

# Thin-layer drying of sawdust mixture

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Drying behaviour of sawdust mixture was investigated in a convective dryer at 0.01 m/s and 25, 60, and 150°C air temperature. Sawdust mixture (60% of spruce and 40% of the second ingredient: beech, willow, ash, alder) and sawdust of spruce, beech, willow, alder and ash was used in the drying experiments. The sawdust mixture drying was affected by the drying of its ingredients. The experimental drying data were fitted to the theoretical, semi-theoretical, and empirical thin-layer models. The accuracies of the models were measured using the correlation coefficient, root mean square error, and reduced chi-square. All semi-theoretical and empirical models described the drying characteristics of sawdust mixture satisfactorily. The theoretical model of a sphere predicts the drying of sawdust mixture better than the theoretical model of an infinite plane. The effect of the composition of the sawdust mixture on the drying models parameters were also taken into account.

**Keywords:** sawdust mixture; drying; modelling.

## INTRODUCTION

In 2008, the European Union Commission put forward a proposal for a new directive on renewable forms of energy. Each of the member states should increase its share of renewable energies in an effort to boost the total share of the EU from current 8.5 to 20% by 2020. To assist in reaching this goal, one solution is to further develop drying techniques of biomasses<sup>1</sup>.

Nowadays the main source of energy in the world is derived from fossil fuels, and their use increases CO<sub>2</sub> emissions, contributing to the greenhouse effect. Biomass is an alternative replacement for these sources<sup>2</sup>. The use of biomass as an energy source is not only very competitive in price and quality compared to fossil fuels, but using biofuels can drastically reduce CO<sub>2</sub> emissions<sup>3</sup>. However, production of biomass is not sufficient to meet the demand<sup>2</sup>. Among others, drying is the biggest challenge in obtaining energy from biomass.

Biomass, mainly in the form of wood, is the oldest form of energy used by humans. Woody biomass originates from different sources such as forest residues, wood processing residues and purpose grown plantations like short rotation coppice willow, poplar and eucalyptus<sup>4</sup>. All freshly cut timber has large quantities of water, which reduces its use in generating power. The calorific value is reduced by 2 MJ/kg for each 10% increase in wood moisture content<sup>2</sup>.

The moisture content of wood is dependent on the wood species, season of the year, location, and duration of storage. Typically, biomass feedstocks have a moisture content of 0.43 to 1.22 kg H<sub>2</sub>O/kg d.m. (30–55% w.b.) and they have to be dried to a typical moisture content of about 0.11 to 0.18 kg H<sub>2</sub>O/kg d.m. (10–15% w.b.), depending on the proposed use. For pellet production, the wood must be dried to the range 0.09 to 0.14 kg H<sub>2</sub>O/kg d.m. moisture content (8–12% w.b.)<sup>1</sup>. Therefore wood drying is one of the most important steps in wood products manufacturing. The drying process consumes roughly 40 to 70% of the total energy in the entire wood products manufacturing process<sup>5</sup>. Thus, drying units need efficient processes. An important aspect of drying technology is the mathematical modelling of the drying process. Mathematical modelling provides a tool

to enable drying rate and efficiency to be predicted under a range conditions. Accurate prediction can determine among others the reduction in process time and in energy consumption. However, there are not so many works concerning mathematical modelling of wood drying process<sup>4, 6–8</sup>. There is also no information on the modelling of drying of sawdust formed by a mixture of different kinds of wood. Processed biofuels (pellets and briquettes) are made among others from dry sawdust formed by one kind of wood or mixture of different kinds of wood.

The objectives of this study were to investigate the drying kinetics of sawdust mixture in a convective dryer and the modelling of thin-layer drying of sawdust mixture.

## MATERIAL AND METHODS

In this research sawdust of spruce (*Picea abies*) (100%), beech (*Fagus silvatica*) (100%), willow (*Salix alba*) (100%), alder (*Alnus glutinosa*) (100%), and ash (*Fraxinus excelsior*) (100%) was used in the drying experiments. The following sawdust mixtures were also dried: spruce and beech (60 and 40%, respectively), spruce and willow (60 and 40%, respectively), spruce and ash (60 and 40%, respectively), and spruce and alder (60 and 40%, respectively). The woody biomass used in the experiments was obtained from sawmill. The size distribution of sawdust was determined through screen analysis. The size of 86% of all particles for spruce, 82% for beech, 84% for willow, and 77% for alder was less than 10 mm and bigger than 0.5 mm. The size of 100% of all particles for ash was 0.5 mm or less. The initial moisture content of samples was ranged from: 0.45 to 0.49 kg H<sub>2</sub>O/kg d.m. (31–32.9% w.b.) for spruce, 0.85 to 0.90 kg H<sub>2</sub>O/kg d.m. (45.9–47.4% w.b.) for beech, 0.77 to 0.82 kg H<sub>2</sub>O/kg d.m. (43.5–45.1% w.b.) for willow, 1.07 to 1.16 kg H<sub>2</sub>O/kg d.m. (51.7–53.7% w.b.) for alder, and 1.06 to 1.12 kg H<sub>2</sub>O/kg d.m. (51.46–52.83 w.b.) for ash. The initial moisture content of mixture samples were ranged from: 0.57 to 0.62 kg H<sub>2</sub>O/kg d.m. (36.31–38.27% w.b.) for spruce and beech, 0.54 to 0.60 kg H<sub>2</sub>O/kg d.m. (35.07–37.50% w.b.) for spruce and willow, 0.46 to 0.51 kg H<sub>2</sub>O/kg d.m. (31.51–33.78% w.b.) for spruce and ash,

and 0.69 to 0.73 kg H<sub>2</sub>O/kg d.m. (41.18–41.86% w.b.) for spruce and alder.

### Drying equipment and experimental procedure

The drying experiments were carried out using Memmert UFP400 (MEMMERT GmbH+Co. KG, Schwabach, Germany) laboratory dryer. The drying experiments were conducted at the drying air temperature of 25, 60, and 150°C and airflow velocity of 0.01 m/s. Measurements of the moisture content changes carried out in the laboratory dryer were conducted in the following way. The sample was put on the tulle stretched on the metal frame (scale) and hung up to the electronic scales WPX 650 (RADWAG, Radom, Poland). The accuracy of the weighing was ±1 mg. Computer connected to the scales was an additional equipment of experimental stand. It recorded mass of dried sample at regular intervals of 60 s. Measurements of mass changes were recorded up to the moment, when mass changes of sample were not observed. Experiments were replicated three times.

Air temperature inside the dryer was measured with 0.1°C accuracy using thermocouple TP-01b-W3 (NiCr-NiAl, CZAKI THERMO-PRODUCT, Raszyn, Poland), placed at a central part of dryer chamber. Temperature reading was done with EMT-08 meter (CZAKI THERMO-PRODUCT, Raszyn, Poland). The velocity of drying air was measured with 3% accuracy using Kestrel®4000 Packet Weather™ Tracker™ (Nielsen-Kellerman Company, Boothwyn, PA, USA).

The initial and final moisture contents of sawdust were determined using the oven method<sup>9</sup>.

### Methods

Sawdust dried in this research consisted of the particles of different shapes. Some of them can be described as a sphere, the other can be considered as an infinite plane. It can be accepted moreover that the water movement inside the dried solid is only a diffusion movement in the convection drying process of biological products. Therefore the equations applied to the description of the sawdust particles drying takes the following form<sup>10, 11</sup>:

– for a sphere of radius  $s$  (m):

$$MR = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp(-n^2 \pi^2 \frac{Dt}{s^2}) \quad (1)$$

– for an infinite plane of thickness  $2s$  (m):

$$MR = \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp[-\frac{\pi^2(2n+1)^2}{4} \frac{Dt}{s^2}] \quad (2)$$

where:

MR is the moisture ratio,  $n$  is the number of terms in the series,  $D$  is the moisture diffusion coefficient (m<sup>2</sup>/s), and  $t$  is the time (s).

The equations (1) and (2) for sphere and infinite plane were obtained using analytical methods for solving the Fick's second law<sup>10, 11</sup>. If the geometry of particles shape is more complicated the solution of Fick's equation can be obtained using numerical methods<sup>12</sup>.

Theoretical thin-layer models presented above take into account fundamentals of the drying process and their parameters have physical meaning. Therefore, they

can give an explanation of the phenomena occurring during drying. On the other hand however theoretical models are time consuming and very often it is difficult to predict the variations of structure and composition occurring during drying of biological products.

In practical drying there is a need for simple thin-layer models, such as semi-theoretical and empirical. These models generally describe fairly well the evolution of the moisture content of the product during drying. The semi-theoretical models are a simplified solution of Fick's second law. The empirical models give a direct relationship between average moisture content and drying time. Such models are easy to use but they neglect the fundamentals of the drying process and their parameters have no physical meaning. Table 1 indicates the semi-theoretical and empirical models used to describe the convection drying kinetics of sawdust and its mixtures.

To plot drying curves a dimensionless variable is used, the moisture ratio, calculated by Eq. (3)

$$MR = \frac{M_t - M_e}{M_0 - M_e} \quad (3)$$

where:

$M_t$  is the moisture content at  $t$  (kg H<sub>2</sub>O/kg d.m.),  $M_e$  is the equilibrium moisture content (kg H<sub>2</sub>O/kg d.m.),  $M_0$  is the initial moisture content (kg H<sub>2</sub>O/kg d.m.).

Drying rate was calculated by Eq. (4)

$$\text{Drying rate} = \frac{M_{t+dt} - M_t}{dt} \quad (4)$$

where:

$M_{t+dt}$  is the moisture content at  $t+dt$ .

Drying curves were fitted to nineteen different models (Table 1 and Eqs. (1) and (2)). A non-linear regression analysis was conducted to fit the models by the Lavenberg-Marquardt method using the computer program STATISTICA 10<sup>29</sup>.

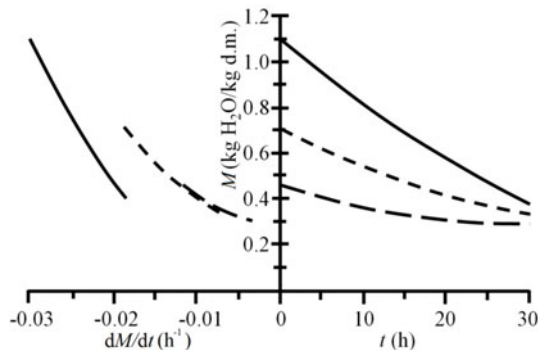
To compare the quality of fit, the coefficient of correlation ( $R$ ), the root mean square error (RMSE), and reduced chi-square ( $\chi^2$ ) were used. High values of  $R$  and low values of RMSE and  $\chi^2$  are needed to show a suitable mathematical model that explains the drying of sawdust mixture<sup>21</sup>.

## RESULTS AND DISCUSSION

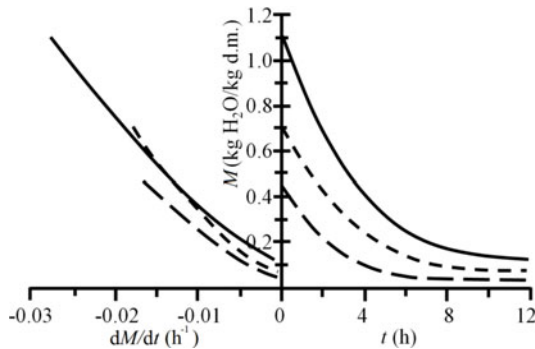
### Drying characteristics

The exemplary results of drying of sawdust mixtures are presented in Figures 1–3. Each of the drying curves  $M(t)$  represents empirical formula which approximates results of the three measurement repetitions of the moisture content changes in time. Each of the drying rate curves  $dM/dt$  was obtained by differentiation of the drying curve.

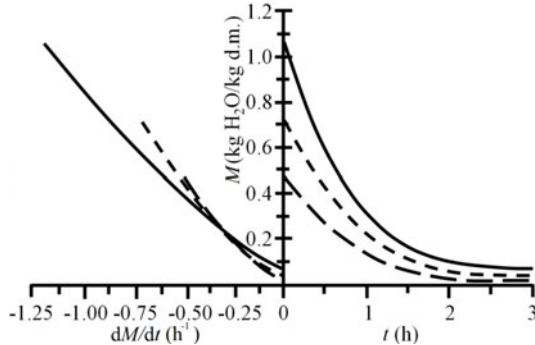
As can be seen the sawdust mixture drying was affected by the drying of its ingredients because drying curve of the mixture was between the drying curves of its ingredients. Such results were obtained at each drying air temperature (25, 60, and 150°C) and for each sawdust mixture investigated (spruce and beech, spruce and willow, spruce and ash, and spruce and alder). It can be stated therefore that the composition of the sawdust mixture influenced the course of its drying.



**Figure 1.** Moisture content vs. time and drying rate vs. moisture content for drying of sawdust at 25°C: (—) alder, (---) mixture of spruce and alder (60% and 40%, respectively), (- -) spruce



**Figure 2.** Moisture content vs. time and drying rate vs. moisture content for drying of sawdust at 60°C: (—) alder, (---) mixture of spruce and alder (60% and 40%, respectively), (- -) spruce



**Figure 3.** Moisture content vs. time and drying rate vs. moisture content for drying of sawdust at 150°C: (—) alder, (---) mixture of spruce and alder (60% and 40%, respectively), (- -) spruce

## Evaluation of the models

The evaluation of the considered semi-theoretical and empirical models was conducted in the following way. The moisture content data obtained for the different drying air temperatures were converted to the dimensionless moisture ratio and then curve fitting computations with the drying time were carried on the seventeen considered drying models. The exemplary results of statistical analyses undertaken on the models are given in Tables 2 and 3. As can be seen from the statistical analysis results, generally high correlation coefficient  $R$  and low values of root mean square error RMSE, and reduced chi-square  $\chi^2$  were observed for all drying models. The  $R$  values varied between 0.9841 and 1, the root mean square error ranged from 0.0007 to 0.0606, and reduced chi-square changed from 0 to 0.0029 for mixture of spruce and beech. The results of statistical analyses for spruce and willow mixture were following:  $R$  ranged from 0.9930 to 1, RMSE varied between 0.0013 and 0.0437, and  $\chi^2$  changed within the range of 0–0.0017. The correlation coefficient ranged from 0.9920 to 1, the root mean square error changed within the range of 0.0025–0.0920, and reduced chi-square changed within the range of 0–0.0077 for mixture of spruce and ash. It turned out from the statistical analyses that for spruce and alder mixture  $R$  varied within the range of 0.9950–0.9999, RMSE changed between 0.040 to 0.0768, and  $\chi^2$  varied between 0 to 0.0053. It can be stated however that the Noomhorm and Verma model ( $R = 0.9920$ –1,  $RMSE = 0.0011$ –0.0883,  $\chi^2 = 0$ –0.0030) gave the best results whereas the Wang and Singh model gave the worst results. The results of statistical analyses undertaken on the Noomhorm and Verma model for examined sawdust mixtures dried at 25, 60, and 150°C are given in Table 4.

The correlation coefficient ranged from 0.9762 to 1, the root mean square error changed within the range of 0–0.1668, and reduced chi-square changed within the range of 0–0.0283 for sawdust consisted of spruce (100%), beech (100%), willow (100%), alder (100%), and ash (100%). The Noomhorm and Verma model can be considered as the best one ( $R = 0.9994$ –1,  $RMSE = 0$ –0.0097,  $\chi^2 = 0$ ), whereas Lewis model as the worst one.

Further regressions were undertaken to account for the effect of the composition of the sawdust mixture

**Table 1.** Semi-theoretical and empirical thin-layer models applied to drying curves

| Model no. | Model equation  | Model name                   | References                                |
|-----------|---|------------------------------|---|
| 1         | $MR = \exp(-kt)$  | Lewis (Newton)               | (Lewis 1921) <sup>13</sup>                |
| 2         | $MR = a \exp(-kt)$                                      | Henderson and Pabis          | (Henderson and Pabis 1961) <sup>14</sup>  |
| 3         | $MR = a \exp(-k_1 t) + b$                               | Logarithmic                  | (Yagcioglu et al. 1999) <sup>15</sup>     |
| 4         | $MR = a \exp(-k_1 t) + b \exp(-k_2 t)$                  | Two-term                     | (Henderson 1974) <sup>16</sup>            |
| 5         | $MR = a \exp(-k_1 t) + b \exp(-k_2 t) + c$              | Noomhorm and Verma           | (Noomhorm and Verma 1986) <sup>17</sup>   |
| 6         | $MR = a \exp(-k_1 t) + b \exp(-k_2 t) + c \exp(-k_3 t)$ | Modified Henderson and Pabis | (Karathanos 1999) <sup>18</sup>           |
| 7         | $MR = a \exp(-kt) + (1-a) \exp(-akt)$                   | Two-term exponential         | (Sharaf-Eldeen et al. 1980) <sup>19</sup> |
| 8         | $MR = a \exp(-kt) + (1-a) \exp(-gt)$                    | Verma et al.                 | (Verma et al. 1985) <sup>20</sup>         |
| 9         | $MR = \exp(-kt^n)$                                      | Page                         | (Page 1949) <sup>21</sup>                 |
| 10        | $MR = a \exp(-kt^n)$                                    | Kaleta et al. I              | (Kaleta et al. 2013) <sup>22</sup>        |
| 11        | $MR = a \exp(-kt^n) + b \exp(-gt^n)$                    | Hii et al.                   | (Hii et al. 2008) <sup>23</sup>           |
| 12        | $MR = a \exp(-kt^n) + (1-a) \exp(-gt^n)$                | Kaleta et al. II             | (Kaleta et al. 2013) <sup>22</sup>        |
| 13        | $MR = \exp[-(kt)^n]$                                    | Modified Page                | (Overhults 1973) <sup>24</sup>            |
| 14        | $MR = a \exp[-(kt)^n]$                                  | Ademiluyi et al.             | (Ademiluyi et al. 2008) <sup>25</sup>     |
| 15        | $MR = a \exp(-kt)^n + b$                                | Demir et al.                 | (Demir et al. 2007) <sup>26</sup>         |
| 16        | $MR = 1 + at + bt^2$                                    | Wang and Singh               | (Wang and Singh 1978) <sup>27</sup>       |
| 17        | $MR = a \exp(-kt^n) + bt$                               | Midilli et al.               | (Midilli et al. 2002) <sup>28</sup>       |

Where  $a$ ,  $b$ ,  $c$ ,  $g$ ,  $n$  are parameters of the models.



**Table 2.** Results of statistical analyses on the modelling of sawdust mixture of spruce and willow drying at 60°C

| Model no. | Model parameters   | R      | RMSE   | $\chi^2$ |
|-----------|--|--------|--------|----------|
| 1         | $k = 0.005652$   | 0.9993 | 0.0196 | 0.0004   |
| 2         | $a = 1.039727; k = 0.005869$   | 0.9987 | 0.0167 | 0.0003   |
| 3         | $a = 1.056878; k = 0.005127; b = -0.044871$  | 0.9999 | 0.0038 | 0.0000   |
| 4         | $a = 9.10853; k_1 = 0.00835;$<br>$b = -8.12431; k_2 = 0.00885$   | 0.9997 | 0.0078 | 0.0001   |
| 5         | $a = 1.892786; k_1 = 0.006392; b = -0.866702; k_2 = 0.008322; c = -0.028137$                           | 1.0000 | 0.0013 | 0.0000   |
| 6         | $a = -0.646432; k_1 = 0.010203;$<br>$b = -0.583080; k_2 = 0.010288;$<br>$c = 2.213714; k_3 = 0.007555$ | 0.9996 | 0.0082 | 0.0001   |
| 7         | $a = 1.603216; k = 0.007058$   | 0.9995 | 0.0091 | 0.0001   |
| 8         | $a = 5.822399; k = 0.008078; g = 0.008808$   | 0.9996 | 0.0083 | 0.0001   |
| 9         | $k = 0.003029; n = 1.115566$   | 0.9995 | 0.0097 | 0.0001   |
| 10        | $a = 0.970080; k = 0.002258; n = 1.164109$   | 0.9996 | 0.0082 | 0.0001   |
| 11        | $a = 0.782506; k = 0.000455; n = 1.418957;$<br>$b = 0.195031; g = 0.003496$                            | 0.9999 | 0.0046 | 0.0000   |
| 12        | $a = 0.919482; k = 0.003029;$<br>$n = 1.115570; g = 0.003030$  | 0.9995 | 0.0097 | 0.0001   |
| 13        | $a = 0.005524; k = 1.115566$   | 0.9995 | 0.0097 | 0.0001   |
| 14        | $a = 0.970081; k = 0.005332; n = 1.164104$   | 0.9996 | 0.0082 | 0.0001   |
| 15        | $a = 1.023401; k = 0.005140;$<br>$n = 1.060700; b = -0.031191$   | 1.0000 | 0.0016 | 0.0000   |
| 16        | $a = -0.003862; b = 0.000004$  | 0.9946 | 0.0376 | 0.0014   |
| 17        | $a = 0.989596; k = 0.003530;$<br>$n = 1.077311; c = -0.000041$   | 1.0000 | 0.0021 | 0.0000   |

**Table 3.** Results of statistical analyses on the modelling of ash sawdust at 60°C

| Model no. | Model parameters   | R      | RMSE   | $\chi^2$ |
|-----------|--|--------|--------|----------|
| 1         | $k = 0.008001$   | 0.9979 | 0.0329 | 0.0011   |
| 2         | $a = 1.064834; k = 0.008495$   | 0.9964 | 0.0276 | 0.0008   |
| 3         | $a = 1.108238; k = 0.006753; b = -0.086572$  | 0.9995 | 0.0089 | 0.0001   |
| 4         | $a = -2.46669; k_1 = 0.00396;$<br>$b = 3.48180; k_2 = 0.00487$                             | 0.9998 | 0.0064 | 0.0000   |
| 5         | $a = 4.42603; k_1 = 0.00324; b = -4.32781;$<br>$k_2 = 0.00179; c = 0.90247$                | 1.0000 | 0.0002 | 0.0000   |
| 6         | $a = 3.71762; k_1 = 0.00029; b = -5.27621;$<br>$k_2 = 0.00076; c = 2.55949; k_3 = 0.00373$ | 1.0000 | 0.0002 | 0.0000   |
| 7         | $a = 1.727166; k = 0.010683$   | 0.9989 | 0.0151 | 0.0002   |
| 8         | $a = -6.23436; k = 0.00406; g = 0.00445$   | 0.9997 | 0.0074 | 0.0001   |
| 9         | $k = 0.002895; n = 1.201833$   | 0.9989 | 0.0144 | 0.0002   |
| 10        | $a = 0.956091; k = 0.001812; n = 1.285297$   | 0.9993 | 0.0117 | 0.0001   |
| 11        | $a = 1.512136; k = 0.003452; n = 1.103919;$<br>$b = -0.526044; g = 0.002003$               | 0.9999 | 0.0034 | 0.0000   |
| 12        | $a = 0.175580; k = 0.006549;$<br>$n = 1.523073; g = 0.000462$                              | 0.9998 | 0.0067 | 0.0000   |
| 13        | $a = 0.007726; k = 1.201834$   | 0.9989 | 0.0144 | 0.0002   |
| 14        | $a = 0.956092; k = 0.007357; n = 1.285297$   | 0.9993 | 0.0117 | 0.0001   |
| 15        | $a = 1.027805; k = 0.006955;$<br>$n = 1.139144; b = -0.045888$                             | 0.9999 | 0.0044 | 0.0000   |
| 16        | $a = -0.005656; b = 0.000008$  | 0.9984 | 0.0203 | 0.0004   |
| 17        | $a = 0.979102; k = 0.003257$<br>$n = 1.161440; c = -0.000091$                              | 0.9998 | 0.0050 | 0.0000   |

on the drying models parameters. The seventeen considered semi-theoretical and empirical models (Table 1) were used to describe the drying process of considered sawdust mixtures. The parameters of the models 1–17 involving the composition of the mixture were determined by investigating the following type of equation

$$Y_{\text{spruce and beech}} = 0.6Y_{\text{spruce}} + 0.4Y_{\text{beech}} \quad (5)$$

$$Y_{\text{spruce and willow}} = 0.6Y_{\text{spruce}} + 0.4Y_{\text{willow}} \quad (6)$$

$$Y_{\text{spruce and ash}} = 0.6Y_{\text{spruce}} + 0.4Y_{\text{ash}} \quad (7)$$

$$Y_{\text{spruce and alder}} = 0.6Y_{\text{spruce}} + 0.4Y_{\text{alder}} \quad (8)$$

The models parameters determined using Eqs. (5)–(8) were then used to estimate the moisture content of sawdust mixtures at any time during process. Validation of the established models was made by comparing the computed moisture content with the measured ones in any particular drying run under certain conditions. It

turned out from the statistical analyses that for all types of sawdust mixture tested only the Hii et al. and Kaleta et al. II models gave the non-acceptable results. It can be assumed however that for examined mixtures the Logarithmic, Noomhorm and Verma, Demir et al., and Midilli et al. models gave the best results ( $R > 0.9570$ ,  $RMSE < 0.5765$ ,  $\chi^2 < 0.2124$ ). The results of statistical analyses undertaken on the Noomhorm and Verma model with parameters involving the composition of the mixture for four examined sawdust mixtures dried at 25, 60, and 150°C are given in Table 5. It should be however stressed that the models with parameters determined using Eqs. (5)–(8) predicted the drying of sawdust mixture worse than the models with parameters determined using curve fitting (Tables 4 and 5). The results of statistical analyses undertaken on the Noomhorm and Verma model given in Tables 4 and 5 allow for the statement that the drying air temperature does not influence the modelling results

**Table 4.** Results of statistical analyses on the modelling using the Noomhorm and Verma model (the best model) of sawdust mixtures drying

| Kind of mixture   | Temperature [°C] | Noomhorm and Verma model parameters |                       |          |                       |          | R      | RMSE   | $\chi^2$ |
|-------------------|------------------|-------------------------------------|-----------------------|----------|-----------------------|----------|--------|--------|----------|
|                   |                  | <i>a</i>                            | <i>k</i> <sub>1</sub> | <i>b</i> | <i>k</i> <sub>2</sub> | <i>c</i> |        |        |          |
| Spruce and beech  | 25               | 4.8893                              | 0.0004                | -2.7693  | 0.0004                | -1.1244  | 1.0000 | 0.0019 | 0.0000   |
|                   | 60               | 1.0513                              | 0.0049                | -0.0003  | 0.1790                | -0.0510  | 1.0000 | 0.0016 | 0.0000   |
|                   | 150              | 0.1554                              | 0.0040                | 0.9566   | 0.0277                | -0.1082  | 1.0000 | 0.0014 | 0.0000   |
| Spruce and willow | 25               | 5.7337                              | 0.0016                | -4.6016  | 0.0018                | -0.1355  | 1.0000 | 0.0013 | 0.0000   |
|                   | 60               | 1.8928                              | 0.0064                | -0.8667  | 0.0083                | -0.0281  | 1.0000 | 0.0011 | 0.0000   |
|                   | 150              | 3.2786                              | 0.0310                | -2.2465  | 0.0360                | -0.0369  | 1.0000 | 0.0025 | 0.0000   |
| Spruce and ash    | 25               | 4.9801                              | 0.0008                | -3.3825  | 0.0009                | -0.5985  | 0.9920 | 0.0883 | 0.0030   |
|                   | 60               | 9.1189                              | 0.0097                | -8.1046  | 0.0102                | -0.0212  | 1.0000 | 0.0025 | 0.0000   |
|                   | 150              | 4.6907                              | 0.0290                | -3.6798  | 0.0331                | -0.0206  | 0.9999 | 0.0034 | 0.0000   |
| Spruce and alder  | 25               | 5.4540                              | 0.0010                | -4.0614  | 0.0011                | -0.3944  | 0.9950 | 0.0690 | 0.0019   |
|                   | 60               | 8.3590                              | 0.0086                | -7.3556  | 0.0093                | -0.0203  | 0.9997 | 0.0064 | 0.0000   |
|                   | 150              | 5.1087                              | 0.0309                | -4.0866  | 0.0350                | -0.0333  | 0.9999 | 0.0043 | 0.0000   |

**Table 5.** Results of statistical analyses on the modelling using the Noomhorm and Verma model of sawdust mixtures drying and Eqs. (5)–(8)

| Kind of mixture   | Temperature [°C] | Parameters of the Noomhorm and Verma model |                       |          |                       |          | R      | RMSE   | $\chi^2$ |
|-------------------|------------------|--|-----------------------|----------|-----------------------|----------|--------|--------|----------|
|                   |                  | <i>a</i>                                   | <i>k</i> <sub>1</sub> | <i>b</i> | <i>k</i> <sub>2</sub> | <i>c</i> |        |        |          |
| spruce and beech  | 25               | 1.1216                                     | 0.0006                | 1.2527   | 0.0006                | -1.3720  | 0.9988 | 0.3640 | 0.0150   |
|                   | 60               | 1.4237                                     | 0.0051                | -0.4850  | 0.0046                | 0.0758   | 0.9995 | 0.0759 | 0.0058   |
|                   | 150              | 0.7219                                     | 0.0232                | 0.3808   | 0.0232                | -0.0799  | 0.9993 | 0.0256 | 0.0007   |
| spruce and willow | 25               | 0.5200                                     | 0.0006                | 1.6039   | 0.0009                | -1.1226  | 0.9992 | 0.3827 | 0.0147   |
|                   | 60               | 0.5004                                     | 0.0060                | 0.5963   | 0.0060                | -0.0741  | 0.9980 | 0.0635 | 0.0041   |
|                   | 150              | 1.4796                                     | 0.0185                | -0.5836  | 0.0153                | 0.1200   | 0.9994 | 0.1508 | 0.0236   |
| spruce and ash    | 25               | 1.5458                                     | 0.0006                | 1.6911   | 0.0006                | -2.2327  | 0.9961 | 0.5515 | 0.0326   |
|                   | 60               | 0.6282                                     | 0.0060                | 0.4653   | 0.0060                | -0.0705  | 0.9992 | 0.0134 | 0.0002   |
|                   | 150              | 0.4487                                     | 0.0211                | 0.6487   | 0.0211                | -0.0709  | 0.9979 | 0.0485 | 0.0024   |
| spruce and alder  | 25               | 3.0336                                     | 0.0007                | -0.1400  | 0.0008                | -1.8929  | 0.9979 | 0.5571 | 0.0390   |
|                   | 60               | 3.6165                                     | 0.0036                | -1.7256  | 0.0039                | -0.8941  | 0.9873 | 0.4625 | 0.2034   |
|                   | 150              | 2.6264                                     | 0.0138                | -0.7425  | 0.0155                | -0.8864  | 0.9947 | 0.3891 | 0.1566   |

although at the temperature of 25°C the obtained results are slightly worse.

The evaluation of the considered theoretical models (Eqs. (1) and (2)) was conducted after the same manner as for semi-theoretical and empirical models 1–17 given in Table 1. The effect of the composition of the sawdust mixture on the moisture diffusion coefficient was determined by using the following type of equation

$$D_{\text{spruce and beech}} = 0.6D_{\text{spruce}} + 0.4D_{\text{beech}} \quad (9)$$

$$D_{\text{spruce and willow}} = 0.6D_{\text{spruce}} + 0.4D_{\text{willow}} \quad (10)$$

$$D_{\text{spruce and ash}} = 0.6D_{\text{spruce}} + 0.4D_{\text{ash}} \quad (11)$$

$$D_{\text{spruce and alder}} = 0.6D_{\text{spruce}} + 0.4D_{\text{alder}} \quad (12)$$

The results of statistical analyses are shown in Tables 6 and 7.

It can be noticed that the theoretical model of sphere drying (Eq. (1)) gave better results for all examined kind of sawdust mixture comparing to the theoretical model of an infinite plane drying (Eq. (2)). It can be accepted moreover that both methods of modelling namely applying curve fitting computations with the drying time

(Table 6) and considering the effect of composition of the sawdust mixture on the moisture diffusion coefficient (Eqs. (9)–(12), Table 7) gave the comparable results.

## CONCLUSIONS

The results of the experiments have shown that the sawdust mixture drying was affected by the drying of its ingredients. The following sawdust mixtures were used in the drying experiments: spruce and beech (60% and 40%, respectively), spruce and willow (60% and 40%, respectively), spruce and ash (60% and 40%, respectively), and spruce and alder (60% and 40%, respectively). Sawdust of spruce (100%), beech (100%), willow (100%), alder (100%), and ash (100%) was also dried.

Theoretical, semi-theoretical, and empirical thin-layer models were investigated for their suitability to describe the drying behaviour of examined sawdust mixtures. All considered semi-theoretical and empirical models may be assumed to represent the drying behaviour of sawdust mixture. The theoretical model of a sphere predicts the drying characteristics of sawdust mixture better than the theoretical model of an infinite plane. Semi-theoretical

**Table 6.** Results of statistical analyses on the modelling using theoretical models (Eqs. (1) and (2)) of sawdust mixtures drying

| Kind of mixture   | Model | R             | RMSE          | $\chi^2$      |
|-------------------|-------|---------------|---------------|---------------|
| Spruce and beech  | lp    | 0.9339–0.9751 | 0.0623–0.1042 | 0.0039–0.0066 |
|                   | S     | 0.9802–0.9977 | 0.0628–0.1145 | 0.0040–0.0069 |
| Spruce and willow | lp    | 0.9531–0.9752 | 0.6290–0.0863 | 0.0040–0.0075 |
|                   | S     | 0.9907–0.9968 | 0.0734–0.0996 | 0.0054–0.0064 |
| Spruce and ash    | lp    | 0.9356–0.9734 | 0.0663–0.1059 | 0.0044–0.0069 |
|                   | S     | 0.9808–0.9959 | 0.0767–0.1173 | 0.0059–0.0074 |
| Spruce and alder  | lp    | 0.9415–0.9724 | 0.0710–0.1021 | 0.0050–0.0080 |
|                   | S     | 0.9833–0.9938 | 0.0839–0.1140 | 0.0070–0.0079 |

Where: lp – infinite plane, S – sphere.

**Table 7.** Results of statistical analyses on the modelling using theoretical models (Eqs. (1) and (2), and Eqs. (9)–(12)) of sawdust mixtures drying

| Kind of mixture   | Model | R             | RMSE          | $\chi^2$      |
|-------------------|-------|---------------|---------------|---------------|
| Spruce and beech  | lp    | 0.9166–0.9690 | 0.0754–0.1987 | 0.0057–0.0256 |
|                   | S     | 0.9551–0.9978 | 0.0630–0.1883 | 0.0040–0.0256 |
| Spruce and willow | lp    | 0.9324–0.9688 | 0.0798–0.1759 | 0.0064–0.0238 |
|                   | S     | 0.9677–0.9940 | 0.0811–0.1640 | 0.0066–0.0227 |
| Spruce and ash    | lp    | 0.9111–0.9709 | 0.0703–0.2148 | 0.0049–0.0306 |
|                   | S     | 0.9467–0.9949 | 0.0789–0.2111 | 0.0062–0.0324 |
| Spruce and alder  | lp    | 0.9091–0.9673 | 0.0814–0.2266 | 0.0066–0.0337 |
|                   | S     | 0.9490–0.9921 | 0.0898–0.2070 | 0.0081–0.0321 |

Where: lp– infinite plane, S – sphere.

and empirical models are more useful in practical drying. To account for the effect of the composition of the sawdust mixture on the drying models parameters all considered models were taken into account. It turned out that for such method of modelling the best results gave the Logarithmic, Noomhorm and Verma, Demir et al., Midilli et al. models, and theoretical model of sphere.

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