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Mechanical-stress analytical modeling for the design of coils in power applications

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Abstract: Modern electrical-power systems are often exploited for transmitting high-frequency carrier signals for communications purposes. Series-connected air-core coils represent the fundamental component allowing such applications by providing a proper filtering in the frequency domain. They must be designed, however, to withstand also the line short-circuit current. When a high-magnitude current flows through a coil, strong mechanical stresses are produced within the conductor, leading to possible damage of the coil. In this paper, an approximate analytical model is derived for the relationship between the maximum mechanical stress and the electrical/geometrical parameters of the coil. Such a model provides the guidelines for a fast and safe coil design, whereas numerical simulations are only needed for the design refinement. The presented approach can be extended to other applications such as, for example, the mechanical stress resulting from the inrush currents in the coils of power transformers.

Key words: coils, mechanical stress, analytical modeling, line traps, PLC

1. Introduction

Modern electrical-power systems are often exploited for transmitting high-frequency carrier signals for communications purposes, such as, for example, remote control, voice communication, remote protection, and so forth. In this case they are usually referred to as Power Line Carrier (PLC) systems. Examples of such systems are high-voltage transmission lines, and high-speed railway systems. Within this framework, air-core coils represent the fundamental component allowing such applications by providing a proper filtering in the frequency domain. In the power systems context such air-core coils are usually referred to as line traps [1]. Thus, line traps are air-core coils used to prevent transmission of high-frequency signals to unwanted directions, without loss of energy at power frequency.

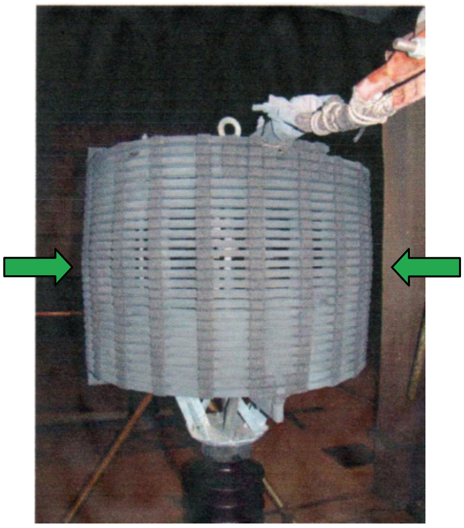

Fig. 1. A line trap coil after a negative test result. The main radial deformation is located around the center of the coil (see the arrows) where the axial magnetic field is maximum

A fundamental aspect is that line traps must be series-connected to a power transmission line, and therefore they must be designed to withstand both the rated power frequency current and the short-circuit current to which the lines are subjected. It is well known that when a current flows through conductor immersed in a magnetic field, a force is acting on the conductor. In fact, the acting Lorentz force per unit volume is given by [2]:

$$\vec{F} = \vec{J} \times \vec{B}, \tag{1}$$

where J and B are the current density and the magnetic flux density, respectively. It follows that when a high-magnitude current flows through a line trap, strong mechanical stresses are produced within the coil due to the interaction between the current and the magnetic field, leading to possible deformation (especially in the radial direction) and/or damaging of the device. A similar problem has been put into evidence with respect to the inrush currents in the coils of high-voltage power transformers [3-4].

For this reason, according to the Standard IEC 60153 [1], line traps must be properly tested with respect to the short-time withstand current and the peak withstand current. In order to avoid negative test results (see Fig. 1 where the radial deformation around the center of the coil is clearly apparent), numerical simulations can be performed to predict the mechanical stress magnitude. Indeed, numerical simulations are commonly used also in the design step when the geometrical parameters of the coil (i.e., height, diameter, conductor cross-section, pitch) must be selected. It would be useful, however, to have at disposal an analytical approximate model in order to provide a guideline for line-trap geometrical design. The analytical model should clearly suggest which geometrical parameters must be changed and in what direction in order to get a decrease in the predicted mechanical stress. Such a guideline would facilitate and optimize the design process, whereas numerical simulations could be performed only at the last stage to assess the design before testing. An approximate analytical model having such characteristics is derived in the paper.

The paper is organized as follows. In Section 2, approximate analytical formulas are given for the inductance calculation of an air-core coil. In Section 3, the tubular model is recalled in order to provide an analytical evaluation of the mechanical stress resulting from a radial pressure inside the coil. In Section 4, the analytical formulas obtained in the previous sections are properly combined in order to provide the analytical relationship between the maximum mechanical stress and the geometrical parameters of a coil. The analytical model is numerically validated in Section 5. Finally, conclusions are drawn in Section 6.

2. Analytical inductance calculation

The inductance of a single-layer air-core coil (see Fig. 2) can be expressed as

$$L = L_s - \Delta L, \tag{2}$$

where L_S is the inductance of the corresponding cylindrical current sheet, and ΔL is a correction term of the order of some percent of L_S [5] such that it can be neglected in the approximate analysis proposed in this paper.

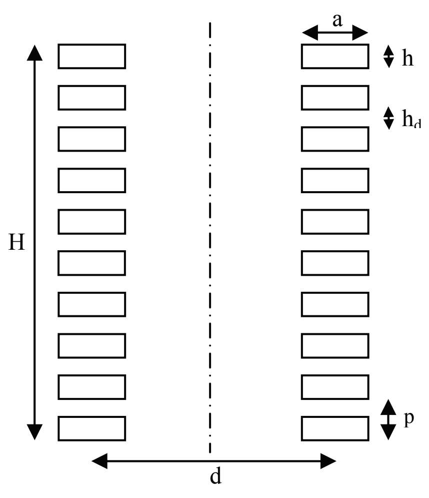

Fig. 2. Longitudinal cross-section of a single-layer coil

The current sheet inductance is a function of the number of turns N, the average diameter d, and the shape ratio $R = H/d$ where H is the height of the coil:

$$L_s = \frac{1}{2}\mu_0 N^2 d\, f(R). \tag{3}$$

The function $f(R)$ can be given in approximate form as [6]

$$f(R)=\begin{cases}\left[\log\left(\frac{4}{R}\right)-\frac{1}{2}\right]f_1\left(R^2\right)+f_2\left(R^2\right), & R<1\\ \frac{\pi}{2R}\left[f_1\left(\frac{1}{R^2}\right)-\frac{4}{3\pi R}\right], & R>1,\end{cases} \tag{4}$$

where

$$f_1(x)\cong\frac{1+0.383901x+0.017108x^2}{1+0.258952x}, \tag{5}$$

$$f_2(x)\cong 0.093842x+0.002029x^2-0.000801x^3. \tag{6}$$

3. Mechanical-stress analytical evaluation

According to (1), the interaction between the current flowing in a coil and the produced magnetic field (mainly directed along the longitudinal axis) results in a radial outward-directed mechanical pressure p_r which is maximum around the central region of the coil (see Fig. 1) where the axial component of the magnetic field is maximum. Such a mechanical load results in a traction stress σ_t within the metallic conductor. The analytical evaluation of the mechanical stress can be readily performed by adopting a tubular model for the coil (see Fig. 3). By considering a longitudinal section of the coil (see the top view of such section in Fig. 4), and by imposing the equilibrium of the forces, the mechanical stress can be derived as [7]-[8]

$$\sigma_t = p_r\frac{r}{a}, \tag{7}$$

where $r = d/2$ is the average radius of the coil.

From (1) it can be readily shown that the radial pressure is given by

$$p_r = JB_a a, \tag{8}$$

where B_a is the average value of the axial (longitudinal) component of the magnetic flux density B evaluated within the central turn of the coil (where the magnetic field is maximum).
By inserting (8) into (7) it can be obtained:

$$\sigma_t = \frac{1}{2}JB_a d. \tag{9}$$

Usually the quantity B_a must be evaluated by means of numerical simulation of the coil [4]. In this paper, an approximate solution is proposed, allowing to avoid the numerical simulation of each coil under consideration during the design stage.

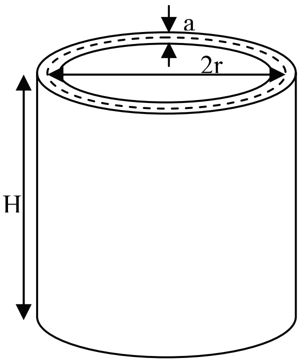

Fig. 3. Tubular model of a cylindrical coil

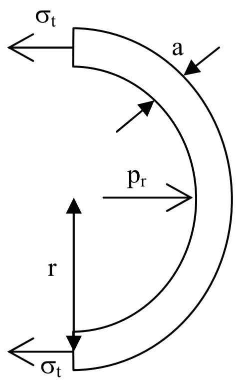

Fig. 4. Evaluation of the mechanical stress σ_t by imposing the equilibrium of the forces acting on one half of the tube in Fig. 3

By considering the cross-section of the central turn (see Fig. 5) and by applying the Ampère's law along the dashed closed line, the following approximate relation between the internal (B_i) and the external (B_e) axial components of the magnetic flux density can be obtained:

$$B_i + B_e \cong \mu_0 \frac{I}{p}, \tag{10}$$

where I is the current flowing in the coil, and p is the coil pitch (see Fig. 2).

In order to obtain the average value B_a of the magnetic flux density, a further relation between B_i and B_e is needed. To this aim, the reference structure in Figure 6

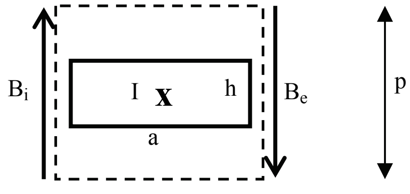

Fig. 5. Transversal cross-section of the central turn of the coil. Approximate application of the Ampère's law along the dashed closed line

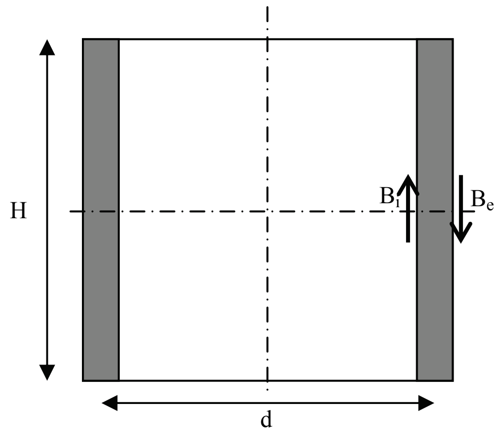

Fig. 6. Reference coil structure for the numerical evaluation of the magnetic flux ratio (11)

was considered. A set of numerical simulations has been performed by changing *d* and *R*, and evaluating the ratio

$$\alpha = \frac{B_e}{B_i}. \tag{11}$$

It was found that, as a first order approximation, such ratio is mainly dependent on the shape ratio *R* as shown in Figure 7.

By combining (11) and (10), the estimate of the average axial magnetic flux density can be obtained as

$$B_a \cong \frac{B_i + B_e}{2} = \frac{\mu_0 I}{2_p} \cdot k(R), \tag{12}$$

where the function

$$k(R) = \frac{1-\alpha(R)}{1+\alpha(R)} \tag{13}$$

is represented in Figure 8.

By inserting (13) into (9) the following expression can be obtained:

$$\sigma_t \cong \frac{\mu_0 I^2}{4ahp} k(R) d. \tag{14}$$

Moreover, by taking into account that the parameter *h* in (14) can be written as

$$h = p - h_d = p\left(1 - \frac{h_d}{p}\right), \tag{15}$$

where h_d is the distance between the turns, the final expression can be obtained:

$$\sigma_t \cong \frac{\mu_0 I^2}{4a(1-\frac{h_d}{p})p^2} k(R) d. \tag{16}$$

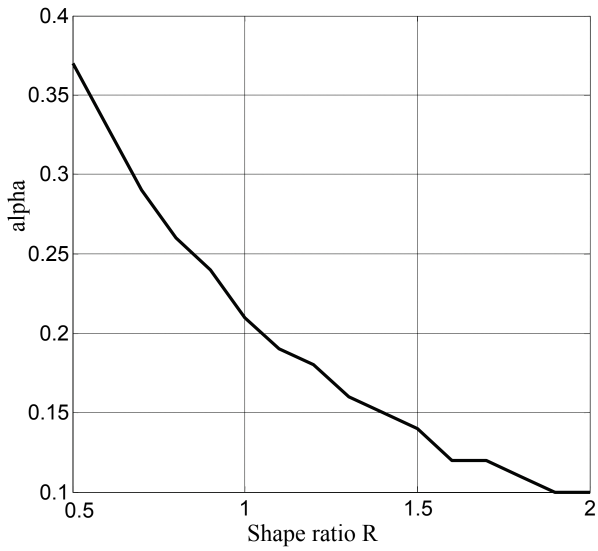

Fig. 7. Numerical evaluation of the magnetic flux ratio α as a function of R for the reference structurein Figure 6

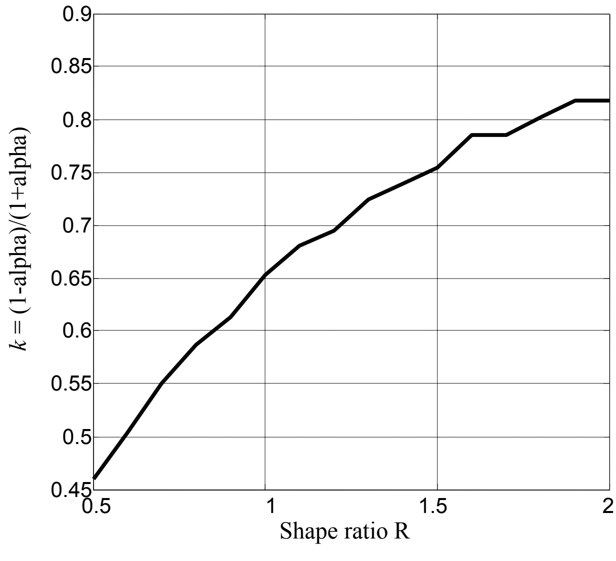

Fig. 8. Behavior of the function $k(R)$ defined in (13)

The result in (16) provides the relationship between geometrical/electrical characteristics of the coil and the mechanical stress. However, such parameters are not independent since they must also obey to the inductance constraint (3). This question is addressed in the next Section.

4. Mechanical stress as a function of inductance

In the design process of a specific coil the inductance is usually a given data. It follows that (3) provides a constraint between the three coil parameters N, d, and R. In (16), however, it can be noticed that the parameter N is not present. In fact, the number of turns N can be written as

$$N = \frac{H}{p} = \frac{Rd}{p} \tag{17}$$

such that (3) becomes:

$$L_s = \frac{1}{2}\mu_0 \frac{R^2}{p^2} d^3 f(R). \tag{18}$$

Now by extracting p^2 from (18)

$$p^2 = \frac{1}{2}\mu_0 \frac{R^2}{L_s} d^3 f(R) \tag{19}$$

and substituting into (16) it can be obtained:

$$\sigma_t \cong \frac{1}{2} L_s I^2 \frac{1}{a(1-\frac{h_d}{p})d^2} \cdot \frac{k(R)}{R^2 f(R)}. \tag{20}$$

Equation (20) provides the mechanical stress σ_t as a function of the desired inductance L_S, and a set of four independent geometrical parameters, i.e., a, h_d/p, d, and R. Of course in (20) the dependence from the square of the current is also put into evidence.

A simpler relationship can be obtained by deriving the least-square approximation of the function

$$g(R) = \frac{k(R)}{R^2 f(R)} \tag{21}$$

given by

$$g(R) \cong \frac{0.57}{R}. \tag{22}$$

Therefore (20) can be rewritten as

$$\sigma_t \cong \frac{1}{2} L_s I^2 \frac{1}{a(1-\frac{h_d}{p})} \cdot \frac{0.57}{d^2 R}. \tag{23}$$

The approximate analytical result (23) is the main result of this paper. It provides the maximum traction stress as a function of two given parameters such that the inductance L_S and the peak current I. The other four parameters, i.e., a, h_d/p, d, and R, can be properly selected to obtain a safe value for the stress σ_t and by taking into account possible other practical constraints such as the available value for the width a or a given distance h_d between the turns. Finally, notice that once the ratio h_d/p is selected such that (23) provides a safe value for the mechanical stress, the two separate values h_d and p can be obtained. To this aim, since the pitch p is given by the square root of (19), also the turns distance h_d is defined.

5. Numerical validation

In this section, the analytical results derived in Section 4 are used, as an example, to design a coil for which a limit on the maximum mechanical stress is enforced. Numerical simulations are also presented to validate the design.

Let us consider the design of a coil with inductance L = 0.5 mH and a peak current I = 102 kA. A maximum mechanical stress equal to 50 MPa is required. The four geometrical parameters in (23) must be now selected in order to fulfill such mechanical limit. Notice that in (23) the geometrical parameter having the most important impact on the mechanical stress is the average diameter d (it contributes with its square value). Therefore, it is reasonable to set arbitrarily the other three parameters, and to treat d as an independent variable to be adjusted such that the limit on the mechanical stress is fulfilled.

The three arbitrary parameters have been selected as a = 40 mm, h_d = 9 mm, and R = 1.2. The fourth parameter, i.e., the average diameter d, was assumed as a variable within the range (800 mm, 1200 mm). In Figure 9, the mechanical stress, given by (23) with p given by (19), as a function of the average diameter is shown. The stress limit equal to 50 MPa is also represented. It is then predicted that d must be greater than 910 mm in order to obtain a stress lower than the limit. For example, by selecting d = 1050 mm it is expected a mechanical stress $\sigma_t \approx 35$ MPa.

The behavior of the pitch p (given by the square root of (19)) as a function of d is shown in Fig. 10. For the selected value of d = 1050 mm the corresponding pitch p is 45 mm. The height h of the conductor (see Fig. 2) is given by $h = p - h_d$ = 36 mm. Finally, from (17) the number of turns can be derived, i.e., N = 28.

The coil considered above has been simulated by means of the commercial FEM software COMSOL Multiphysics®. A magnetostatic simulation has been performed by injecting a constant current I = 102 kA into a coil having the geometrical properties derived above, corresponding to a current density $J = 7.08 \cdot 10^7 \mathrm{A/m^2}$. The actual inductance was 0.46 mH, instead of the design value 0.5 mH. This point has been already considered when (2) was introduced. Figure 11 shows the spatial distribution of the longitudinal (z axis) component of the magnetic flux density B. The value of B around the central turn is responsible of the maximum mechanical stress as shown in the previous sections. The average value of B on the central turn is obtained by numerical integration as 0.986 T, in good agreement with the analytical estimate 0.992 T provided by (12). Inserting the numerical value of the average B into (9) the mechanical stress obtained through numerical simulations is 36.6 MPa, in good agreement with the estimated analytical value equal to 35 MPa.

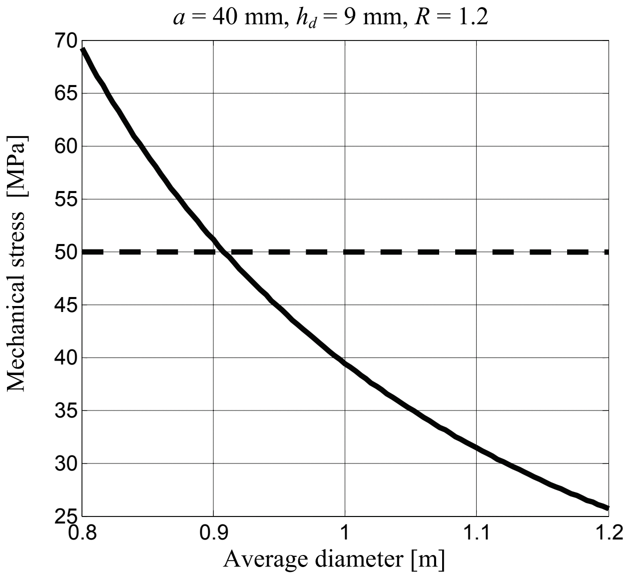

Fig. 9. Mechanical stress as a function of the average diameter according to (23). The other parameters are defined in the text. The dashed line represents the assumed limit for the stress

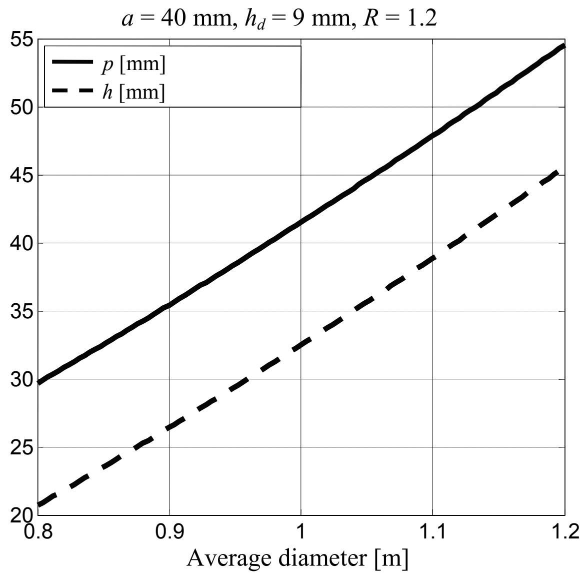

Fig. 10. Behavior of the pitch p as a function of the average diameter d, according to (19) (solid line). The corresponding conductor height h is also represented according to (15) (dashed line)

In Table 1 the comparison between analytical and numerical estimates of the maximum mechanical stress is shown for different coils and currents. The differences are always lower than 10%.

Table 1. Comparison between analytical and numerical values for the mechanical stress, for different coils

Coil	**σ_t analytical [MPa]**	**σ_t numerical [MPa]**
$d = 0.8$ m, $H = 0.684$ m, $L = 0.5$ mH, $I = 63.75$ kA	53	53
$d = 0.81$ m, $H = 0.7375$ m, $L = 0.5$ mH, $I = 80.325$ kA	65	67
$d = 0.93$ m, $H = 0.6885$ m, $L = 0.5$ mH, $I = 80.325$ kA	54	51
$d = 0.81$ m, $H = 0.767$ m, $L = 0.5$ mH, $I = 102$ kA	86	84
$d = 0.93$ m, $H = 0.6885$ m, $L = 0.5$ mH, $I = 102$ kA	77	74
$d = 0.93$ m, $H = 0.9405$ m, $L = 0.5$ mH, $I = 102$ kA	71	74

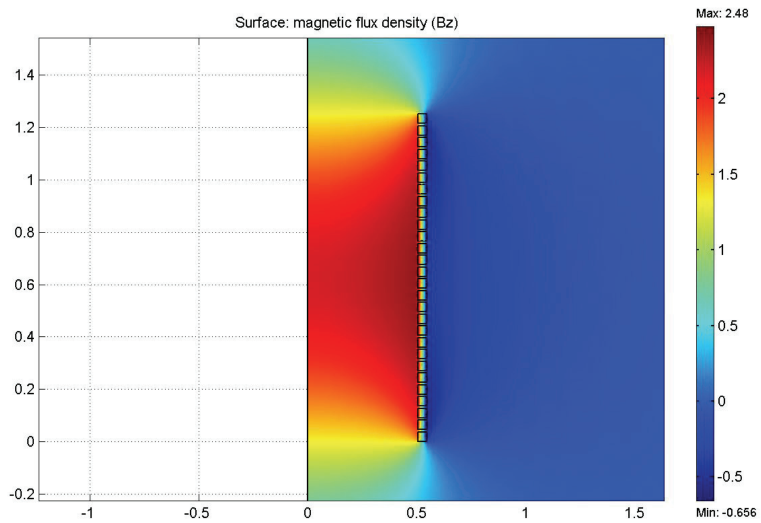

Fig. 11. Numerical evaluation (COMSOL Multiphysics®) of the spatial distribution of the longitudinal component of the magnetic flux density

6. Conclusions

In this paper an analytical approximate expression for the maximum mechanical traction stress due to the radial electromagnetic pressure in an air-core coil has been derived as a function of the electrical/geometrical parameters of the coil. In the design stage, for a given inductance and peak current, the analytical model allows to select proper values for the coil geometric parameters a, h_d/p, d, and R, in order to obtain a safe value for the mechanical stress. The main advantage is that the numerical simulation of the coil is needed only in the final stage of the design for validation and refinement.

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