 \right\rangle = \left\langle Q^2 \right\rangle - Q\left\langle Q \right\rangle - \left\langle Q \right\rangle Q + (\left\langle Q \right\rangle)^2 = \left\langle Q^2 \right\rangle - \left\langle Q \right\rangle^2 \qquad 2\text{-}1$$

We consider two Hermitian operators P and Q .And then we define operator $\Delta P\,\Delta Q$ that could be written as;

$$\Delta P\,\Delta Q = \frac{1}{2}[\Delta P, \Delta Q] + \frac{1}{2}\{\Delta P, \Delta Q\} \qquad 2\text{-}2$$

Since Pand Q are Hermitian their commutator is anti-Hermitian, and their anti-commutator is Hermitian.

Now we consider the expectation value of $\Delta P\,\Delta Q$

$$\left\langle \Delta P\,\Delta Q \right\rangle = \frac{1}{2}\left\langle [\Delta P, \Delta Q] \right\rangle + \frac{1}{2}\left\langle \{\Delta P, \Delta Q\} \right\rangle \qquad 2\text{-}3$$

Now we know that Hermitian operators have real expectation values, and anti-Hermitian operators have imaginary values. So the expectation value of $[P,Q]$ commutator is imaginary and the expectation value, then the right hand side of eq. (2-3) could be considered as $u + iv$. So the magnitude squared of both sides gives:

$$\left|\left\langle \Delta P\,\Delta Q \right\rangle\right|^2 = \frac{1}{4}\left|\left\langle [P,Q] \right\rangle\right|^2 + \frac{1}{4}\left|\left\langle \{P,Q\} \right\rangle\right|^2 \qquad 2\text{-}4$$

Using Schwarzinequality, we get;

{Schwarz inequality: $\forall |a\rangle, |b\rangle \Rightarrow \langle a|a\rangle\langle b|b\rangle \geq |\langle a|b\rangle|^2$ }

We assume $|a\rangle = \Delta P\,|x\rangle$ and $|b\rangle = \Delta Q\,|x\rangle$,

Then

$$\langle a|a\rangle\langle b|b\rangle = (\Delta P)^2\langle x|x\rangle(\Delta Q)^2\langle x|x\rangle = \left\langle \Delta P \right\rangle^2\left\langle \Delta Q \right\rangle^2 \geq \left|\left\langle \Delta P\,|\Delta Q \right\rangle\right|^2 = \left|\left\langle \Delta P\,\Delta Q \right\rangle^2\right| = \frac{1}{4}\left|\left\langle [P,Q] \right\rangle\right|^2 + \frac{1}{4}\left|\left\langle \{P,Q\} \right\rangle\right|^2 \qquad 2\text{-}5$$

The term on the left side is; $\sigma_P^2\sigma_Q^2$, (if the variance of p that we considered as ΔP is equals to σ_P and the variance of Q we considered as ΔQ is equals to σ_Q) so we get:

$$\sigma_P^2\sigma_Q^2 \geq \frac{1}{4}\left|\left\langle [P,Q] \right\rangle\right|^2 \qquad 2\text{-}6$$

Since both terms on the right side of eq. (2-5) are positive, dropping the anti-commutator only strengthens the inequality, so it could be denied.

This gives us a limit on the variance of the P or Qoperator. Now if $[P,Q] = 0$,the two Hermitian operators commute with each other and determined states are shared, it means that we can measure property P and Qsimultaneously.

The commentator relation between **x** and **p** is considered as: $[x, P] = i\hbar$ So we can write the position – momentum uncertainty relation as:

$$\sigma_x^2 \sigma_P^2 \geq \frac{\hbar^2}{4} \rightarrow \sigma_x \sigma_P \geq \frac{\hbar}{2} \qquad 2\text{-}7$$

This relation is supposed to hold for any state. We consider the example of harmonic oscillator as for checking out validity of the eq. (2-7)
We got the variance of **x** for the Eigen state $|n\rangle$:

$$\sigma_x^2 = \frac{(2n+1)\hbar}{2m\omega}$$

The variance of the momentum operator is considered as;

$$\sigma_P = P = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$

So we get;

$$\langle n|P^2|n\rangle = -\frac{\hbar m\omega}{2}\langle n|a_+^2 - a_+a_- - a_-a_+ + a_-^2|n\rangle = \frac{\hbar m\omega}{2}(\langle n|a_+a_-|n\rangle + \langle n|a_-a_+|n\rangle) = \frac{\hbar m\omega}{2}(2n+1)$$

Then: $$\sigma_x^2 \sigma_P^2 = \frac{\hbar^2(2n+1)^2}{4} \qquad 2\text{-}8$$

and for $n = 0$, the ground state, we actually achieve the lower band that we see it consists with the basic position – momentum uncertainty relation. In classical mechanics, the time derivative of a function $J(x, p, t)$ could be written as;

$$\frac{dJ}{dt} = [J, H] + \frac{\partial J}{\partial t} \qquad 2\text{-}9$$

The definition of the Poisson bracket is considered as;

$$[H, J] = \frac{\partial H}{\partial x^\alpha}\frac{\partial J}{\partial P_\alpha} - \frac{\partial J}{\partial x^\alpha}\frac{\partial H}{\partial P_\alpha} \qquad 2\text{-}10$$

and the Hamiltonian equations of motion:

$$\dot{P_\alpha} = -\frac{\partial H}{\partial x^\alpha} \quad \dot{x^\alpha} = \frac{\partial H}{\partial P_\alpha} \qquad 2\text{-}11$$

So the total time derivative is;

$$\frac{dJ}{dt} = \frac{\partial J}{\partial x^\alpha}\dot{x^\alpha} + \frac{\partial J}{\partial P_\alpha}\dot{P_\alpha} + \frac{\partial J}{\partial t} = \frac{\partial J}{\partial x^\alpha}\frac{\partial H}{\partial P_\alpha} - \frac{\partial J}{\partial P_\alpha}\frac{\partial H}{\partial x^\alpha} + \frac{\partial J}{\partial t} = [J, H] + \frac{\partial J}{\partial t} \qquad 2\text{-}12$$

This relation is used, when **J** is not explicitly a function of time. Consider an operator $\hat{Q}(x, p, t)$ then the total time derivative of its expectation value is [3]:

$$\frac{d}{dt}\langle\psi|\hat{Q}|\psi\rangle = \langle\dot{\psi}|\hat{Q}|\psi\rangle + \langle\psi\left|\frac{\partial\hat{Q}}{\partial t}\right|\psi\rangle + \langle\psi|\hat{Q}|\dot{\psi}\rangle \qquad 2\text{-}13$$

with $|\dot{\psi}\rangle \equiv \frac{d}{dt}|\psi\rangle$.

We know from Schrodinger's equation, that

$$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle \Rightarrow -i\hbar\langle\dot{\psi}| = \langle\psi|H\rangle \qquad 2\text{-}14$$

So inputting this into the total time derivative of **Q** gives;

$$\frac{d}{dt}\langle\hat{Q}\rangle = -\frac{1}{i\hbar}\langle\psi|\hat{H}\hat{Q}|\psi\rangle + \langle\psi\left|\frac{\partial\hat{Q}}{\partial t}\right|\psi\rangle + \frac{1}{i\hbar}\langle\psi|\hat{Q}\hat{H}|\psi\rangle = \frac{1}{i\hbar}\langle[\hat{Q},\hat{H}]\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle \qquad 2\text{-}15$$

This result can be used to calculate the time-dependence, in particular, if we take **Q = p**, we recover Ehrenfest's theorem. Now, we consider the relation (2-15), considering the original uncertainty principle and that we know there is no explicit time – dependence and with $P = \hat{H}\ \&Q = \hat{Q}$

$$\sigma_H^2\sigma_Q^2 \geq \frac{1}{4}\left|\langle[H,Q]\rangle\right|^2 = \frac{\hbar}{4}\left|\frac{d\langle\hat{Q}\rangle}{dt}\right|^2 \qquad 2\text{-}16$$

In fact we define a deviation as;

$$\sigma_H\Delta t = \frac{\hbar}{2} \rightarrow \sigma_H\left|\frac{d\langle\hat{Q}\rangle}{dt}\right|\Delta t = \frac{\hbar}{2}\left|\frac{d\langle\hat{Q}\rangle}{dt}\right| \rightarrow \sigma_H\left|\frac{d\langle\hat{Q}\rangle}{dt}\right|\Delta t = \sigma_Q\sigma_H$$

$$\left|\frac{d\langle\hat{Q}\rangle}{dt}\right|\Delta t = \sigma_Q \qquad 2\text{-}17$$

So Δt is a natural time scale induced by the measurement properties of the operator $\hat{Q}(x, p)$. If we refer to the standard deviation of the Hamiltonian as

$$\Delta\text{E}.\,\Delta E\,\Delta t \geq \frac{\hbar}{2} \qquad 2\text{-}18.$$

This is called the "energy-time" uncertainty relation. It relates the spread in energy to the time.

3. THE ALGEBRAIC STRUCTURE OF THE GENERALIZED UNCERTAINTY PRINCIPLE

Measurements in quantum gravity are controlled by a generalized uncertainty principle [4].

$$\Delta x \geq \frac{\hbar}{\Delta P} + cst.G\Delta P \qquad 3\text{-}1$$

(G is Newton's constant). At energies much below the Planck mass M_{PL} the extra term in eq.(1-4) is irrelevant and the Heisenberg relation is recovered. It is responsible for the existence of a minimal observable length on the order of the Planck length.

The result (1-4) was first suggested in the context of string theory in the kinematical region where 2GE is smaller than the string length. However, heuristic arguments suggest that this formula might have a more general validity in quantumgravity, and it is not necessarily related to strings. It is there for natural to ask whether there is an algebraic structure which reproduces eq. (1-4). (Or more in general which reproduces the existence of a minimal observable length). So to obtain generalized uncertainty relations we define a new algebra. The commutator [x, p] = ih controls the algebra used in obtaining Heisenberg uncertainty principle. But with this commutator relation the extra term in eq. (1-4) won't be reproduced; so we look forward an algebra which produces this extra term. So we need deformated algebra because eq. (1-4) would not recover by using the mentioned algebra.

Deformated algebra is an associative algebra where it is defined a commutator which is non-linear in the elements of the algebra; and there is a deformation parameter such that, in an appropriate limit, a Lie algebra is recovered. We therefor look for the most general deformed algebra which can be constructed from coordinates x_i and momenta p_i(I = 1,2,3). We restrict making the following assumptions.

1. The three dimensional rotation group is not deformed; the angular momentum J satisfies the undeformed *SU(2)* commutation relations, and coordinate and momenta satisfy the undeformed commutation relations: $\left[J_i, P_j\right] = i\,\varepsilon_{ijk} P_k$ &$\left[J_i, x_j\right] = i\,\varepsilon_{ijk} x_k$

2. The momenta commutes between themselves: $\left[P_i, P_j\right] = 0$

So that also the transition group is not deformed.

3. The [x,x] and [x,p] commutators depend on a deformation parameter *k* with dimensions of mass. In the limit $k \to \infty$ (that is,*k* much larger than any energy), the canonical commutation relations are recovered. The commutator between x's is non-zero. If *k~Planck mass* the non-commutativity shows up only at the levels of the Plancklength.With these assumptions, the most general form of the κ-deformed algebra is;

$$\left[x_i, x_j\right] = \frac{\hbar^2 a(E)}{\kappa^2} i\,\varepsilon_{ijk} J_k \qquad 3\text{-}2$$

$$\left[x_i, P_j\right] = i\hbar\delta_{ij} f(E) \qquad 3\text{-}3$$

Here *a (E)* and *f (E)* are real, dimensionless functions of E/*k*, and $E^2 = p^2 + m^2$; the angular momentum J is defined as dimensionless, so on the right hand side the dimensions are carried by $\hbar$ and *k* only. The fact that this is the most general form compatible with our assumptions is clear from the following considerations: the factors of iare determined by the condition of hermiticity of x_i,p_i and *Ji*. The tensor ε_{ijk} in eq. (3-2) appears because we assume that the three dimensional rotation group is undeformed and then it is the only tensor antisymmetric in *i, j*.

A term proportional to x_ip_j-$x_jp_i$might also be added to the right hand side of eq. (3-2). In the second equation, again δ_{ij} must appear because it is the only available tensor under rotation. In order to recover the undeformed limit, we further require that f(0) = 1 and that a (E) is less singular than E^{-2}as E→0. We neglect the possibility that the functions *a,f* depend on also other scalars like x^2or *x.p* .

Of course the form of functions *a(E), f(E)* is severely restricted by the Jacobi identities. We consider first the Jacobi identity;

$\left[x_i,\left[x_j, x_k\right]\right] + cyclic = 0$. Using $\left[x_i, E\right] = i\hbar(E)\frac{P_i}{E}$, and

$\left[x_i, a(E)\right] = i\hbar(E)(\frac{P_i}{E})\frac{da}{dE}$ we get : $\frac{da}{dE} P.J = 0$ 3-4

Since the Jacobi identity must be satisfied independently of the particular representation of the algebra, that is independently of wether the condition p. J = 0 holds or not, we conclude that *a(E) = constant*. with a redefinition of *k* we can set this constant to ±1 .

The Jacobi identity $\left[x_i,\left[x_j, P_k\right]\right] + cyclic = 0$ gives

$$\frac{f(E)}{E}\frac{df}{dE} = \frac{1}{\kappa^2} \qquad 3\text{-}5$$

where the negative and positive signs correspond to the choice *a = +1(-1).* Since *f(0) = 1,* eq.

(3-5) gives ; $f(E) = (1 \mp (\frac{E^2}{k^2}))^{\frac{1}{2}}$ 3-6

All other Jacobi identities are automatically satisfied. So

i) there exists a solution: we can have a deformated algebra that it turns to Heisenberg algebra under limited conditions.

ii) Thesolution is unique .(within our assumptions)

Now we only consider the positive sign and rewrite the *k*-deformed Heisenberg algebra as:

$$\left[x_i, x_j\right] = -\frac{\hbar}{\kappa^2} i\,\varepsilon_{ijk} J_k \qquad 3\text{-}7$$

$$\left[x_i, p_j\right] = i\hbar\delta_{ij}(1 + \frac{E^2}{\kappa^2})^{\frac{1}{2}} \qquad 3\text{-}8$$

The generalized uncertainty principle is derived from eq.(8-4) as;

$$\Delta x_i \Delta P_j \geq \frac{\hbar}{2}\delta_{ij}\left\langle (1+\frac{E^2}{\kappa^2})^{\frac{1}{2}}\right\rangle \qquad 3\text{-}9$$

Expanding the square root in powers of $(E/k)^2$ and using $\left\langle P^2\right\rangle = P^2 + (\Delta P)^2$ where $(\Delta P)^2 = \left\langle (P - \left\langle P\right\rangle)^2\right\rangle$, at first order we obtain;

$$\Delta x_i \Delta P_j \geq \frac{\hbar}{2}\delta_{ij}\,(1+\frac{E^2+(\Delta P)^2}{2\kappa^2}) \qquad 3\text{-}10$$

$(1+E^2/k^2)$ could be considered as a function named *f* and the generalized uncertainty principle simplifies as;

$$\Delta x_i\ \Delta p_j \geq \frac{\hbar}{2}\delta_{ij}\left\langle f\right\rangle \qquad 3\text{-}11$$

4. AN APPLICATION OF THE GENERALIZED UNCERTAINTY PRINCIPLE

(The generalized uncertainty principle and black holeremnants)

Small black holes are believed to emit blackbody radiation at the Hawking temperature, at least until they approach Plancksize. A small black hole should emit black body radiation, there by becoming lighter and hotter, and so on, leading on to an explosive end when the mass approaches zero. Does a small black hole evaporate entirely to photons and other ordinary particles and vacuum, or would something be left behind, which we refer to as a remnant [5-35].

Since there is no evident symmetry or quantum number preventing it, a black hole should radiate entirely away to photons and other ordinary stable particles and vacuum, just like any unstable quantumsystem. The total collapse of a black hole may be prevented by dynamics, and not by symmetry. Just as we may consider the hydrogen atom to be prevented from collapse by the uncertainty principle. The generalized uncertainty principle may prevent a black hole from complete evaporation. The uncertainty principle argument for the stability of the hydrogen atom can be stated very briefly. The energy of the electron is $p^2/2m\text{-}e^2/r$, so the classical minimum energy is very large and negative, since $p = r = 0$, the result is not compatible with the uncertainty principle.

If we assume that $P = \frac{\bar{\hbar}}{r}$ we see that; $E = \frac{\bar{\hbar}^2}{2mr^2} - \frac{e^2}{r}$ thus $r_{\min} = \frac{\bar{\hbar}^2}{me^2}$ & $E_{\min} = -\frac{me^4}{2\bar{\hbar}^2}$ which means the energy has a minimum.

The (GUP) gives the position uncertainty as

$$\Delta x \geq \frac{\bar{h}\hbar}{\Delta P} + L_p^2\frac{\Delta P}{\bar{\hbar}}\ \&\ L_P = \sqrt{\frac{G\hbar}{c^3}} \qquad 4\text{-}1$$

This is a result of string theory or more general considerations of quantum mechanics and gravity. The usual Heisenberg argument leads to an electron position uncertainty given by the

first term of 4-1. But we should add to this a term due to the gravitational interaction of the electron with photon, and that term must be proportional to G times the photon energy, or Gpc. Since the electron momentum uncertainty Δp will be of the order of *p*, we see that on dimensional grounds the extra term must be of order $G\Delta p/c^3$, as given in 4-1.

The position uncertainty has a minimum value of $\Delta x = 2L_p$, so the Planck distance plays the role of a minimum or fundamental distance.But the generalized uncertainty principle may prevent a black hole from complete evaporation. One of the applications of the generalized uncertainty principle is that using this we may get to the Hawking temperature of a spherically symmetric black hole or its general properties. There is quantumvacuum around a black hole, which meas that a fluctuating sea of virtual particles, near the surface of a black hole the effective potential energy can negate the rest energy of a particle, and the surface itself is a one-way membrane which can swallow particles.

The net effect is that for a pair of photons one photon may be absorbed by the black hole with effective negative energy –E, and the other may be emitted to asymptotic distances with positive energy +E . The characteristic energy **E** of the emitted photons may be estimated from the standard uncertainty principle. In the vicinity of the black hole surface there is an intrinsic uncertainty in the position of any particle of about the Schwarzschildradius, r_s, due to the behavior of its field lines. So the momentum uncertainty would be as;

$$\Delta P \approx \frac{\bar{h}\hbar}{\Delta x} = \frac{\bar{h}\hbar}{2r_s} = \frac{\hbar^2}{4GM} \;\&\; \Delta x = r_s = \frac{2GM}{c^2} \qquad 4\text{-}2$$

and the energy uncertainty is $\Delta pc = \frac{\hbar^3}{4GM}$ 4-3

This is the energy of the emitted photon. and the Hawking temperature would be as;

$$T_H = \frac{\hbar^3}{8\pi GM} = \frac{M_P^2 c^3}{8\pi M}, M_P = \sqrt{\frac{\hbar}{G}} \qquad 4\text{-}4$$

we show that the emitted photons should have a thermal black body spectrum. From (4-1) we solve for the momentum uncertainty in terms of the distance uncertainty. This gives the following momentum and temperature for radiated photons;

$$\frac{\Delta P}{\bar{h}\hbar} = \frac{\Delta x}{2L_P^2}\left[1 \mp \sqrt{1 - \frac{4L_P^2}{\Delta x^2}}\right] \qquad 4\text{-}5$$

$$T_{GUP} = \frac{Mc^2}{4\pi}\left[1 \mp \sqrt{1 - \frac{M_P^2}{M^2}}\right] \qquad 4\text{-}5$$

5. CONCLUSIONS

We shown that a deformation of the algebra commutator (base of physics effects) which depends on a dimensionful parameter lead to the generalized uncertainty principle in quantum

gravity. We know that the deformed algebra and therefore the form of the generalized uncertainty principle are fixed uniquely by rather simple assumptions. In this regard, we can use this mechanism for study other physics subjects such as Black hole.

References

[1] Kenneth S. Krane, Modern Physics, John Wiley & Sons, 1996.

[2] Generalized Uncertainty Principle, Saurav Dwivedi, November 5, 2010.

[3] Uncertainty Principles, Lecture 20, Physics 342 (2010).

[4] M. Maggiore, *Phys. Lett. B* 304 (1993) 65.

[5] R. J. Adler, P. Chen, D. I. Santiago, The Generalized Uncertainty Principle and Black Hole Remnants, Winner of 3rd place in the 2001 Gravity Research Foundation essay competition, 2001.

[8] A. Einstein, *Annalen der Physik* 354(7) (1916) 769-822.

[9] S. W. Hawking, *Communications in Mathematical Physics* 87(3) (1982) 395-415.

[10] S. W. Hawking, Gary T. Horowitz, *Class. Quant. Grav.* 13 (1996) 1487-1498.

[11] Tomasz Borowski, *International Letters of Chemistry, Physics and Astronomy* 1 (2012) 1-5.

[12] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 4 (2012) 8-16.

[13] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 2 (2012) 28-34.

[14] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 3 (2012) 1-10.

[15] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 3 (2012) 11-23.

[16] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 4 (2012) 1-*7.*

[17] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 5 (2012) 35-45.

[18] Tomasz Borowski, *International Letters of Chemistry, Physics and Astronomy* 11 (2013) 44-53.

[19] Michael A. Persinger, *International Letters of Chemistry, Physics and Astronomy* 12 (2013) 67-71.

[20] V. Vishal, B. Siddharth, C. Venkatachalam, *International Letters of Chemistry, Physics and Astronomy* 13 (2013) 8-12.

[21] Dipo Mahto, Amresh Kumar Singh, Kumari Vineeta, Ashok Kumar, *International Letters of Chemistry, Physics and Astronomy* 13(2) (2014) 95-103.

[22] Michael A. Persinger, *International Letters of Chemistry, Physics and Astronomy* 8(1) (2013) 8-19.

[23] S. A. Koren, M. A. Persinger, *Journal of Physics, Astrophysics and Physical Cosmology* 4 (2010) 1-4.

[24] Michael A. Persinger, *Journal of Cosmology* 14 (2010) 113-119.

[25] Mukul Chandra Das, Rampada Misra, *International Letters of Chemistry, Physics and Astronomy* 9(1) (2013) 13-16.

[26] R. Aldrovandi, J. G. Pereira, K. H. Vu, *Foundations of Physics* 37(10) (2007) 1503-1517.

[27] I. Ciufolini, J. A. Wheeler, *Gravitation and Inertia*. Princeton University Press, Princeton, 1995.

[28] Cahill R.T., Kerrigan D., *Progress in Physics* 4 (2011) 79-82.

[29] Lewis A., Challinor A., Lasenby A., *Astrophysical Journal* 538(2) (2000) 473-476.

[30] D. L. Bulathsinghala, K. A. I. L. Wijewardena Gamalath, *International Letters of Chemistry, Physics and Astronomy* 9(2) (2013) 103-115.

[31] Perlmutter S., Aldering G., Goldhaber G., et al. *ApJ* 517 (1999) 565.

[32] Perlmutter S., Gabi S., Goldhaber G., et al., *ApJ* 483 (1997) 565-581.

[33] Riess A. G. et al., *AJ* 116 (1998) 1009.

[34] Michael A. Persinger, *International Letters of Chemistry, Physics and Astronomy* 2 (2014) 11-14.

[35] Michael A. Persinger, *International Letters of Chemistry, Physics and Astronomy* 2 (2014) 15-21.

(Received 02 September 2014; accepted 12 September 2014)