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The use of modal analysis to examine the lattice structure

Key words: modal analysis, natural vibration frequency, stabilization diagram, structural vibrations

Introduction

Modern building structures, production of silent-running machines and devices are associated with a high precision level of their manufacturing and appropriate selection of materials that greatly influence their quality, reliability and durability (Ibrahim & Mikulcik, 1977; M. Żółtowski, 2007, 2014b).

In investigating real systems (structures, buildings, machines, devices) the main problem is to determine quantity of energy stored, dissipated and transmitted by particular elements of the systems. Knowledge of the quantities serves to assessing material effort, fatigue, diagnostic investigations as well as predicting noise levels, and also to facilitate designing system's elements, e.g. vibration isolation (Shih, Tsuei, Allemang & Brown, 1988; Brunarski, 1996; Batel, 2002; B. Żółtowski, Łukasiewicz & Kałaczyński, 2012; M. Żółtowski, 2014b).

Development of measurement methods, especially those for measuring energy quantities, has substantially extended possibility of research on sound radiation by structures as well as made it possible to calculate sound power radiated to a remote field on the basis of close-field measurements. Methods for quantitative and qualitative research on vibroacoustic energy propagation within space of complex boundary areas have been developed (Bishop & Johnson, 1980; Allemang & Brown, 1983; Vold, Schwarz & Richardson, 2000; Pickrel, 2002; M. Żółtowski, 2014b).

It is necessary to improve methods for research on dynamic characteristics of structures especially those exposed on large dynamic loads. New materials and technology methods have been introduced to building engineering as well as novel structural solutions make it possible to increase productivity and quality of products, however they are accompanied with large, often dangerous dynamic loads (Wiliams, Crowley & Vold, 1985; M. Żółtowski, 2005, 2007; B. Żółtowski et al., 2012; M. Żółtowski, B. Żółtowski & Castaneda, 2013).

Modal analysis is widely used for investigating degradation state and fault location, modification of dynamics of tested structures, description and updating analytical model, as well as monitoring structural vibrations in aircraft and civil engineering. In the subject-matter literature the following notions can be found: modal analysis, experimental modal analysis and operational modal analysis (Allemang & Brown, 1983; Uhl, 1997; Peeters & Ventura, 2001; Pickrel, 2002; M. Żółtowski, 2011a).

In this paper are presented research results of differentiated state of lattice structure. For this aim was used the LMS Test.Lab software performing the tests and visualizing their results (M. Żółtowski, 2007).

Vibration in description of structures

Vibroacoustics is a domain of science which deals with any vibration, acoustic and pulsation processes occurring in nature, building engineering, technology, machines, devices, communication and transport means, i.e. in the environment.

Vibroacoustic process may be presented as:

- generation of time-varying forces acting onto a structure and its environment;
- propagation and transformation of energy in different environment structures;
- sound radiation through material elements of environment.

In analysis of vibroacoustic processes the following is taken into account:

- time – space distribution of run of energy coming from a (primary), source;
- response of a system (structure, liquid) as well as energy transmission through propagating media;
- mutual relations between sources.

The notion of measurement means a process of acquisition and transformation of information about a measured quantity to get – by comparing it with measurement unit – a quantitative result in a form most comfortable to be acquired by human sense organs, its transmission in space or time (recording), mathematical processing or application to steering. To carry out such measurements is necessary for (Bishop & Johnson, 1980; M. Żółtowski, 2011a):

- the determining of time runs of vibrations and their parameters to determine kinds of the vibrations, their characteristic quantities and to perform detail analysis;
- the finding of vibration sources and places of their occurrence;
- the determining of characteristic features of systems (e.g. determining loads during vibrations and their dependence on object's parameters, its shape, dimensions, material properties etc.);
- the minimizing of vibrations harmful for reliable operation of devices and their human operators;
- the determining of harmfulness level of occurring vibrations and the implementing of preventive measures.

In practice, vibration signal is more often used than noise one, due to its easiness of transferring and exactness of measuring (Vold, Kundrat, Rocklin & Russell, 1982; Brunarski, 1996).

System's vibrations resulting from upsetting state of equilibrium of an object which then moves under action of elastic, gravity or friction forces, are called free vibrations. In one-degree-of-freedom (d.o.f.) systems the upsetting of state of equilibrium is characterized by the initial conditions: the initial position (x_0) and initial velocity (v_0). If the system is of one d.o.f. (single mass *m*) and linear characteristics of elasticity (*k*) and damping (*c*) – Figure 1, and the harmonic excitation force *F*(*t*) acts onto it, then its motion equation is expressed by the following formula:

$$m * \dot{x} + c\dot{x} + kx = F(t) \tag{1}$$

which represents the equation of harmonic vibrations or harmonic oscillator vibrations.

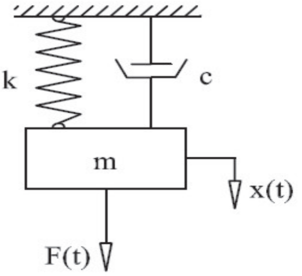


FIGURE 1. One-d.o.f. system to perform translation motion (own studies)

As results from it, natural vibration of one-d.o.f. system is entirely determined by natural frequency of vibration. Amplitude of the vibration depends on initial conditions, but natural frequencies and vibration period do not depend on them.

The parameters: *a*, *v*, *x* – are those of vibration process, which convinces that the vibrations properly describe state of structure.

In the low frequency range, building structures can be modelled by means of discrete systems of a few d.o.f.s – and rather often – one – d.o.f. system. The discrete system – in contrast to continuous one – is characterized by point distribution of mass, stiffness and damping and dimensions of the elements do not play any role. Number of d.o.f.s determines number of independent coordinates which should be introduced to get unambiguous description of system's motion (number of d.o.f.s is equal to number of mass elements in the system in question). In practice, the system presented in Figure 1 can model:

- the building machine of mass ***m***, seated on shock absorbers (***k***, ***c***) and fastened to a big mass foundation;
- the work machine of mass ***m***, seated on shock absorbers (***k***, ***c***) and moving along an even road;
- the high building structure (high chimney, masts) under wind action.

Many systems can be preliminarily modelled by using one – d.o.f. system, to search for its properties by means of mathematical description and analysis of solutions of equations which describe it. It is possible to investigate system's properties by using the vibration parameters ($\ddot{x}, \dot{x}, x$) which – being results of solutions of mathematical description of the model – interchangeably describe the same properties but from the viewpoint of the system's vibration measuring process.

Many possible occurrences of randomness and disturbances result in additional assumptions dealing with inputs and occurring transformations of structures destruction states. As a result of existence of the input and realization of transformation of states, which represent processes occurring in structure, many measurable characteristic symptoms contained in output processes emitted from structure, are obtained. The processes form the basis for elaboration of a signal generation model which determines a way of forming, functioning and changing states of object's destruction (M. Żółtowski, 2011b, 2014b).

The output signal received in an arbitrary point of structure is the weighted sum of responses to all elementary events (t, θ, r) which occur always in the same sequence in particular points of the dynamic system of the pulse transition function $h(t, \theta, r)$. The influences sum up together and subject to additional transformation along different reference axes, and a change of signal reception point r is associated also with change of transmittance (Fig. 2).

Model of vibration signal transmitting through tested structures or brick wall elements is described practically by FRF function which is determined by means of experimental modal analysis in the form of ratio of vibration excitation force and vibration acceleration amplitude at output. The transmittance $H(f)$ defined as the response-to-excitation ratio is inversion of the FRF function.

The indicated properties of the elaborated model of signal transition through tested materials were further used for assessing changes of lattice structures.

Theoretical modal analysis is defined as a matrix eigenvalue problem dependent on matrices of mass, stiffness and damping. It requires the eigenvalue problem for an assumed structural model of investigated structure to be solved (Uhl, 1997; M. Żółtowski, 2014a, 2014b). The determined sets of natural frequencies, damping coefficients for the natural frequencies and forms of natural vibrations make it possible to simulate behaviour of structure under arbitrary excitations, choice of steering means, structural modifications and other issues.

Analysis of natural frequencies and vectors is obtained on the basis of motion equations (after neglecting terms which contain damping matrix and external load vector). Then the motion equation of natural vibrations obtains the following form:

$$B\ddot{q} + Kq = 0 \qquad (2)$$

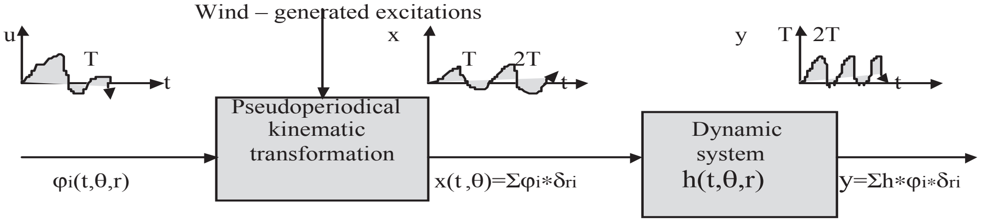


FIGURE 2. Transformation of the characteristic signal φi(*) into the output signal y(*), considered to be a model of signal generation in objects under environmental excitation (Ibrahim & Mikulcik, 1977; M. Żółtowski, 2011b)

For one d.o.f. system its solution is as follows:

$$q(t) = \vec{q}\sin(\varpi t + \phi) \tag{3}$$

where $\vec{q}$ – vector of amplitudes of natural vibrations.

On substitution of the above given equation and second derivative to the motion equation the following is obtained:

$$(-\varpi^2 B + K)\vec{q}\sin(\varpi t + \phi) = 0 \tag{4}$$

The equation is to be satisfied for arbitrary instant (t), then the set of algebraic equations is yielded as follows:

$$(K - \varpi^2 B)\vec{q} = 0 \tag{5}$$

$$(k_{11} - \omega^2 m_{11})q_1 + (k_{12} - \omega^2 m_{12})q_2 + \cdots$$
$$+ (k_{1n} - \omega^2 m_{1n})q_n = 0$$

$$(k_{21} - \omega^2 m_{21})q_1 + (k_{22} - \omega^2 m_{22})q_2 + \cdots$$
$$+ (k_{2n} - \omega^2 m_{2n})q_n = 0$$

….. …… ……. ……. ……

$$(k_{41} - \omega^2 m_{41})q_1 + (k_{42} - \omega^2 m_{42})q_2 + \cdots$$
$$+ (k_{nn} - \omega^2 m_{nn})q_n = 0$$

This way was produced the set of linear homogeneous algebraic equations, which has non-zero solution only when the condition:

$$\det(K - \varpi^2 B) = 0 \tag{6}$$

is fulfilled.

On transformations the n-order polynomial is obtained. Among its roots multifold ones may be present, and the vector built from the set of frequencies ω^2 ordered according to increasing value sequence is called the frequency vector, and the first frequency is called the fundamental one (M. Żółtowski, 2014b):

$$\omega = [\omega_1, \omega_2, \ldots, \omega_n] \tag{7}$$

Measurement software

For the measurement waveforms extortion and response system and determine the most used functions FRF measurement equipment purchased for the project company under the name of LMS Test.Xpress. Figure 3 shows printscreen of sensor calibration via measurement equipment software.

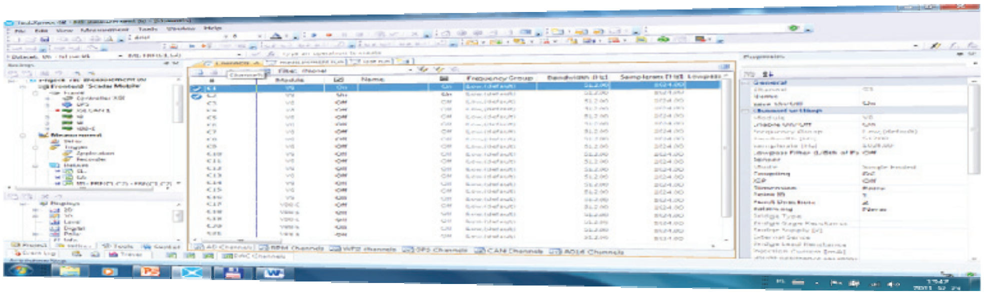


FIGURE 3. Calibration of sensor connection (own studies)

Creating a real model

This model was created based on measuring the vibration structure at a given forcing. Operational modal analysis is based on the measurement of vibration caused by natural forcing operational characteristic of the work of a machine or mechanical design.

The next step is to choose the appropriate response measurement points and enforce proper place where this goes arouse the greatest amount of vibration studied mechanical design.

Figure 4 shows the mounting of the sensor responses and how enforcement structures modal hammer. Figures 5 and 6 show two graphs in the time domain from the modal hammer response of the sensor.

Modal hammer is equipped with a force sensor which enables the value of a given force. Reaction force the truss to the question is measured vibration acceleration sensor. Figure 5 shows the force value in the form of a pulse at a level above 100 N. The signal caused by forcing is received by a piezoelectric sensor response (Fig. 6). During the measure-

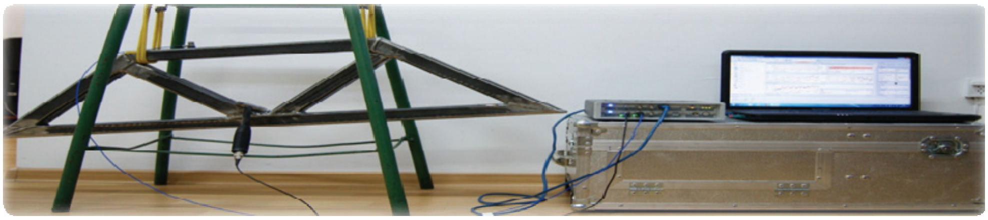


FIGURE 4. Measuring position (own studies)

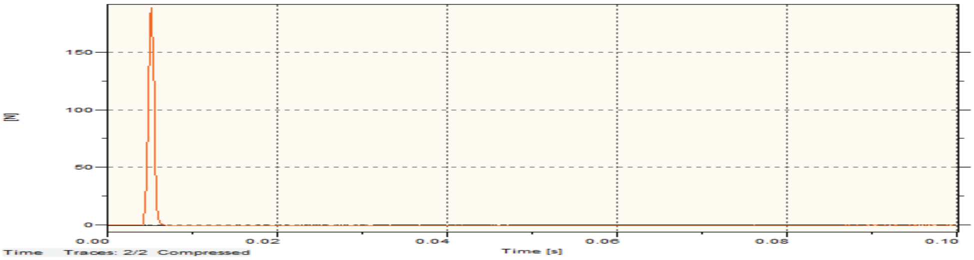


FIGURE 5. The signal from the modal hammer response (own studies)

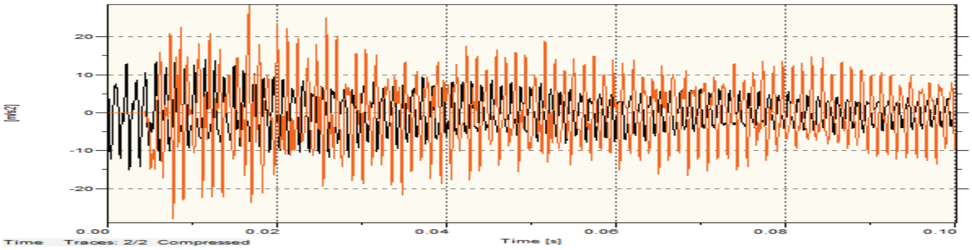


FIGURE 6. The signal from the sensor response (own studies)

ment of vibration with increasing frequency forcing amplitude changes occur at the measuring point.

Despite constant during the exciting force, the answer to the question of the extortion strengthened in some frequencies grill until achieving full compatibility between the frequency and force of its resonant frequency. Processing time signal to a frequency signal by fast Fourier transform (FFT) allows to determine the so-called spectral transfer function (FRF). This form of the signal allows for a much simpler form of the object of determining the resonant frequency. The appointment of these frequencies is even easier if you superimpose on the chart FRF plot coherence (Fig. 7). Data obtained in the form of spectral transfer function are used to estimation of individual modal parameters including mode shapes. Fashion vibrations examined structure takes different forms depending on the frequency of extortion (Fig. 8).

Modal parameters of the model estimates of individual spectral transfer function (FRF). Each FRF is presented in the form of a graph, which is established to analyse the frequency range. This process is shown in Figure 9.

Modal parameter estimation can be carried out in two areas: time and frequency. After preparing the measurement results for further analysis, their estimation is done by creating so-called stabilisation diagram. This diagram consists of different fields marked: *s* – the field is stable, *v* – vector modal, *f* – frequency field, *d* – the field attenuation, *o* – blank (Fig. 10).

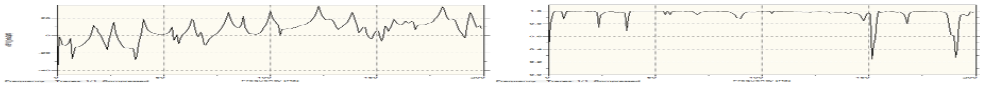


FIGURE 7. Chart (from left) FRF spectral transfer function and coherence function chart COH (own studies)

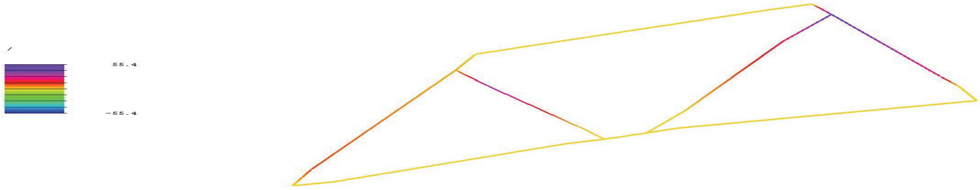


FIGURE 8. Form lattice vibrations at a frequency of 2,803.25 Hz and attenuation at 0.06% (own studies)

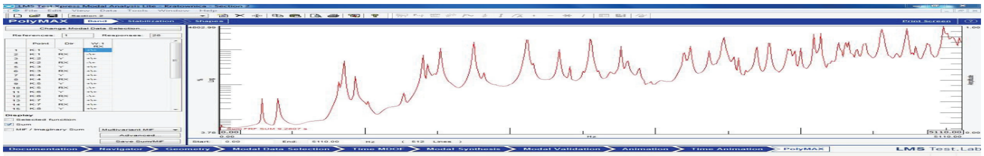
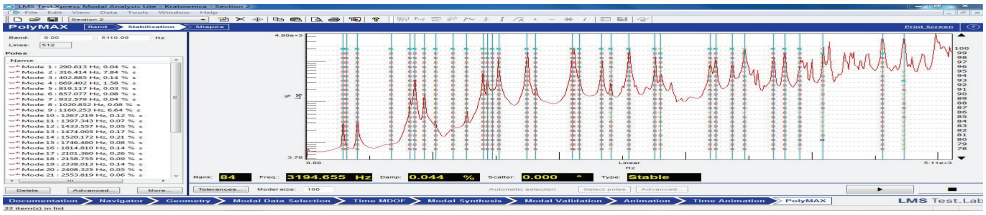


FIGURE 9. Summary and preparation spectral estimation function FRF modal parameters (own studies)



Band: 0.00 5110.00 Hz
Lines: 512

Poles

Name
Mode 1 : 290.613 Hz, 0.04 % s
Mode 2 : 316.414 Hz, 7.84 % s
Mode 3 : 402.885 Hz, 0.14 % s
Mode 4 : 669.402 Hz, 1.58 % s
Mode 5 : 819.117 Hz, 0.03 % s
Mode 6 : 857.077 Hz, 0.08 % s
Mode 7 : 932.579 Hz, 0.04 % s
Mode 8 : 1020.852 Hz, 0.08 % s
Mode 9 : 1160.253 Hz, 6.64 % s
Mode 10 : 1267.219 Hz, 0.12 % s
Mode 11 : 1397.343 Hz, 0.07 % s
Mode 12 : 1433.557 Hz, 0.05 % s
Mode 13 : 1474.005 Hz, 0.17 % s
Mode 14 : 1520.172 Hz, 0.21 % s
Mode 15 : 1746.460 Hz, 0.08 % s
Mode 16 : 1814.810 Hz, 0.14 % s
Mode 17 : 2101.360 Hz, 0.26 % s
Mode 18 : 2158.755 Hz, 0.09 % s
Mode 19 : 2338.013 Hz, 0.14 % s
Mode 20 : 2408.325 Hz, 0.05 % s
Mode 21 : 2553.819 Hz, 0.06 % s

Rank: 84 Freq.: 3194.655 Hz Damp: 0.044 % Scatter: 0.000 ° Type: Stable
Model size: 100
33 item(s) in list

FIGURE 10. Stabilization diagram (own studies)

By his own assessment of the stabilization diagram obtained these fields choose stable (*s*), which, in our discretion properly will describe the state of our object. If you clear each individual field in the diagram, we obtain the modal parameters in the form of natural frequencies, damping factor and the form of vibrations, respectively. For analysis of grid summarizes the first two modal parameters in Table 1.

For each shown in Table 1 as a vibration is also assigned its graphical form, an example of which is shown in Figure 11. Other forms of vibration are summarised in the next section, as compared with figures obtained from the analysis of the vibration theoretical model.

Characteristic vibrations that were obtained were also validated to eliminate these forms, which are largely dependent on each other. Individual fashion vibration may be different in nature: the torsional and flexural – torsion. The position of the natural frequencies and mode shapes due to the properties of the test structures described by the parameters such as mass, stiffness and damping. Vibration analysis of individual charac-

TABLE 1. Summary of natural frequencies and damping coefficients for the analysed structure after validation – frequency range of 0.7–5,000 Hz (own studies)

Form vibrations	Frequency [Hz]	Damping [%]
1	316.414	7.48
2	402.885	0.14
3	857.077	0.08
4	932.579	0.04
5	1 020.852	0.08
6	1 160.253	6.64
7	1 433.557	0.05
8	1 474.005	0.17
9	1 814.810	0.14
10	2 338.013	0.14
11	2 408.325	0.05
12	2 553.819	0.06
13	2 803.254	0.06
14	3 465.221	0.06
15	3 784.573	0.04
16	3 907.980	0.07

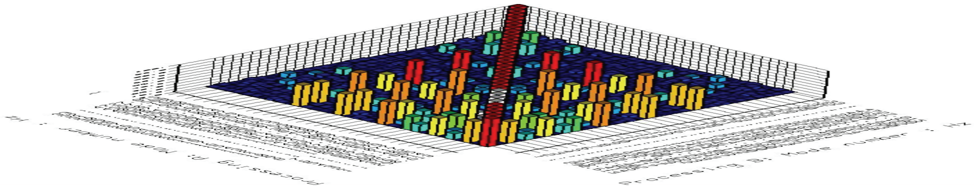


FIGURE 11. Validation mode shapes obtained (own studies)

ters but not all the time is ambiguous. Therefore, to assess the correctness of the selected poles modal model method is used.

Validated results are shown in Table 2 and Figure 12. After validation of the form 33 resulting vibrations is selected 16, which in the best extent reflect the current state of the test grid. Sixteen statement of mode shapes are shown in Table 3.

Criterion involves checking the condition of orthogonality of eigenvectors for the analysed modal model. Thanks to the analysis carried out it was possible to reject part of the poles, which are chosen subjectively, leading to the final form of the modal model describing the dynamic state of the object of research. Step validation of foreclosing the creation of a model based on measurements on the actual object.

TABLE 2. Tabulated results validate certain mode shapes (own studies)

	Mode No.	Frequency	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10	Mode 11	Mode 12	Mode 13	Mode 14	Mode 15	Mode 16	Mode 17	Mode 18	Mode 19	Mode 20	Mode 21	Mode 22	Mode 23	Mode 24	Mode 25	Mode 26	Mode 27	Mode 28	Mode 29	Mode 30	Mode 31	Mode 32	Mode 33
1	Mode 1	290.613	100	2,753	78,057	0,125	20,621	0,021	63,245	1,277	27,139	47,894	0,045	0,116	82,66	0,087	0,221	5,077	0,015	77,448	0,03	0	0,057	0,267	76,49	0	0,108	6,131	0	1,556	0,861	3,053	1,077	0,617	0,533
2	Mode 2	316.414	2,753	100	1,771	2,444	1,013	19,99	0,347	4,22	0,958	1,925	0,324	12,018	1,983	0,18	2,177	0,22	9,036	0,123	0,696	0,28	1,063	2,802	0,051	0,63	0,814	6,24	0,512	0,188	0,026	6,957	1,73	0,077	2,705
3	Mode 3	402.885	78,057	1,771	100	0,014	26,582	0,051	58,246	0,186	19,415	34,546	0,292	0,14	68,913	0,117	0,294	3,648	0,217	75,507	0,107	0,007	0,076	0,353	75,275	0,062	0,055	1,321	0,009	0,132	0,013	4,164	0,072	0,765	0,001
4	Mode 4	669.402	0,125	2,444	0,014	100	0,065	15,998	18,534	0,112	47,288	16,608	7,803	0,353	4,387	0,23	11,558	1,92	19,85	7,827	0,813	0,04	0,035	1,824	5,906	1,723	2,244	0,732	0,744	0,182	0,435	2,024	6,771	8,158	0,209
5	Mode 5	819.117	20,621	1,013	26,582	0,065	100	1,278	23,698	0,598	10,81	15,735	0	2,353	24,387	0,44	0,089	5,808	0,531	31,612	0,274	0,198	0,013	0,004	28,998	2,042	0,005	11,03	0,028	11,58	0,06	2,745	4,446	0,298	4,16
6	Mode 6	857.077	0,021	19,99	0,051	15,998	1,278	100	0,167	2,745	0,111	0,032	5,092	3,751	0,044	0,004	12,875	0,279	2,487	0,013	0,181	5,556	0,438	1,685	0,183	1,909	0,106	1,926	0,44	0,012	0,568	14,55	0,047	1,314	1,322
7	Mode 7	932.579	63,245	0,347	58,246	18,534	23,698	0,167	100	0,006	1,316	66,252	0,035	0,378	86,026	0,065	0	10,203	2,216	83,852	0,029	0,011	0,039	0,077	85,472	0,319	0,036	2,779	0	0,755	2,827	1,205	1,442	0,133	0,004
8	Mode 8	1020.852	1,277	4,22	0,186	0,112	0,598	2,745	0,006	100	0,514	0,238	0,056	6,1	0,477	17,692	2,24	0,049	6,464	0,003	25,248	0,091	4,929	2,782	0,091	0,027	1,205	2,202	0,281	0,204	0,626	6,027	0,632	16,374	0,87
9	Mode 9	1160.253	27,139	0,958	19,415	47,288	10,81	0,111	1,316	0,514	100	0,156	9,129	0,202	11,425	0,75	2,601	0,199	7,405	5,758	1,68	2,922	0,034	1,171	7,132	0,061	0,653	7,821	0,61	0,502	3,409	0,053	4,474	5,141	3,969
10	Mode 10	1267.219	47,894	1,925	34,546	16,608	15,735	0,032	66,252	0,238	0,156	100	0,002	1,073	53,572	0,047	0,007	2,84	1,407	61,358	0,006	0,004	0,074	0,022	60,996	0,488	0,004	2,932	0	1,055	6,178	9,284	12,395	0,133	0,328
11	Mode 11	1397.343	0,045	0,324	0,292	7,803	0	5,092	0,035	0,056	9,129	0,002	100	13,743	0,152	0,318	1,786	0,429	0,1	0,01	1,024	4,292	2,669	0,467	0,02	0,004	0,097	0,873	20,203	0	0,138	2,522	0,005	0	1,413
12	Mode 12	1433.557	0,116	12,018	0,14	0,353	2,353	3,751	0,378	6,1	0,202	1,073	13,743	100	0,02	0,896	2,316	0,739	1,166	0,424	0,01	0,13	13,719	11,16	0,412	1,307	0,015	0,2	1,114	0,121	0,013	21,421	0,262	1,321	0,993
13	Mode 13	1474.005	82,66	1,983	68,913	4,387	24,387	0,044	86,026	0,477	11,425	53,572	0,152	0,02	100	0,048	0,078	10,101	0,366	88,399	0,041	0	0,026	0,125	90,473	0,272	0,078	4,173	0,11	0,246	1,742	0,72	0,04	0,014	0,762
14	Mode 14	1520.172	0,087	0,18	0,117	0,23	0,44	0,004	0,065	17,692	0,75	0,047	0,318	0,896	0,048	100	0,263	0,232	3,577	0,003	1,963	1,71	0,176	4,511	0,019	0,245	9,032	0,545	1,861	1,965	0,015	0,482	0,594	6,885	0,373
15	Mode 15	1746.460	0,221	2,177	0,294	11,558	0,089	12,875	0	2,24	2,601	0,007	1,786	2,316	0,078	0,263	100	1,332	13,051	0,493	0,802	13,417	16,779	0,509	0,003	0,043	0,379	0,652	9,288	0,277	0,899	0,063	0,121	0,567	1,302
16	Mode 16	1814.810	5,077	0,22	3,648	1,92	5,808	0,279	10,203	0,049	0,199	2,84	0,429	0,739	10,101	0,232	1,332	100	0	12,438	0,831	0,321	0,661	0,192	8,886	0,195	0,364	8,448	0,223	4,852	0,397	0,051	1,134	0,669	2,917
17	Mode 17	2101.360	0,015	9,036	0,217	19,85	0,531	2,487	2,216	6,464	7,405	1,407	0,1	1,166	0,366	3,577	13,051	0	100	3,263	0,524	2,935	10,34	0,03	0,244	0,006	0,654	0,125	0,961	0,017	0	0,133	0,03	0,603	3,01
18	Mode 18	2158.755	77,448	0,123	75,507	7,827	31,612	0,013	83,852	0,003	5,758	61,358	0,01	0,424	88,399	0,003	0,493	12,438	3,263	100	0,358	0,06	0,087	0,01	92,013	0,167	0,08	7,902	0,058	4,602	0,011	4,44	0,21	0,167	0,587
19	Mode 19	2338.013	0,03	0,696	0,107	0,813	0,274	0,181	0,029	25,248	1,68	0,006	1,024	0,01	0,041	1,963	0,802	0,831	0,524	0,358	100	0,003	2,242	2,393	0,025	0,126	2,627	0,148	0,061	3,148	0,02	3,543	0,002	0,058	0,615
20	Mode 20	2408.325	0	0,28	0,007	0,04	0,198	5,556	0,011	0,091	2,922	0,004	4,292	0,13	0	1,71	13,417	0,321	2,935	0,06	0,003	100	35,821	2,158	0,06	0,556	0,103	0,142	0,063	0,44	0,137	3,603	0,496	0,635	2,383
21	Mode 21	2553.819	0,057	1,063	0,076	0,035	0,013	0,438	0,039	4,929	0,034	0,074	2,669	13,719	0,026	0,176	16,779	0,661	10,34	0,087	2,242	35,821	100	0,072	0,011	0,444	0,071	0,448	8,95	0,02	0,534	3,997	0,077	2,886	0,679
22	Mode 22	2759.615	0,267	2,802	0,353	1,824	0,004	1,685	0,077	2,782	1,171	0,022	0,467	11,16	0,125	4,511	0,509	0,192	0,03	0,01	2,393	2,158	0,072	100	0,67	0,038	14,116	0,539	10,192	0,417	2,645	0,134	0	7,634	0,011
23	Mode 23	2803.254	76,49	0,051	75,275	5,906	28,998	0,183	85,472	0,091	7,132	60,996	0,02	0,412	90,473	0,019	0,003	8,886	0,244	92,013	0,025	0,06	0,011	0,67	100	0,264	0,016	4,74	0,106	0,834	0	1,972	0,534	0,027	0,002
24	Mode 24	3194.469	0	0,63	0,062	1,723	2,042	1,909	0,319	0,027	0,061	0,488	0,004	1,307	0,272	0,245	0,043	0,195	0,006	0,167	0,126	0,556	0,444	0,038	0,264	100	0,047	0,112	0,085	4,133	20,372	11,646	25,825	0	0,58
25	Mode 25	3238.807	0,108	0,814	0,055	2,244	0,005	0,106	0,036	1,205	0,653	0,004	0,097	0,015	0,078	9,032	0,379	0,364	0,654	0,08	2,627	0,103	0,071	14,116	0,016	0,047	100	1,21	1,514	1,644	0,294	0,724	0,176	0,953	0,005
26	Mode 26	3369.014	6,131	6,24	1,321	0,732	11,03	1,926	2,779	2,202	7,821	2,932	0,873	0,2	4,173	0,545	0,652	8,448	0,125	7,902	0,148	0,142	0,448	0,539	4,74	0,112	1,21	100	0,215	38,318	0,323	0,109	0,438	0,123	1,967
27	Mode 27	3465.221	0	0,512	0,009	0,744	0,028	0,44	0	0,281	0,61	0	20,203	1,114	0,11	1,861	9,288	0,223	0,961	0,058	0,061	0,063	8,95	10,192	0,106	0,085	1,514	0,215	100	0,759	1,074	1,761	0,166	0,586	0,49
28	Mode 28	3695.568	1,556	0,188	0,132	0,182	11,58	0,012	0,755	0,204	0,502	1,055	0	0,121	0,246	1,965	0,277	4,852	0,017	4,602	3,148	0,44	0,02	0,417	0,834	4,133	1,644	38,318	0,759	100	0,348	0,408	2,945	0,331	8,178
29	Mode 29	3784.573	0,861	0,026	0,013	0,435	0,06	0,568	2,827	0,626	3,409	6,178	0,138	0,013	1,742	0,015	0,899	0,397	0	0,011	0,02	0,137	0,534	2,645	0	20,372	0,294	0,323	1,074	0,348	100	18,822	42,324	0,162	0,048
30	Mode 30	3907.980	3,053	6,957	4,164	2,024	2,745	14,55	1,205	6,027	0,053	9,284	2,522	21,421	0,72	0,482	0,063	0,051	0,133	4,44	3,543	3,603	3,997	0,134	1,972	11,646	0,724	0,109	1,761	0,408	18,822	100	13,053	0,584	7,31
31	Mode 31	4090.162	1,077	1,73	0,072	6,771	4,446	0,047	1,442	0,632	4,474	12,395	0,005	0,262	0,04	0,594	0,121	1,134	0,03	0,21	0,002	0,496	0,077	0	0,534	25,825	0,176	0,438	0,166	2,945	42,324	13,053	100	0,263	0,231
32	Mode 32	4557.588	0,617	0,077	0,765	8,158	0,298	1,314	0,133	16,374	5,141	0,133	0	1,321	0,014	6,885	0,567	0,669	0,603	0,167	0,058	0,635	2,886	7,634	0,027	0	0,953	0,123	0,586	0,331	0,162	0,584	0,263	100	1,809
33	Mode 33	4727.815	0,533	2,705	0,001	0,209	4,16	1,322	0,004	0,87	3,969	0,328	1,413	0,993	0,762	0,373	1,302	2,917	3,01	0,587	0,615	2,383	0,679	0,011	0,002	0,58	0,005	1,967	0,49	8,178	0,048	7,31	0,231	1,809	100

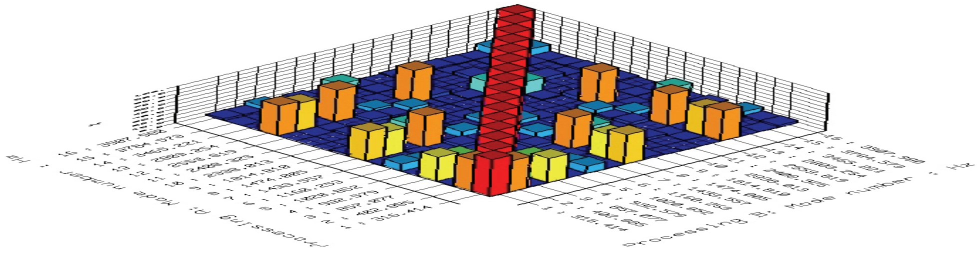


FIGURE 12. Validation mode shapes obtained (own studies)

TABLE 3. Tabulated results validate certain mode shapes (own studies)

	Mode No.	Frequenc	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10	Mode 11	Mode 12	Mode 13	Mode 14	Mode 15	Mode 16
1	Mode 1	316.414	100	84,502	0,494	66,049	0,289	19,989	1,096	79,749	4,461	0,003	0,056	0	81,348	0,004	0,661	5,22
2	Mode 2	402.885	84,502	100	0,002	57,477	0	16,321	0,298	66,847	3,65	0,155	0,146	0,451	75,194	0,005	0,004	4,031
3	Mode 3	857.077	0,494	0,002	100	0,228	3,385	7,889	4,608	0,044	0,023	0,182	5,742	0,384	0,179	0,439	0,559	14,379
4	Mode 4	932.579	66,049	57,477	0,228	100	0,339	3,723	0,277	84,451	10,669	0,038	0,004	0,114	85,591	0	2,866	1,279
5	Mode 5	1020.852	0,289	0	3,385	0,339	100	0,893	7,386	0,134	0,196	23,73	0,305	7,648	0,03	0,052	0,566	6,382
6	Mode 6	1160.253	19,989	16,321	7,889	3,723	0,893	100	4,298	17,57	0,254	1,263	0,4	4,317	10,132	2,943	1,041	0,179
7	Mode 7	1433.557	1,096	0,298	4,608	0,277	7,386	4,298	100	0,015	0,369	0,104	0,209	14,862	0,306	0,359	0,002	24,488
8	Mode 8	1474.005	79,749	66,847	0,044	84,451	0,134	17,57	0,015	100	10,874	0,221	0,018	0,024	88,974	0,037	1,637	1,056
9	Mode 9	1814.810	4,461	3,65	0,023	10,669	0,196	0,254	0,369	10,874	100	0,991	0,021	0,159	9,56	0,056	0,323	0,047
10	Mode 10	2338.013	0,003	0,155	0,182	0,038	23,73	1,263	0,104	0,221	0,991	100	0,006	2,226	0,017	0,055	0,021	3,53
11	Mode 11	2408.325	0,056	0,146	5,742	0,004	0,305	0,4	0,209	0,018	0,021	0,006	100	35,463	0,079	0,067	0,129	3,567
12	Mode 12	2553.819	0	0,451	0,384	0,114	7,648	4,317	14,862	0,024	0,159	2,226	35,463	100	0,005	8,957	0,547	4,029
13	Mode 13	2803.254	81,348	75,194	0,179	85,591	0,03	10,132	0,306	88,974	9,56	0,017	0,079	0,005	100	0,079	0	2,02
14	Mode 14	3465.221	0,004	0,005	0,439	0	0,052	2,943	0,359	0,037	0,056	0,055	0,067	8,957	0,079	100	1,105	1,746
15	Mode 15	3784.573	0,661	0,004	0,559	2,866	0,566	1,041	0,002	1,637	0,323	0,021	0,129	0,547	0	1,105	100	19,22
16	Mode 16	3907.980	5,22	4,031	14,379	1,279	6,382	0,179	24,488	1,056	0,047	3,53	3,567	4,029	2,02	1,746	19,22	100

A summary of the results of the actual and theoretical model

Modal study aims to determine the dynamic properties of lattice elements commonly used in the construction to identify possible ways to diagnose and even modify these properties through structural changes, which would ensure a high quality of these objects. To perform modal analysis lattice structure created a three-dimensional model of the selected item. For truss structures consist of suitable types of shapes or profiles associated disjoint or inseparable connections. Similarly implemented in Inventor. Modelled the selected element, which has been associated with a geometric relationship, according to the nature of cooperation between these elements. Thus, created were analysed element via the "Stress Analysis". This module is one of the analytical computing subsystems Inventor and comes with the possibility of using the finite element method to carry out the theoretical modal analysis. Pre-preparation step in the calculation include:

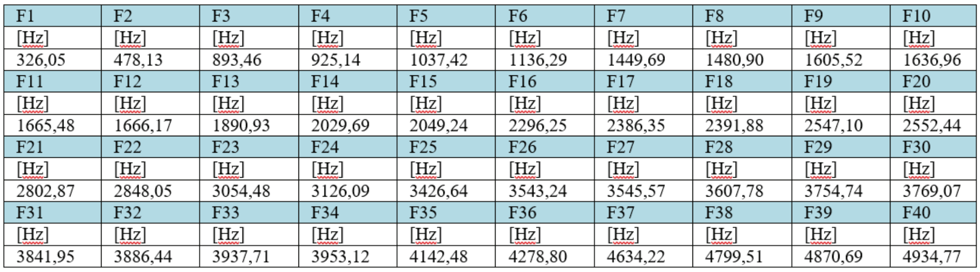
- define how to support the test piece;
- conversion of the bonds resulting from the assembly and method of assembling blocks of individual elements to a form suitable for and determine the number of mode shapes.

Depending on the type of connection being present between the construction elements have been replaced by so-called contact bound in the case of static and spring-type contacts for mobile connections. Contact is related to bonding material equivalent to the combined elements such as welds. When modelling the damping effect is bypassed in the case of flexing, which results in that the model is greatly simplified.

Inventor analysis is based on the finite element method (FEM). The result of this analysis is the natural frequency and vibration forms without attenuation coefficient. Table 4 shows the results of the analysis of the truss structure in the environment of Autodesk Inventor. The assumptions of the analysis determined the frequency range from 0.7 to 5,000 Hz and a maximum of 40 characters vibration.

Figures vibration frequency for these are shown in comparison to results obtained in real model. Based on experimental studies identified modal parameters. The results obtained were compared with the results of FEA grid and compared in Table 5.

TABLE 4. Forty mode shapes for truss (own studies)



F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]
326,05	478,13	893,46	925,14	1037,42	1136,29	1449,69	1480,90	1605,52	1636,96
F11	F12	F13	F14	F15	F16	F17	F18	F19	F20
[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]
1665,48	1666,17	1890,93	2029,69	2049,24	2296,25	2386,35	2391,88	2547,10	2552,44
F21	F22	F23	F24	F25	F26	F27	F28	F29	F30
[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]
2802,87	2848,05	3054,48	3126,09	3426,64	3543,24	3545,57	3607,78	3754,74	3769,07
F31	F32	F33	F34	F35	F36	F37	F38	F39	F40
[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]
3841,95	3886,44	3937,71	3953,12	4142,48	4278,80	4634,22	4799,51	4870,69	4934,77

TABLE 5. Comparison the results of analysis natural frequencies for the FEM model and the actual model (own studies)

Shape	Vibration frequency [Hz]		Discrepancy results [%]
	experimental studies	FEM analysis	
1	316.414	326.05	2.96
2	402.885	478.13	15.74
3	857.077	893.46	4.07
4	932.579	925.14	0.80
5	1 020.852	1 037.42	1.60
6	1 160.253	1 136.29	2.07
7	1 433.557	1 449.69	1.11
8	1 474.005	1 480.90	0.47
13	1 814.810	1 890.93	4.03
16	2 338.013	2 296.25	1.79
18	2 408.325	2 391.88	0.68
19	2 553.819	2 547.10	0.26
21	2 803.254	2 802.87	0.01
25	3 465.221	3 426.64	1.11
30	3 784.573	3 769.07	0.41
33	3 907.980	3 937.71	0.76

Comparing the results of FEA and operational studies, it was found that the results are satisfactory, and the resulting discrepancy can be traced resulting in a slightly different way of restraint lattice with finite element analysis in relation to the restraint of the grid used in the experimental study. Nevertheless, it is possible to observe significant similarities between the different forms of vibration.

Graphical representation of individual normal modes for the FEM model and experimental studies are compared in Figures from 13 to 16.

The need to improve the dynamic performance of mechanical structures, in particular, port cranes, force designers need to identify the dynamic characteristics of the design already on the road. The studies support the use of FEA, the results of which are specific feedback during the design phase. Results of FEA can be the basis for changing the geometry of the structure.

As commonly used in the practice of testing technique of dynamic properties, modal analysis allows the identification parameters of the mechanical properties, and hence possible to predict their behaviour as a result of imbalances.

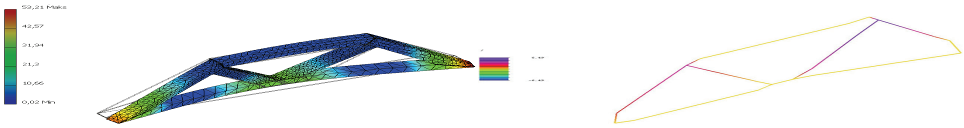


FIGURE 13. Comparison of the results (from left) FEA and experimental studies of the first form of vibrations (own studies)

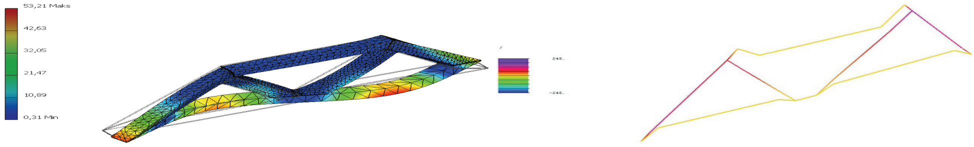


FIGURE 14. Comparison of the results (from left) FEA and experimental studies of the third form of vibrations (own studies)

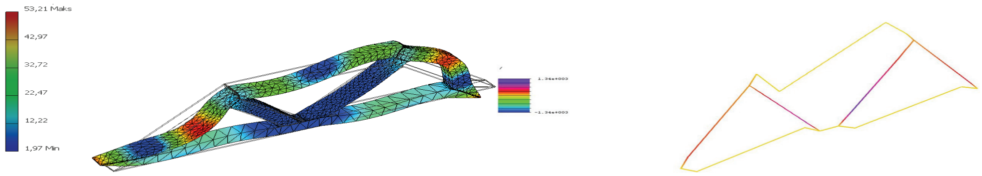


FIGURE 15. Comparison of the results (from left) FEA and experimental studies sixteenth mode shapes (own studies)

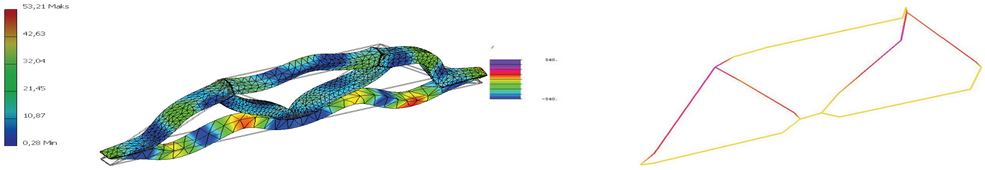


FIGURE 16. Comparison of the results (from left) FEA and experimental studies thirty-third of the normal modes (own studies)

Conclusions

The results point to the fact that it is possible to distinguish between material properties, which has an impact on the ability to distinguish between their mechanical properties. The study also confirmed the usefulness of the LMS test apparatus using operational modal analysis performed on the lattice steel structure.

By obtaining graphical charts, and a later their comparison it is possible to observe their diversity. These charts are different for materials that are in good condition, and damaged, which demonstrates the ability to assessment of the destruction of a lattice steel structure.

It practically verified the sensitivity of assessment of modal analysis to degree of brick structure degradation. It becomes possible to determine hazards to a building structure on the basis of examining values of frequencies.

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Summary

The use of modal analysis to examine the lattice structure. Steel structures are subject to large dynamic loads clearly reflected by generated vibration processes. The vibrations may affect state of serviceability of structures by lowering comfort of persons working there as well as possible reaching the level hazardous to safety of the structures. The effect of vibrations to structure is mainly manifested by additional stresses in a given cross-section, which are summed up with those resulting from static loads. The dynamic loads may cause damaging effects in buildings of various structural types or even lead to their destruction. Judging the necessity of improving the quality assessment methods of building structures for purposes of estimation of their state as well as safety factors for lattice structures, the author of this work undertook an attempt to investigate destruction process of selected object by using the modal analysis method.

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