Article citation info:
QUEZADA DEL VILLAR AV, RODRÍGUEZ-PICÓN LA, PÉREZ-OLGUÍN IJC, MÉNDEZ-GONZÁLEZ LC. Stochastic modelling of the temperature increase in metal stampings with multiple stress variables and random effects for reliability assessment. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2019; 21 (4): 654–661, http://dx.doi.org/10.17531/ein.2019.4.15.

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STOCHASTIC MODELLING OF THE TEMPERATURE INCREASE IN METAL STAMPINGS WITH MULTIPLE STRESS VARIABLES AND RANDOM EFFECTS FOR RELIABILITY ASSESSMENT

OCENA NIEZAWODNOŚCI Z WYKORZYSTANIEM STOCHASTYCZNEGO MODELU WZROSTU TEMPERATURY W METALOWYCH WYTŁOCZKACH, UWZGLĘDNIAJĄCEGO WIELORAKIE ZMIENNE NAPRĘŻENIOWE ORAZ EFEKTY LOSOWE

Many products wear out over time even before they fail or stop working, therefore, through accelerated degradation tests one is able to make inferences about statistical parameters or the distributions of a product useful life. Since many devices experience different types of variation due to unobservable factors during the manufacturing processes or under certain operating conditions; these situations lead to the need in developing accelerated degradation models with several variables of acceleration and random effects. The proposed model in this paper, is a model based on the gamma process with random effects to have a better analysis of degradation. This model is applied to the analysis of the temperature increase of metal stampings that are affected by multiple explanatory variables. In addition, a statistical inference method based on a Bayesian approach is used to estimate the unknown parameters to then perform a reliability analysis after obtaining the first-passage time distributions.

Keywords: *accelerated degradation test, gamma process, random effects, reliability assessment.*

Wiele produktów zużywa się z upływem czasu zanim nawet ulegną uszkodzeniu lub przestaną działać. Badania przyspieszonego starzenia pozwalają wyciągać wnioski na temat parametrów statystycznych lub rozkładów okresu użytkowania produktu. Wiele urządzeń podlega różnym rodzajom zmienności pod wpływem działania nieobserwowalnych czynników występujących podczas procesu produkcyjnego lub w pewnych warunkach pracy; sytuacje te wymagają opracowania modeli przyspieszonego starzenia uwzględniających wielorakie zmienne przyspieszenia oraz efekty losowe. Zaproponowany w przedstawionym artykule model opiera się na procesie gamma z efektami losowymi, dzięki czemu pozwala na lepszą analizę degradacji. Model ten zastosowano do analizy wzrostu temperatury w metalowych wytłoczkach, na które oddziałuje wiele zmiennych objaśniających. Ponadto do oszacowania nieznanych parametrów wykorzystano metodę wnioskowania statystycznego opartą na podejściu bayesowskim. Umożliwiło to analizę niezawodności po uzyskaniu rozkładów czasu pierwszego przejścia.

Słowa kluczowe: *badanie przyspieszonego starzenia, proces gamma, efekty losowe, ocena niezawodności.*

1. Introduction

Currently, the level of quality and reliability of the products offered to customers is essential to maintain competitiveness in the market. Furthermore, to obtain highly reliable products, very large experimentation times are required, an alternative that mitigates this drawback are the accelerated degradation tests (ADT), in which a group of products suffer high levels of environmental stress [17]. In this way, it is possible to obtain the degradation measurements of a particular performance characteristic and the failure times in the shortest time possible. In addition, by including random effects on the variables [32], it is possible to describe the variation of the initial conditions of the devices as a function of the parameters of the model and to improve the accuracy of the predictions of reliability under conditions of normal use [22].

According to Peng & Tseng [26] in this situation the degradation models that are described by stochastic processes (gamma process, Wiener process, Gaussian inverse process), are useful for the analysis of the performance of one or several magnitudes that vary randomly as a function of time. Where some of its parameters can be affected and modified in order to incorporate the variability and the explanatory variables that affect the increase in degradation. Finally, with the correct specification of the model and the adequate supervision of the degradation processes, it is possible to predict the remaining useful life of the product, precise when maintenance or replacements are necessary or appropriate, be certain of the products useful life time, and provide greater reliability to the client [46,43].

Therefore, manufacturers must constantly innovate and produce value-added components [18] and in order to avoid or reduce failures, stochastic process models are naturally applicable and allow to counteract the previous problems. Among them is the Wiener process, [33] this stochastic process has been used to model the light intensity of LED lamps; Joseph & Yu [15] to improve reliability; Wang, et al. [42] proposed an adaptive method and a numerical example about cracks (low intensity) due to fatigue; Barker & Newby [3] to describe the degradation of a multi-component system and develop an optimal non-periodic inspection policy; in areas such as medicine, to describe a series of biomarkers that represent the deterioration of systems over time, in a population of individuals infected with HIV to predict the residual time since the entry into the study until the moment in which a critical limit is reached [9]. Another model is the inverse Gaussian process which has been used by Wang & Xu [40] to fit the laser GaAs data; Peng [25] and Ye et al. [45] discussed when the underlying degradation follows the Gaussian inverse process; Zhang, et al. [50] describe a model to characterize the growth of depth defects by corrosion in underground energy pipelines; in fatigue cracks of structural components from metallic aircrafts, mainly in aluminum parts [19]; in the location of wireless sensor networks [28]; for the analysis of the degradation of industrial bearings [13]; in a series of biomarkers that represent the deterioration of the systems over time, in a population of individuals infected with HIV to predict the latency time from the moment of infection until the moment in which it is detected [9]. The Poisson process has also been presented as a model to deal with the reliability estimation of systems and processes. Andrzejcza et al. [2] presented a Poisson based model to estimate the cost of corrective maintenance of public mass transport vehicles.

On the other hand, the gamma process has been used extensively in the literature, this due to its important characteristics that it can be used when a large number of product failures is caused by the impact of external random factors that tend to be very small, with independent increases, with base of zero and up to infinity (not negative) and where performance can only decrease with respect to time [43, 16 10]. Therefore, it is adequate to model gradual damage where a stochastic monotonous deterioration accumulates [35], the increments are stationary [37]. Some applications have been presented as worn-out, with propagations of fissures, crack growth, erosion, consumption, creep, swelling, degrading health index, corrosion, consumption and fatigue, among other factors [29,36]; by Iervolino, et al. [14] to model the effect produced by earthquakes; van Noortwijk [36] studied the application in maintenance Wang, et al. [41] proposed an adaptive method applied to the growth of fatigue cracks; Bordes, et al. [4] considered a degradation model that consists of two independent processes, in addition they illustrated their method through a study and an application to a set of real data presented in the article by Takeda & Suzuki [31]; Pan, et al. [21] proposed reliability models for systems with two degrading components depending on an example of a railway.

This article presents a case study related to the increase of temperature in metal stampings which were exposed to different levels of electric current, apart from considering different explanatory variables during an ADT. The analysis of this case study is carried using the gamma process by considering the use of a life-stress relationship such as the exponential link function. This life-stress relationship is commonly used when the case of multiple stress variables is presented, some important applications can be found in: Park & Padgett [24] that suggested a hyper-cuboidal volume approach as a measure of acceleration that can incorporate several acceleration variables. A special case that includes the Weibull model and the law of power [20]. By considering the proportional hazard model [6] as an extended model with a weak link, with hyper-cuboidal volume and the consideration of random effects. In different applications such as bridge beam data that includes degradation due to the entry of chloride ions [39]. To compare a set of fatigue-cracks growth data [47]; to evaluate the effectiveness of laser photocoagulation to delay visual loss in patients with diabetic retinopathy [49]. As a model that has applications in maintenance [7], and in the degradation of light intensity of an electronic device [48]. On the other hand, random effects are considered in the gamma process model in order to include the unit to unit variation. The parameters of interest are estimated based on a Bayesian approach via Markov chain Monte Carlo (MCMC). Likewise, a comparison was made with other stochastic processes in order to obtain the one that best fits the data that was used.

The rest of this document is organized as follows. In Section 2, the characteristics, functions and the first-passage time distribution of the gamma process are presented. First, the gamma process is introduced with random effects, and then the exponential link relationship is presented as a life stress function that best fits the explanatory variables that affect the case study. In Section 3, an estimation scheme for the gamma process parameters is presented, under a Bayesian analysis with MCMC approach. In Section 4, a case study based on the temperature increase of metal stampings is provided to illustrate the application and utility of the proposed model; the estimation of the model was made based on Bayesian inference, to obtain the first passage time distributions; finally, a comparison with other stochastic processes is discussed. The conclusions can be seen in Section 5.

2. Stochastic model based on the gamma process

2.1. General Characteristics

In this paper, it is considered that the degradation of a performance characteristic at time t $\{Z(t);t\geq 0\}$ is governed by a gamma process. The gamma process has been widely used in the literature as described in Section 1. The main properties of this process are:

1. $Z(0)\equiv 0$;
2. $Z(t)$ has independent increments.
3. For any $t>s;Z(t)-Z(s)=\Delta Z(t)$ follows a gamma distribution $Ga(v(t-s),u)=Ga(v\Delta t,u)$

where $v(t);t\geq 0$ is a non-negative increasing function with $t\geq 0$ and $v(0)\equiv 0$, also known as the shape function. And, $u>0$ is the scale parameter. Supposing that the stochastic gamma process describe the degradation level of some performance characteristic at time t, then the probability density function (PDF) can be denoted as in (1), with mean $vt\cdot u$, variance $vt\cdot u^2$, and compact notation as $Z(t)\sim Ga(vt,u)$.

$$f\left(Z(t)|vt,u\right)=\frac{Z(t)^{vt-1}}{u^{vt}\Gamma(vt)}e^{-\left(\frac{Z(t)}{u}\right)}. \tag{1}$$

The moment of failure is an important characteristic that can be obtained from (1) and occurs when the degradation $Z(t)$ reaches a critical level of degradation ω [5]. This moment of failure is a random variable T_ω with expression denoted as:

$$T_\omega = inf\{Z(t)\geq\omega\}. \tag{2}$$

On the other hand, the cumulative distribution function (CDF) of T_ω can be derived based on:

$$\begin{aligned}F_{Ga}(t_\omega)&=P(T_\omega\leq t_\omega)=P(Z(t)\geq\omega)\\&=\int_\omega^\infty f_{Z(t)}(z)dz=\int_\omega^\infty\frac{z^{vt-1}}{u^{vt}}e^{-\left(\frac{z}{u}\right)}dz\\&=\frac{1}{\Gamma(vt)}\int_{\omega/u}^\infty x^{vt-1}e^{-x}dx,\end{aligned}$$

where $x = z/u$, the integral in the last previous equation can be simplified by considering the incomplete gamma function, $\Gamma(a,b) = \int_b^{\infty} \zeta^{a-1} e^{-\zeta} d\zeta$. Thus, under the notation $a = vt$ and $b = \omega/u$, the CDF results as denoted in (3).

$$F_{Ga}(t_\omega) = \frac{\Gamma(vt, \omega/u)}{\Gamma(vt)}. \quad (3)$$

Now suppose that an ADT has been performed to N devices that are observed during M inspections until the test termination time T. Then, $Z_i(t_j)$ degradation measurements are observed for the trajectories $i = 1,2,\ldots,N$ at the corresponding times $t_j, j = 1,2,\ldots,M$. Considering the independent increment property of the gamma process, and $\Delta Z_i(t_j) = Z_i(t_j) - Z_i(t_{j-1}), t_0 = 0$ and $\Delta t_j = t_j - t_{j-1}$, thus the variable $\Delta Z_i(t_j)$ is governed by a gamma process model as denoted in (4) with compact notation $\Delta Z_i(t_j) \sim Ga(v\Delta t_j, u)$.

$$f\left(\Delta Z_i(t_j) | v\Delta t_j, u\right) = \frac{\Delta Z_i(t_j)^{v\Delta t_j} e^{-\left(\frac{\Delta Z_i(t_j)}{u}\right)}}{u^{v\Delta t_j}\Gamma(v\Delta t_j)}. \quad (4)$$

2.2. Random effects in the gamma process

Random effects are considered in the gamma process when it is suspected that non observable factors can cause variations in the observed degradation. Specifically random effects are incorporated to describe unit to unit variability, *i.e.,* heterogeneity among unities under test [46,10]. According to Rodríguez Picón, et al. [30], when performing a degradation test each unit under test may be affected by different sources of variation which denotes the need of incorporate random effects in the model. In the case of the gamma process, random effects can be incorporated as a function of the scale parameter. Then, it is considered that u_i is a random variable for the $i = 1,2,\ldots,N$ devices under test. It can be noted that as the mean of the gamma process is described as $vt \cdot u$ and the variance is described as $vt \cdot u^2$, the randomness of u_i have an impact over the mean degradation and the variance degradation. This means that it is expected that the degradation paths exhibit a large variation among paths (the mean degradation is a function of u_i) and a large variation within each path (the variance is also a function of u_i).

In the literature, it has been found that the random scale parameter (u_i) is described by a gamma distribution $Ga(\delta,\varphi)$ [38]. This model is known as the classical gamma process model with random effects. By considering this, the PDF of the degradation $\Delta Z_i(t_j)$ is represented as:

$$f\left(\Delta Z_i(t_j)\right) = \int_0^{\infty} f_{Ga}\left(\Delta Z_i(t_j) | v\Delta t_j, u_i\right) \cdot f_{Ga}(u_i|\delta,\varphi) du_i. \quad (5)$$

Thus, the CDF of the lifetime when the degradation path reaches the critical level of degradation ω can be obtained by solving the integral presented in (6):

$$F_R(t_\omega) = \int_0^{\infty} F_{Ga}(t_\omega) \cdot f_{Ga}(u_i|\delta,\varphi) du_i. \quad (6)$$

2.3. The exponential link relation as a function of the shape parameter

The ADTs are used to accelerate the degradation process and thus accelerate the failure time in the aims of obtaining a reliability estimation in less time. Of course, these type of tests consist in submitting a device to high levels of stress during a determined period of time, which allows to obtain the degradation increments that lead to the failure in a short time [34]. Furthermore, a product may be exposed to multiple covariates (s) that affects the degradation process, such as temperature, humidity, voltage, etc., and other characteristics such as material type, geometrical characteristics, etc. Then, it is important to incorporate such covariates in the model. Specifically as a function of a parameter of the proposed model [46].

The exponential link function has been used in the cases when multiple covariates have an effect on the performance of a characteristic of interest. This function relates a life characteristic and the stress variables through exponential functions [11]; the general form of this function is presented in (7) [44]:

$$h(s_k) = \beta_0 e^{\beta_k s_k}, \quad (7)$$

where $h(s_k)$ can repesent a life characteristic that is observed under the effect of $s_k = s_1, s_2, \ldots s_p$ stress variables, where p represents the number of covariates. On the other hand, β_0 and $\beta_k (k = 1,2,\ldots p)$ are constant parameters to be estimated [23]. In the case of the gamma process, in the literature it has been found that the shape parameter describes the effect of stress on the performance of products, such that v can relate the effect of $h(s_k)$. Thus, it is considered that $v(s_k) = h(s_k) = \beta_0 e^{\beta_k s_k}$ for $k = 1,2,\ldots p$. In this way, the PDF of the gamma process with the kth stress variable can be expressed as in (8):

$$f\left(\Delta Z_i(t_j) | \delta,\varphi,\beta_0,\beta_k\right) = \int_0^{\infty} f_{Ga}\left(\Delta Z_i(t_j) | v(s_k)\Delta t_j, u_i\right) \cdot f_{Ga}(u_i|\delta,\varphi) du_i;$$
$$k = 1,2,\ldots p;\ i = 1,2,\ldots,N;\ j = 1,2,\ldots,M \quad (8)$$

The CDF of the moment of failure when the degradation path reaches the critical value ω can be found by solving the integral in (9):

$$F(t_\omega | s_k) = \int_0^{\infty} F_{Ga}\left(t_\omega | v(s_k) t_j, u_i\right) \cdot f_{Ga}(u_i|\delta,\varphi) du_i \quad (9)$$

According to Lawless & Crowder[16], the integral in (9) results in terms of the Fisher distribution. Then, by considering the exponential link function, the CDF of the first passage times result as in (10):

$$F(t_\omega | s_k) = 1 - F_{v(s_k)t_j, 2\delta}\left(\frac{\delta\omega}{\varphi v(s_k) t_j}\right) \quad (10)$$

3. Estimation of parameters

Certainly, it results of interest to estimate the parameters $(\beta_0, \beta_k, \delta, \varphi)$ from the model presented in (8), such that it is possible to obtain reliability estimations through the function presented in (10). However, it can be noted from the integral presented in (8) that the classical methods of estimation, such as the maximum likelihood estimation (MLE) method, result too complicated to implement given the complexity of the function in (8). On the other hand, in the last years there have been important advances in numeric approximation techniques to deal with complex functions via MCMC [12]. Specifically, the estimation of complex functions via a Bayesian approach with MCMC has been presented as an important alternative. Definitely, the implementation of a numerical technique such as a Bayesian MCMC estimation scheme to solve complex functions requires the use of a specialized software. Fortunately, there several open source softwares such as OpenBUGS and R which can be used. In this paper, we consider an estimation scheme based on a Bayesian approach with MCMC to estimate the parameters in the model (8). This approach has the practical advantage that it can incorporate subjective information in a natural way when there is little information about the historical behavior of the parameters on interest.

In Figure 1, we present the general estimation scheme based on a Bayesian approach. For the parameters of interest $(\beta_0, \beta_k, \delta, \varphi)$, we consider non-informative prior distributions (which are denoted as $\pi(\beta_0), \pi(\beta_k), \dots$), given that no prior knowledge of these parameters is available. Specifically, the next prior distributions were considered:

$$\pi(\beta_k) \sim N(0, \tau^2); k = 1, 2, \dots p$$

$$\pi(\beta_0) \sim N(0, \tau^2)$$

$$u_i \sim Ga(\delta, \varphi)$$

$$\pi(\delta); \delta \sim f_{Ga}(0.01, 0.01)$$

$$\pi(\varphi); \varphi \sim f_{Ga}(0.01, 0.01)$$

Non-informative prior normal distributions $N(0, \tau^2)$ were considered for the parameters of the exponential link function $(\beta_0, \beta_k; k = 1, 2, \dots p)$ with mean 0 and $\tau^2 = 0.001$, where $\tau^2 = 1/\sigma^2$ is know as a precision parameter. While, non-informative prior gamma distributions were considered for the parameters of the random scale $u_i \sim Ga(\delta, \varphi)$, with shape parameters $a_\delta = 0.01, a_\varphi = 0.01$ and scale parameters $b_\delta = 0.01, b_\varphi = 0.01$, respectively. Considering these prior distributions and the likelihood function of (8), the posterior distribution can be expressed as in (11). This estimation scheme was programmed in OpenBUGS and solved via MCMC.

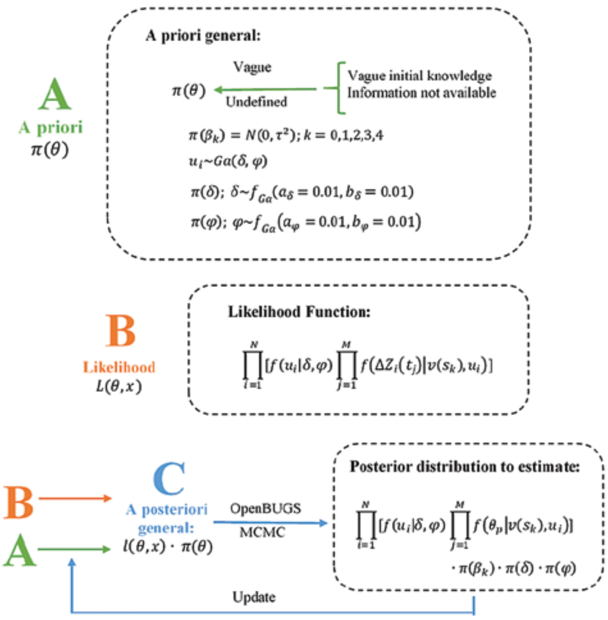

Fig. 1. Bayesian estimation scheme for the stochastic gamma process with random effects and link exponential function. Based on [27]

$$f\left(\beta_0, \beta_k, \delta, \varphi | \Delta Z_i(t_j)\right)$$
$$\propto \prod_{i=1}^{N}[f(u_i|\delta,\varphi)\prod_{j=1}^{M} f\left(\Delta Z_i(t_j)|v(s_k),u_i\right)] \cdot \pi(\beta_0) \cdot \pi(\beta_k) \cdot \pi(\delta) \cdot \pi(\varphi);$$
$$k = 1, 2, \dots p; i = 1, 2, \dots, N; j = 1, 2, \dots, M \tag{11}$$

4. Case study

The case study consists in an ADT performed to obtain the temperature increase of metal stampings that are incorporated in printed circuit boards (PCB). These PCBs are used in the fuse box of a certain automobile. The ADT was performed by applying different levels of current into the stamping as 60, 80 and 100 amperes. In addition, three types of materials were used in the test, which are: CDA151, CDA210 and CDA425, and four lengths of the stamping were also considered as 100, 150, 200 and 250 mm. This test configuration resulted in a total of five replicates each with 28 combinations of the described levels of the three factors. A total of 140 metal stampings, *i.e.* $i = 1, 2, \dots, 140$, were subjected to the different levels of current during 30 minutes. Thus, all the devices were observed simultaneously at $t_0 = 30, t_1 = 60, t_2 = 90, t_3 = 120, t_4 = 150$ and $t_5 = 180$. Given that the fuse box has a limit of temperature of 170°C, and for safety reasons,

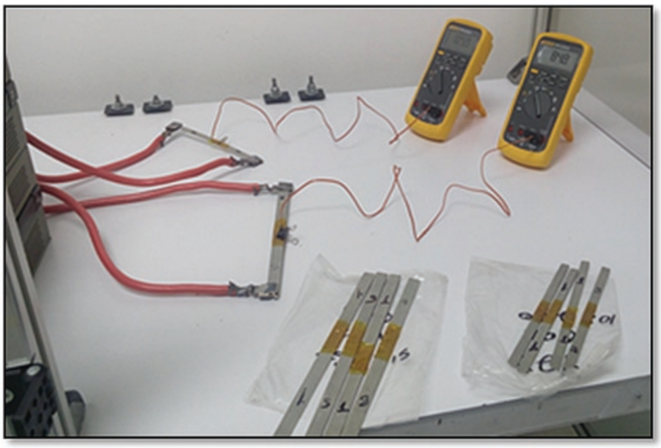

Fig. 2. Test configuration to obtain the temperature increase of the metal stampings

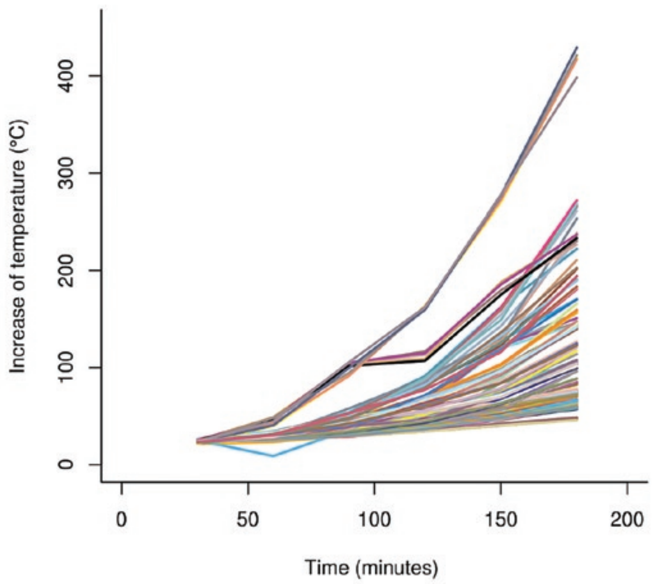

Fig. 3. Temperature increase paths for a sample of metal stampings

the maximum allowance for the temperature was set at 145°C (maximum design temperature). In Figure 2, the general configuration of the test is presented, the material type and the test equipment can also be seen. Furthermore, as can be noted in Figure 2 all the temperature readings were obtained in the middle of the stamping, this in the aims of obtaining homogenous readings for all the stampings under test. In Figure 3, a sample of paths of the temperature increase of different metal stampings is presented.

4.2. Estimation of the model

Considering the stochastic gamma process with random effects and the exponential link function for the current, material type and the length of the stamping, then the posterior distribution described in (11) results in:

$$f\left(\beta_0,\beta_k,\delta,\varphi|\Delta Z_i\left(t_j\right)\right)$$
$$\propto \prod_{i=1}^{N}\left[f\left(u_i|\delta,\varphi\right)\prod_{j=1}^{M} f\left(\Delta Z_i\left(t_j\right)|v\left(s_k\right),u_i\right)\right]\cdot\pi\left(\beta_0\right)\cdot\pi\left(\beta_k\right)\cdot\pi\left(\delta\right)\cdot\pi\left(\varphi\right);$$
$$k=1,2,3;\ i=1,2,\ldots,140;\ j=1,2,3,4,5$$

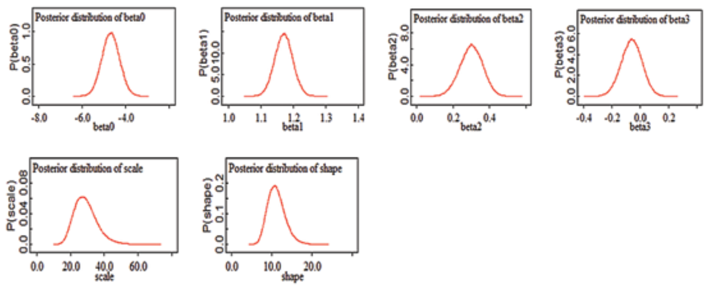

Fig. 4. Posterior distributions for $\left(\hat{\beta}_0,\hat{\beta}_1,\hat{\beta}_2,\hat{\beta}_3,\hat{\delta}\text{ and }\hat{\varphi}\right)$

Table 1. Obtained estimations of parameters under the Bayesian approach

	Mean	Sd	MC error	$t_{0.025}$	Median	$t_{0.975}$
$\hat{\beta}_0$	-4.688	0.4074	0.004095	-5.479	-4.69	-3.887
$\hat{\beta}_1$	1.171	0.02764	2.99E-04	1.117	1.171	1.225
$\hat{\beta}_2$	0.3002	0.06246	9.42E-04	0.1745	0.3014	0.4188
$\hat{\beta}_3$	-0.06096	0.07387	8.00E-04	-0.2082	-0.06007	0.08192
$\hat{\delta}$	28.99	6.769	0.1169	17.94	28.29	44.48
$\hat{\varphi}$	11.1	2.185	0.0356	7.403	10.91	15.96

The estimation scheme proposed in Figure 1 was considered for the estimation of the parameters of the model described above $\left(\beta_0,\beta_1,\beta_2,\beta_3,\delta,\varphi\right)$. According to the proposed scheme, the OpenBUGS software was used for the implementation of the MCMC method. The algorithm was constructed considering the model presented in (8) and non-informative prior distributions as follows: non-informative normal distributions were considered for the parameters of the link exponential function as $\beta_0,\beta_1,\beta_2,\beta_3 \sim N\left(0,0.0001\right)$. In addition, non-informative prior gamma distributions were considered for the parameters of the random effects parameter u_i as $\delta,\varphi \sim Ga\left(0.01,0.01\right)$. A total of 400,000 iterations were considered, from which a total of 50,000 were disregarded for burn-in purposes. In Table 1, a summary of the obtained estimations is presented. While, in Figure 4 the posterior distributions for $\left(\beta_0,\beta_1,\beta_2,\beta_3,\delta,\varphi\right)$ are presented.

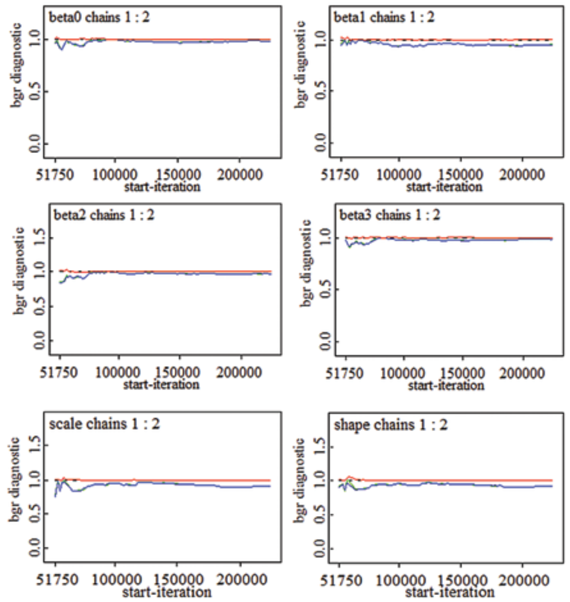

Fig. 5. BGR diagnostic for the parameters of interest

The Brooks-Gelman Rubin (BGR) diagnostic statistic was used in the aims of determining convergence in the estimation of parameters. For this, two chains of initial values were defined for the parameters of interest. The first chain of initial values was defined as $\beta_0=0.005,\beta_1=0.006,\beta_2=0.007,\beta_3=0.002,\delta=2$ and $\varphi=0.001$, while the second chain the next initial values were considered $\beta_0=0.1,\beta_1=0.1,\beta_2=0.1,\beta_3=0.1,\delta=1$ and $\varphi=0.1$. In Figure 5, the BGR graphs are presented for the parameters of interest under the two chains of initial values. Given that for all the

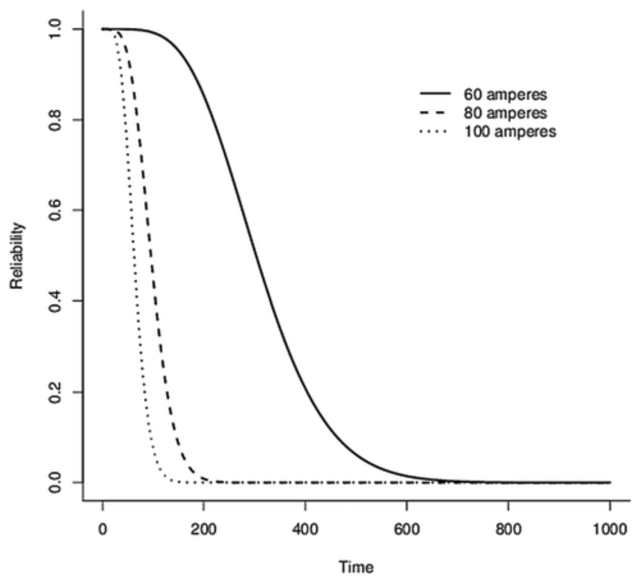

Fig. 6. Reliability functions under different levels of current

parameters of interest, the behavior of the two chains is around 1, then it can be said that convergence was achieved.

4.3. First passage time distribution

Considering the estimated parameters of interest that are presented in table 1, it results necessary to characterize the first passage time distributions when the degradation trajectories cross the critical level 145°C. As presented in equation (10) the CDF of the first passage times is described as:

$$F\left(t_\omega|s_k\right)=1-F_{v(s_k)t_j,2\delta}\left(\frac{\delta\omega}{\varphi v\left(s_k\right)t_j}\right)$$

Thus, the reliability function can be described as:

$$R\left(t_\omega|s_k\right)=F_{v(s_k)t_j,2\delta}\left(\frac{\delta\omega}{\varphi v\left(s_k\right)t_j}\right) \quad (12)$$

where the parameter $v\left(s_k\right), k=1,2,3$ is related to the link exponential function as $v\left(s_k\right)=\beta_0 e^{1.171s_1+0.3002s_2-0.06096s_3}$, where $s_1=current$, $s_2=material\,type$ and $s_3=length$. For this case study, we coded the material types as $CDA151=1$, $CDA210=2$ and $CDA425=3$. Of course different first passage time reliability functions can be characterized depending of the levels of $s_k, k=1,2,3$. In this case, we are interested in the material type $CDA151$ with coded value as 1, and the stamping length of 100. Under these two fixed levels of these two factors and the different levels of the current, we evaluated the function presented in equation (12). In figure 6, the different reliability functions are presented.

The effect of the current over the temperature of the stamping can be noted in the behavior of the reliability functions. In addition, the mean time to failure (MTTF) can be obtained as:

$$MTTF=\int_0^\infty R\left(t_\omega|s_k\right).$$

By considering the reliability function in (12) and the previously discussed levels of the material type and stamping length, the MTTF under the different levels of current were obtained as $MTTF=313.8065\,minutes$ for 60 amperes, $MTTF=98.74441\,minutes$ for 80 amperes and $MTTF=64.33426\,minutes$ for 100 amperes.

4.4. Comparison with other stochastic process

Besides the gamma process, we also considered the Wiener and the inverse Gaussian process to model the dataset obtained from the ADT. Equivalent stochastic degradation models with random effects from these two stochastic processes were considered, specifically the models proposed by Cheng & Peng [5] and Ye & Chen [46]. The comparison was carried out based on the Akaike information criterion (AIC), which is obtained via the formula $\text{AIC}=(-2)\text{loglikelihood}+2\,(\text{number of parameters})$ [8,1].

The obtained AIC values for the three stochastic processes resulted in: for the Wiener process a value of 239.5724, for the gamma process a value of 232.2521 was obtained, while for the inverse Gaussian process a value of 234.3279 was obtained. Based on these results, it can be noted that the stochastic gamma process is more adequate to model the degradation dataset of the presented case study. This, given that the AIC value was the lowest compared to the other two stochastic processes.

5. Conclusions

In this paper, a stochastic degradation model based on the gamma process was proposed, in which random effects and multiple stress variables are incorporated. The estimation of parameters was carried out using a Bayesian approach considering MCMC. The proposed model as well as the estimation scheme were implemented in a case study that consisted in the increase of temperature on metal stamping under different electric current levels, different types of material and different plate lengths were also considered. Given that non-informative a priori distributions were considered for the parameters of interest, two chains of initial values were defined in order to evaluate the convergence using a BGR graph. Given the behavior of the BGR graph it was noted that convergence was observed in all parameters. With the parameters obtained it was possible to estimate the reliability under the different electric current levels for the material type CDA-151 and a length of 100 mm, in the same way it was possible to obtain the MTTF values. The effect of the current and the increase in the temperature on the metal stamping can be seen both in the reliability functions and in the MTTF's. Finally, a comparison was made with the Wiener and inverse Gaussian processes based on the AIC value, given the obtained results it can be said that the gamma process turned out to be the best. As a future work, other characteristics of the stamping may be considered, to add them to the exponential link function, such as the width and thickness. These characteristics can be easily added to the proposed model.

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