Adam Słowik
Department of Electronics and Computer Science
Koszalin University of Technology
aslowik@ie.tu.koszalin.pl

Modified Fan Roulette Selection Method for Application in Evolutionary Algorithms

Key words: artificial intelligence, evolutionary algorithms, selection methods.

1. Introduction

The phase of selection is an important element among particular steps occurring in genetic or evolutionary algorithms. Due to selection operation, new populations i.e. sets of potential solutions are created. In literature we can find different methods of selection used in genetic algorithms, for example roulette method [1, 2, 3], elitist method [4], deterministic methods [5], random choice method according to the rest [1] (with repetition and without repetition), randomly tournament method [6]. But the most common selection methods used in practice are roulette and elitist methods. According to the schemata theorem, more copies are generated from the best individuals (chromosomes), the same number of copies are generated from average quality individuals, and the worse individuals are dying. However in the roulette selection the best chromosome (solution) can be destroyed and schemata coded in it will stop to spread out. To avoid this situation the elitist selection is used, in which the best individual found is remembered and replaces an individual with the worst fitness, in the next generation (when the best individual did not survive). With such an approach we know for sure that the best solution found will not be destroyed. In papers [7, 8] a modification of the roulette method has been presented. This modification depended on increasing survival probability for the best individual (surviving schemata existing in it) without guarantee that the best individual will pass to the next population for sure (thus, we assure a certain random factor during selection). This modification has been named a fan roulette selection (FRS). The results of test function minimization obtained using fan roulette selection presented in papers [7, 8] have been promising in relation to the results obtained using roulette selection, and elitist selection method. The fan roulette selection described in papers [7, 8] depends on increase of selection probability of the best individual in selection to the next population by simultaneous decrease of selection chances for other solutions (individuals). In the fan roulette selection the relative fitness values for particular individuals i.e. the probability selection values of individuals passing to the next population have been modified using formula (1), and (2):

* for the best individual

$$rfitness'_{max} = rfitness_{max} + (1 - rfitness_{max}) \cdot a \quad (1)$$

* for other individuals

$$rfitness' = (1 - rfitness'_{max}) \cdot \left(rfitness + \frac{rfitness_{max}}{M-1}\right) \quad (2)$$

where:

rfitness'$_{max}$ – new relative fitness of the best individual; *rfitness*$_{max}$ – relative fitness of the best individual; *a* – parameter causing the "fan expansion" ∊ [0, 1]; *rfitness'* – new relative fitness of chosen individual; *rfitness* – relative fitness of chosen individual; *M* – number of individuals in population.

The value changes of selection probabilities for given individual (potential solution) for different values of parameter *a* are shown in Figure 1.

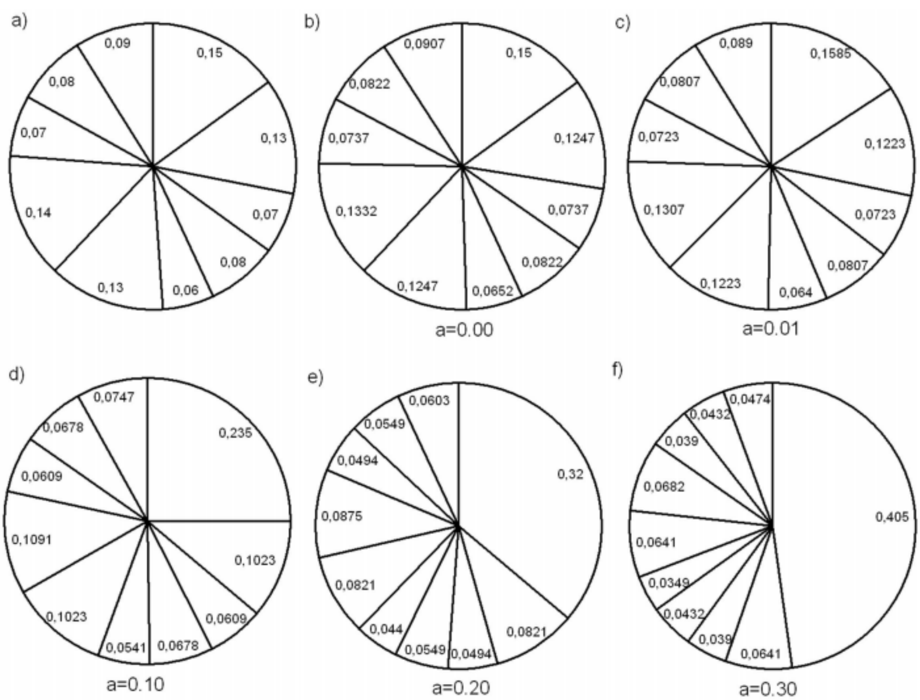

Figure 1. Roulette wheel: roulette selection (a), fan roulette selection for different values of *a* (b, c, d, e, f)

However, the fan roulette selection algorithm presented in papers [7, 8] has a inconvenience, that is for low values of a parameter, besides increasing of selection probability for the best individual, the selection probability values of the worse individuals selected to the new population are also increased at the expense of average quality individuals [9]. In this paper a modification of the fan roulette selection is presented. This modification has been named proportional fan roulette selection (PFRS). The proposed method eliminates early described inconvenience. The effectiveness of PFRS method has been checked by minimization of ten test functions chosen from literature. Results obtained using PFRS method have been compared with results obtained using roulette method, elitist method, and the fan roulette selection method (FRS).

2. Proportional Fan Roulette Selection – PFRS

The proportional fan roulette selection depends on increasing the selection probability for the best individual selected to the new population with simultaneous proportional decreasing of selection chances for other individuals selected to the new population. In PFRS method the formula (1) remains not changed, but the formula (2) is changed to the following form:

$$rfitness' = rfitness \cdot \left(\frac{rfitness_{max} - rfitness'_{max}}{\sum_{i=1}^{M} rfitness_i - rfitness_{max}} + 1 \right) \tag{3}$$

where:

rfitness'$_{max}$ – new relative fitness of the best individual; *rfitness*$_{max}$ – relative fitness of the best individual; *rfitness'* – new relative fitness of chosen individual; *rfitness* – relative fitness of chosen individual; M – number of individuals in population.

Depending on the value of parameter a, the values of selection probabilities for given individuals are changed as is shown in Figure 2.

It can be seen from Figure 2, that probability of survival of the best solution in the next generation is increasing and the probabilities of survival for all other solutions diminish proportionally in the PFRS method.

In the Figure 3, the relative fitness RF values of individuals shown in Figure 1a, and Figure 2a are presented for the purpose of more careful comparison of both methods. However, the β scaling coefficient values (see Appendix) for different values of a parameter are presented in particular columns of Figure 3. The β scaling coefficient is defined as follows:

$$\beta = \frac{rfitness'}{rfitness} \tag{4}$$

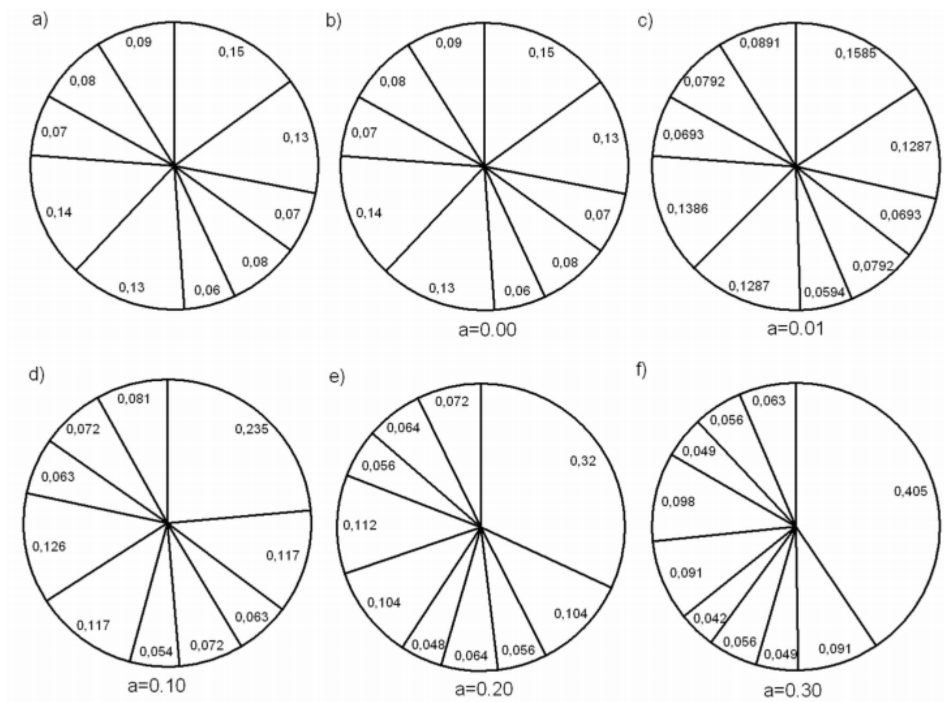

Figure 2. Roulette wheel: roulette selection (a), proportional fan roulette selection for different values of *a* (b, c, d, e, f)

and points out the ratio of relative fitness of individual after scaling (*rfitness'*), and before scaling (*rfitness*).

The symbols used in Figure 3, are the same as in equations (1-3).

It can be seen from Figure 3, PFRS method has better properties than FRS method.

No.	RF	β (for a=0.0)		β (for a=0.01)		β (for a=0.1)	
		FRS	PFRS	FRS	PFRS	FRS	PFRS
1	0.15	1.0	1.0	1.0566	1.0566	1.5667	1.5667
2	0.13	0.9590	1.0	0.9410	0.99	0.7866	0.9
3	0.07	1.0524	1.0	1.0334	0.99	0.8706	0.9
4	0.08	1.0271	1.0	1.0084	0.99	0.8479	0.9
5	0.06	1.0861	1.0	1.0668	0.99	0.9010	0.9
6	0.13	0.9590	1.0	0.9410	0.99	0.7866	0.9
7	0.14	0.9512	1.0	0.9333	0.99	0.7796	0.9
8	0.07	1.0524	1.0	1.0334	0.99	0.8706	0.9
9	0.08	1.0271	1.0	1.0084	0.99	0.8479	0.9
10	0.09	1.0074	1.0	0.9889	0.99	0.8302	0.9
No.	RF	β (for a=0.2)		β (for a=0.3)		β (for a=0.9)	
		FRS	PFRS	FRS	PFRS	FRS	PFRS
1	0.15	2.1333	2.1333	2.7	2.7	6.1	6.1
2	0.13	0.6312	0.8	0.4928	0.7	0.0194	0.1
3	0.07	0.7059	0.8	0.5582	0.7	0.0287	0.1
4	0.08	0.6857	0.8	0.5405	0.7	0.0262	0.1
5	0.06	0.7329	0.8	0.5818	0.7	0.0321	0.1
6	0.13	0.6312	0.8	0.4928	0.7	0.0194	0.1
7	0.14	0.6250	0.8	0.4873	0.7	0.0186	0.1
8	0.07	0.7059	0.8	0.5582	0.7	0.0287	0.1
9	0.08	0.6857	0.8	0.5405	0.7	0.0262	0.1
10	0.09	0.6699	0.8	0.5267	0.7	0.0242	0.1

Figure 3. Comparison of relative fitness RF and β coefficient values for different values of a parameter in fan roulette selection FRS and proportional fan roulette selection PFRS methods

3. Description of Experiments

Experiments were performed using evolutionary algorithm with individual representations in the form of lists of real numbers (each gene was represented by a real number from a given range). One point crossover and uniformly distributed mutation are used. The several test functions (from literature [1, 7, 8]) are chosen for verification and comparison of different selection methods (abbreviation GM stands for global minimal value):

a) De Jong function F1

$$\sum_{i=1}^{3} x_i^2 \text{ ; } -5.12 \le x_i \le 5.12\text{; GM=0 in } (x_1, x_2, x_3) = (0, 0, 0)$$

b) De Jong function F2

$100 \cdot \left(x_1^2 - x_2\right)^2 + (1 - x_1)^2$; $-2.048 \le x_i \le 2.048$; GM=0 in $(x_1, x_2) = (1, 1)$

c) De Jong function F3

$\sum_{i=1}^{5}$ *integer* (x_i); $-5.12 \le x_i \le 5.12$;

GM=–25 for all $-5.12 \le x_i \le -5.0$

d) De Jong function F4

$\sum_{i=1}^{30} i \cdot x_i^4$; $-1.28 \le x_i \le 1.28$; GM=0 in $(x_1, x_2, ..., x_{30}) = (0, 0, ..., 0)$

e) De Jong function F5

$$\frac{1}{1/K + \sum_{j=1}^{25} f_j^{-1}(x_1, x_2)}, \text{ where } f_j(x_1, x_2) = c_j + \sum_{i=1}^{2} \left(x_i - a_{ij}\right)^6,$$

and $-65.536 \le x_i \le 65.536$, K=500, $c_j = j$, and

$$\left[a_{ij}\right] = \begin{bmatrix} -32 & -16 & 0 & 16 & 32 & -32 & -16 & ... & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & -16 & -16 & ... & 32 & 32 & 32 \end{bmatrix}$$

GM=0.998 in $(x_1, x_2) = (-32, -32)$

f) Schaffer function F6

$$0.5 + \frac{\sin^2 \sqrt{x_1^2 + x_2^2} - 0.5}{\left[1.0 + 0.0001 \cdot \left(x_1^2 + x_2^2\right)\right]^2}; \ -100 \le x_i \le 100;$$

GM=0 in $(x_1, x_2) = (0, 0)$

g) Schaffer function F7

$\left(x_1^2 + x_2^2\right)^{0.25} \cdot \left[\sin^2\left(50 \cdot \left(x_1^2 + x_2^2\right)^{0.1}\right) + 1.0\right]$; $-100 \le x_i \le 100$;

GM=0 in $(x_1, x_2) = (0, 0)$

h) Goldstein-Price function F8

$$\left[1 + \left(x_1 + x_2 + 1\right)^2 \cdot \left(19 - 14 \cdot x_1 + 3 \cdot x_1^2 - 14 \cdot x_2 + 6 \cdot x_1 \cdot x_2 + 3 \cdot x_2^2\right)\right] \cdot$$
$$\left[30 + \left(2 \cdot x_1 - 3 \cdot x_2\right)^2 \cdot \left(18 - 32 \cdot x_1 + 12 \cdot x_1^2 + 48 \cdot x_2 - 36 \cdot x_1 \cdot x_2 + 27 \cdot x_2^2\right)\right]$$

$-2 \le x_i \le 2$; GM=3 in $(x_1, x_2) = (0, -1)$

i) Six-humps camel back function F9

$$\left(4 - 2.1 \cdot x_1^2 + \frac{x_1^4}{3}\right) \cdot x_1^2 + x_1 \cdot x_2 + \left(-4 + 4 \cdot x_2^2\right) \cdot x_2^2$$
;

$-3 \le x_1 \le 3$ and $-2 \le x_2 \le 2$

GM=–1.0316 in $(x_1, x_2) = (-0.0898, 0.7126)$ and $(0.0898, -0.7126)$

j) Coldville function F10

$$100 \cdot \left(x_2 - x_1^2\right)^2 + \left(1 - x_1\right)^2 + 90 \cdot \left(x_4 - x_3^2\right)^2 + \left(1 - x_3\right)^2 +$$
$$+ 10.1 \cdot \left(\left(x_2 - 1\right)^2 + \left(x_4 - 1\right)^2\right) + 19.8 \cdot \left(x_2 - 1\right) \cdot \left(x_4 - 1\right),$$

$-10 \le x_i \le 10$; GM=0 in $(x_1, x_2, x_3, x_4) = (1, 1, 1, 1)$

In Figures 4-8, the 3D graphical representations of each test function are shown. In the case of test functions having more variables than two (for example function F4 or F10), the graphical function representation based only on their two first variables have been shown. For test function F10, it is assumed, that variables x_3, and x_4 are equal to 1.

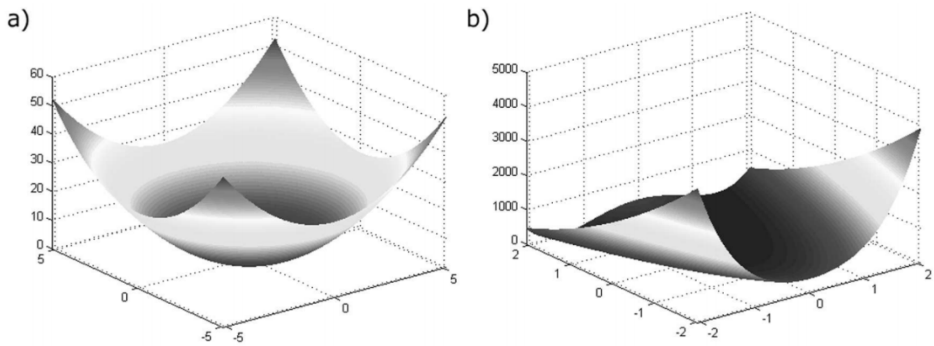

Figure 4. Graphical representation of De Jong function F1 (a), and De Jong function F2 (b)

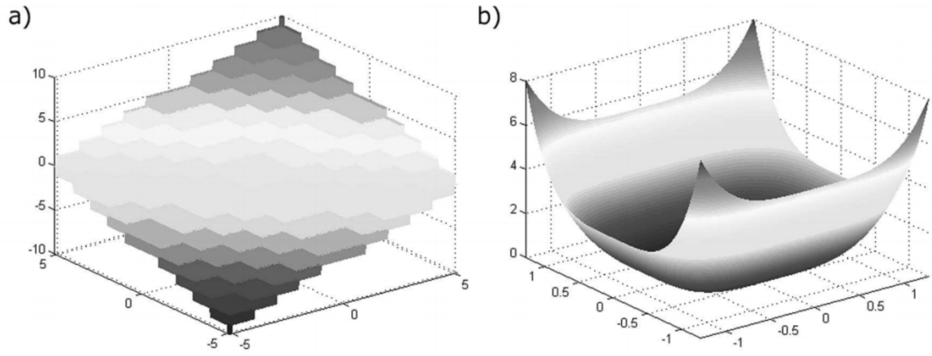

Figure 5. Graphical representation of De Jong function F3 (a), and De Jong function F4 (b)

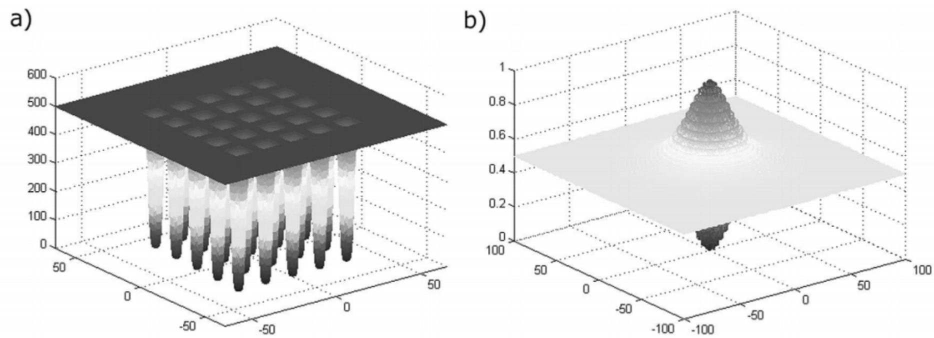

Figure 6. Graphical representation of De Jong function F5 (a), and Schaffer function F6 (b)

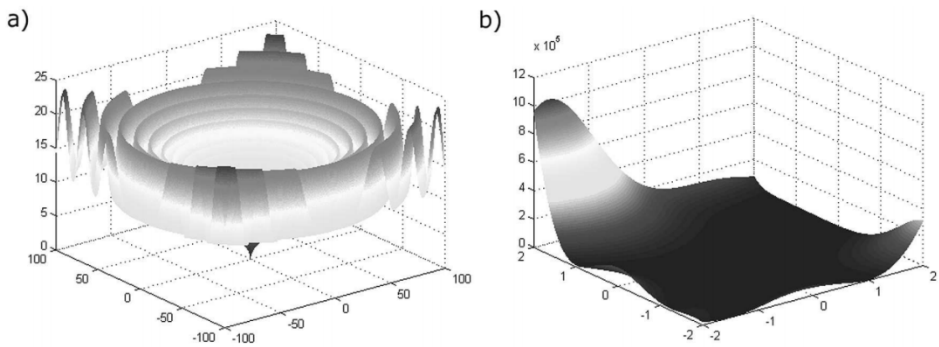

Figure 7. Graphical representation of Schaffer function F7 (a), and Goldstein-Price function F8 (b)

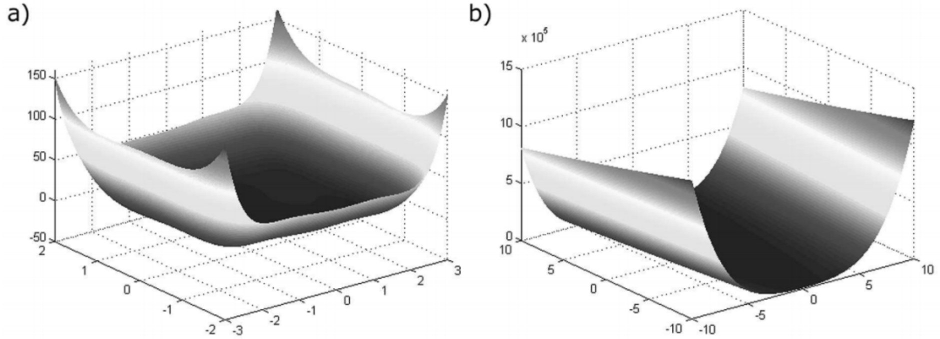

Figure 8. Graphical representation of Six-humps camel back function F9 (a), and Coldville function F10 (for x_3=1 and x_4=1) (b)

The evolutionary algorithms searched extremes of the test functions for different selection methods (roulette, elitist, fan roulette, proportional fan roulette). In the first experiment an evaluation of function minimum values which have been found by algorithms were performed. The evolutionary algorithm used for this purpose had following parameters: cross-over probability 0.5, mutation probability 0.1, number of individuals in population 100, the fan "expansion" *a* parameter value 0.3, number of generations 100. The computation has been repeated 100-fold. In Figure 9 the best function minimum values obtained after 100-fold repetition of evolutionary algorithm are shown.

TF	GM	RS	ES	FRS	PFRS
F1	0	$1.1293 \cdot 10^{-3}$	$4.2520 \cdot 10^{-4}$	$\mathbf{9.2702 \cdot 10^{-7}}$	$9.8702 \cdot 10^{-7}$
F2	0	$9.5024 \cdot 10^{-3}$	$3.3844 \cdot 10^{-4}$	$\mathbf{1.0770 \cdot 10^{-4}}$	$3.1188 \cdot 10^{-4}$
F3	-25	**-25**	**-25**	**-25**	**-25**
F4	0	23.1877	3.7063	0.0911	**0.0664**
F5	0.998	**0.9980**	**0.9980**	**0.9980**	**0.9980**
F6	0	$9.7285 \cdot 10^{-3}$	$9.7160 \cdot 10^{-3}$	$1.7518 \cdot 10^{-3}$	$\mathbf{9.5864 \cdot 10^{-4}}$
F7	0	0.05503244	0.03640455	0.01906307	**0.01650163**
F8	3	3.506820	3.003203	3.000057	**3.000003**
F9	-1.0316	-1.0311873	**-1.0316**	**-1.0316**	**-1.0316**
F10	0	41.1205836	0.91838732	0.13542065	**0.04534423**

Figure 9. The best minimal function values after 100-fold evolutionary algorithm repetition

TF	GM	RS	ES	FRS	PFRS
F1	0	0.26796329	0.01061643	0.00045582	**0.00041223**
F2	0	0.05785460	0.01170290	**0.00466204**	0.00507883
F3	-25	-24.36	**-25**	**-25**	**-25**
F4	0	43.6227861	11.5860708	0.63902525	**0.54373476**
F5	0.998	3.97313152	1.05175371	0.99825404	**0.99824610**
F6	0	0.04817742	0.02215023	**0.01745605**	0.01819210
F7	0	0.22630675	0.12623553	0.12286859	**0.11739955**
F8	3	27.3174015	3.2903303	3.0099151	**3.0098951**
F9	-1.0316	-0.92488558	-1.0300348	**-1.0315323**	-1.0314822
F10	0	863.908505	20.456860	5.24611426	**4.48156882**

Figure 10. The average minimal function values after 100-fold evolutionary algorithm repetition

In Figure 10 the average values of function minima after 100-fold repetition of evolutionary algorithm are shown. In Figure 9, and Figure 10, the symbols are as follows: TF - test function, GM - global minimum value, RS - roulette selection method, ES - elitist selection method, FRS – fan roulette selection method, PFRS - proportional fan roulette selection method. In both Tables the bold fonts represent the best obtained results.

It follows from Figure 9, that the solutions found using the proportional fan roulette selection (after 100 generations) are much better than solutions found in the same run-time using roulette selection, and are better (or comparable) than solutions found using elitist selection. In comparison to fan roulette selection method (FRS), the better or comparable results in 8 cases on 10 possible have been obtained using proposed proportional fan roulette selection (PFRS) method. Also, average values after 100 repetitions (Figure 10) show that the proportional fan roulette selection (for selected parameter *a*) is more stable, than roulette selection or elitist selection. Described PFRS selection method has the least deviations of obtained results from the best obtained solution. It is understandable, because larger part of the best individuals has a chance to enter to the next population. The highest differences we can find for De Jong function F4, Goldstein-Price function F8, and Coldvill function F10. Those differences refer to both the best solutions found after 100 generations and average values of solutions found in 100 subsequent tests. In the case of De Jong function F4 it is probably caused by the fact, that this function has 30 variables, what with mutation probability of order of 0.1, and population size of order of 100 causes that during one generation, approximately 300 genes can be mutated. This means that each individual in the population will undergo mutation, that is the searching will have more random character. It is possible to conclude from this, that the proportional fan roulette selection gives much better results, than roulette selection, and elitist selection in the case of existence of large number of mutated genes in population. The results obtained in Figure 10 for PFRS selection are in 7 cases (on 10 possible) better or comparable with results obtained using FRS selection. In the second experiment it has been examined how fan "expanding" parameter *a* influences the solution quality found by evolutionary algorithm. Here only the value of parameter *a* was changed in the range [0; 1], and other algorithm parameters were as in the first experiment. In order to obtain more diverging results, two test functions have been chosen to this experiment: F4, and F10, for which the highest variations of average value have been observed (after 100 generations). The average values of test functions minima obtained after 100 repetitions of evolutionary algorithm (with FS selection, and PFS selection) for different values of *a* parameter are shown in Figure 11. In Figure 11, the bold fonts represent the best obtained results.

Value of a	Test function F10		Test function F4	
	FRS	PFRS	FRS	PFRS
0.00	923.787454	**825.831571**	43.22775589	**41.27742876**
0.05	9.34704799	**6.05478458**	**16.32133735**	17.43088281
0.10	**4.01422242**	4.51124504	7.05015349	**6.75264382**
0.15	**3.42510477**	3.64843891	3.84245710	**3.75743812**
0.20	3.59535739	**3.49873288**	1.87237231	**1.59304921**
0.25	4.47301575	**3.45493020**	1.01711616	**0.96485874**
0.30	**3.93893284**	5.72613486	0.52734645	**0.51963446**
0.35	**4.40365514**	4.93385538	**0.35324468**	0.38431538
0.40	5.47608763	**4.31626349**	**0.16633081**	0.19688269
0.45	7.85601993	**5.40837456**	0.15065552	**0.10133293**
0.50	7.09333457	**6.97917105**	0.10778160	**0.07065058**
0.55	7.67483995	**6.32359892**	**0.05067826**	0.05873887
0.60	**5.88777219**	7.04008018	0.04277946	**0.03591072**
0.65	**8.06077111**	8.54346201	**0.02588992**	0.02944295
0.70	8.31152387	**8.14839001**	**0.02306716**	0.02453017
0.75	10.46178631	**10.32998750**	**0.01562465**	0.01929378
0.80	10.07995614	**6.90307361**	0.01415020	**0.01368041**
0.85	**9.93050283**	10.23908552	0.01122326	**0.01112236**
0.90	11.31274074	**7.20104095**	0.00912869	**0.00857236**
0.95	11.83710844	**11.60276270**	**0.00773982**	0.00774064
1.00	11.16117024	**9.52263277**	0.00632409	**0.00610230**

Figure 11. The value of *a* parameter influence on obtained average values of functions minima

It can be seen from Figure 11, that in the case of function F10 better results in (lower function values) have been obtained using proposed PFRS method compared to FRS method in 14 cases on 21 possible cases. However, for function F4, better results have been obtained using PFRS method compared to FRS method in 13 cases on 21 possible. Also, in the case of function F10, we can determine an approximate range of *ax* parameter values, for which better results have been obtained. This range is between 0.1 and 0.4. In the case of function F4, it has been observed, that the increase of *a* parameter values causes considerable improvement of obtained minimum values, and improves the algorithm convergence (more and more better results are found in the same time period).

4. Conclusions

In this paper the modification of fan roulette selection, named proportional fan roulette selection has been presented. Due to application of formula (3) the disadvantage (occurring in the fan roulette selection) depending on promotion of worst individuals at the cost of average quality individuals has been eliminated. Results obtained using proposed PFRS selection are in all cases better or comparable to results obtained using roulette selection, and elitist selection. The results obtained using proposed proportional fan roulette selection are in more cases (42 obtained results on 62 possible) better or comparable to results obtained using fan roulette selection.

References

1. Z. Michalewicz, *Genetic Algorithms + Data Structures = Evolution Programs*, WNT, Warsaw, 1999, (in Polish)
2. D. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, WNT, Warsaw, 1998, (in Polish)
3. J. Arabas, *Lectures of evolutionary algorithms*, WNT, Warsaw, 2001, (in Polish)
4. S. Zen, C. T. Zhou Yang, *Comparison of steady state and elitist selection genetic algorithms*, Proceedings of 2004 International Conference on Intelligent Mechatronics and Automation, August 26-31, 2004, pp. 495-499
5. N. Takaaki, K. Takahiko, Y. Keiichiro, *Deterministic Genetic Algorithm*, Papers of Technical Meeting on Industrial Instrumentation and Control, IEE Japan, pp. 33-36, 2003
6. T. Blickle, L. Thiele, *A Comparison of Selection Schemes used in Genetic Algorithms*, Computer Engineering and Communication Networks Lab, Swiss Federal Institute of Technology, TIK Report, No. 11, Edition 2, December 1995

7. A. Slowik, M. Bialko, *Modified Version of Roulette Selection for Evolution Algorithm - The Fan Selection*, Proceedings of Seventh International Conference on Artificial Intelligence and Soft Computing, ICAISC 2004, Lecture Notes in Artificial Intelligence, Volume 3070/2004, pp. 474-479, Springer-Verlag, 2004
8. A. Slowik, *Design and Optimization of Digital Electronic Circuits Using Evolutionary Algorithms*, Ph. D. Thesis, Koszalin University of Technology, Department of Electronics and Computer Science, Koszalin, 2007, (in Polish)
9. Private information from Prof. W. Jedruch from Gdansk University of Technology, Department of Electronics, Telecommunications and Informatics, 2007

Abstract

In the paper modified version of fan roulette selection method named proportional fan roulette selection is presented. This modification depends on increase of survive probability of the best individual at the expense of worse individuals and often gives better results compared to other selections. Test functions chosen from literature are used for determination of quality of proposed method. Results obtained using proportional fan roulette selection are compared with results obtained using roulette selection, elitist selection, and fan roulette selection.

Streszczenie

W artykule przedstawiono proporcjonalną selekcję wachlarzową będącą zmodyfikowaną wersją selekcji wachlarzowej. Wprowadzona modyfikacja polega na zwiększeniu prawdopodobieństwa przeżycia najlepszego osobnika kosztem osobników gorszych, często dając lepsze rezultaty w porównaniu do innych metod selekcji. Do sprawdzenia jakości utworzonej metody zastosowano funkcje testowe wybrane z literatury. Wyniki uzyskane przy użyciu proporcjonalnej selekcji wachlarzowej porównano z wynikami uzyskanymi przy użyciu selekcji ruletkowej, elitarnej oraz wachlarzowej.

Słowa kluczowe: sztuczna inteligencja, algorytmy ewolucyjne, metody selekcji