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Semi-markovian approach to modelling air pollution

Keywords

pollution, sulphur dioxide, carbon monoxide, nitrogen dioxide, particular matter, semi-Markov process

Abstract

The air pollution assessment based on concentration's changes of sulphur dioxide, carbon monoxide, nitrogen dioxide, ozone, benzene, and particulate matter is discussed in the chapter. The semi-Markov model of the environmental pollution process is introduced and its characteristics are determined. Next the proposed model is practically applied to examine and characterized air pollution in Gdańsk (Poland) as the exemplary industrial agglomeration. The main parameters and characteristics of the air pollution process are determined, such as concentration states of particular kinds of air pollutants, the limit values of transient probabilities and the mean total sojourn times staying at the air pollutants' concentration states, for the fixed time interval.

1. Introduction

The phenomenon defined as a presence of harmful, toxic substances or their mixtures in the air at the concentration level exceed a certain one causing detrimental changes to the quality of life and possessing a health risk is called the air pollution. Commonly the air pollution is evaluated based on either one kind of pollutant concentration in air such as: sulphur dioxide – SO_2, carbon monoxide – CO, nitrogen dioxide – NO_2, ozone – O_3, benzene – C_6H_6, and particulate matter with a diameter less than 2.5 µm or between 2.5 and 10 µm – $PM_{2.5}$ and PM_{10} respectively or all together.

There are some methods of air quality assessment and forecast. These approaches are usually based on the historical statistical data as the background of the future pollutant concentration prediction. Generally, the statistical forecasting methods, recently reviewed in (Bai et al., 2018), include linear or nonlinear regression (Dalal, 2015; Huebnerova & Michalek, 2014; (Kaboodvandpour et al., 2015; Shafabakhsh et al., 2018), dispersion (PriyaDarshini et al., 2016; Shadab et al., 2019; Sivacoumar et al., 2001), neural network (Bai et al., 2016; Feng et al., 2015; Fu et al., 2015; Park et al., 2020; Rahman et al., 2015; Sarwat & El-Shanshoury, 2018; Wongsathan & Seedadan, 2016; Yan et al., 2018), fuzzy logic (Bouharati et al., 2014; Dunea et al., 2011; Olvera-Garcia et al., 2016; Xu & Xu, 2018; Yadav et al., 2015; Yang et al., 2020) or hybrid systems (Chen et al., 2015; Qin et al., 2014; Wang et al., 2015; Wu & Lin, 2019; Yang et al., 2017; Zhou et al., 2014; Zhu et al., 2018).

In this chapter, the approach based on the semi-Markov process for the environmental pollution assessment is proposed. The semi-Markov process theory was developed by Lévy (Lévy, 1954) and Smith (Smith, 1955). The semi-Markov process is a stochastic one evolves its states over time and provides modelling real systems, commonly applied in the safety and reliability fields (Bogalecka, 2020; Grabski, 2015; Iosifescu, 1980; Kołowrocki, 2004, 2014; Kołowrocki & Soszyńska-Budny, 2011; Korolyuk & Turbin, 1976; Limnios & Oprisan, 2001). The semi-markovian approach has never used before to model the air pollution. The model will be further applied to optimization that allows the mitigation of the air pollution and motivate to using this class of model.

Safety and Reliability of Systems and Processes, Summer Safety and Reliability Seminar 2021.

DOI: 10.26408/srsp-2021-03.

The chapter is organized into 5 parts, this Introduction as Section 1, Sections 2–4 and Conclusion as Section 5. Section 2 is devoted to air pollution problems and methods of air quality assessment based on the pollutants' concentration. In Section 3, the semi-Markov model of air pollution process is introduced and presented. In Section 4, the proposed air pollution model is applied to the exemplary industrial agglomeration. The model is examined and its parameters and characteristics are determined such as concentration states of particular kinds of air pollutants, the limit values of transient probabilities and mean total sojourn times staying at the air pollutants' concentration states, for the fixed time interval. Finally, the evaluation of results is discussed. The possibility of the presented model's wider applications in the field considered in this chapter is suggested in Conclusion.

2. Pollutants in air quality assessment

The adverse impacts of air pollutants on biosphere and human and animal health as well have been carried out for years. The quality of air is based on the concentration in air some representative pollutants such as: SO_2, CO, NO_2, O_3, C_6H_6, $PM_{2.5}$ and PM_{10}.

SO_2 is a colourless and non-inflammable gas with the specific pungent odour. SO_2 combines with water vapour in the atmosphere to produce acid rain. Its wet and dry deposition has a negative impact on the ecosystems' condition and causes destruction (corrosion) of materials. SO_2 can affect human health, particularly in those suffering from asthma and chronic lung diseases.

CO is a colourless, odourless and toxic gas that is emitted into the atmosphere mainly as a result of combustion processes of coal, fuels and other organic compounds when there is not enough oxygen to produce carbon dioxide – CO_2. CO combines with haemoglobin and block carries oxygen, leaving it ineffective for delivering oxygen to bodily tissues.

NO_2 is a reddish brown gas, toxic by inhalation and skin absorption. NO_2, similarly as SO_2, combines with water vapour in the atmosphere to produce acid rain. NO_2 is a product of nitric oxide – NO and oxygen (from air) synthesis. NO is formed during high temperature combustion processes when nitrogen and oxygen present in the atmosphere combines each other. Therefore the road traffic is recognized as a principal source of nitrogen oxides: NO and NO_2, collectively known as NO_x which concentrations are greatest in urban areas where traffic is heaviest.

O_3 is a colourless or bluish gas with a characteristic odour. Ground-level ozone is formed primarily from photochemical reactions between two other pollutants: NO_x and volatile organic compounds – VOC (e.g. benzene described below) and also sunlight. Because of sunlight provides the energy to initiate ozone formation, the highest O_3 concentration is observed during hot, sunny, summertime weather. O_3 is a highly reactive oxidiser hazardous to health and also destroys materials. O_3 when inhaled causes an inflammatory response to the eyes, the respiratory tract and decreasing lung capacity. In the environment O_3 contributes to "smog".

C_6H_6 is a colourless liquid with a gasoline-like odour. It belongs to the group of VOC that are released in vehicle exhaust gases. C_6H_6 causes chronic health effects include cancer, liver and kidney damage, central nervous system disorders, reproductive disorders, and birth defects.

PM can be made up of hundreds of different chemicals and contains microscopic solids or liquid droplets that are so small that they can be inhaled and cause serious health problems. The impact of PM depends on its size (small – PM_{10} and very small – $PM_{2.5}$) and the number of particles retained in various areas in the respiratory system. $PM_{2.5}$ is more likely to travel and to penetrate into the deepest sections of the lungs, where they are accumulated or dissolved in biological liquids, causing aggravation of asthma, acute respiratory responses and impairment of the lung activity. On the other hand PM_{10} is more likely to deposit on the surfaces of the larger airways of the upper region of the lung. The road traffic emissions, especially from diesel vehicles is the principal source of airborne $PM_{2.5}$ and PM_{10} in cities.

Because of the harmful properties of above mentioned pollutants, their limit values for the ambient concentration correspond to different levels of health concern are distinguished and given in Table 1. These values are also used as components of the air quality indicators. The pollution levels presented in Table 1 correspond to Polish ones published by the Main Inspectorate for Environmental Protection.

Table 1. Air quality according to pollutants' concentration

(Pollution level) Air quality	Pollutant's concentration [μg/dm³] SO_2	CO	NO_2	O_3	C_6H_6	$PM_{2.5}$	PM_{10}
(1) very good	0-50	0-2·10³	0-40	0-70	0-5	0-12	0-20
(2) good	50.1-100	2.1·10³-6·10³	40.1-100	70.1-120	5.1-10	12.1-36	20.1-60
(3) moderate	100.1-200	6.1·10³-1·10⁴	100.1-150	120.1-150	10.1-15	36.1-60	60.1-100
(4) sufficient	200.1-350	1.1·10⁴-1.4·10⁴	150.1-200	150.1-180	15.1-20	60.1-84	100.1-140
(5) bad	350.1-500	1.41·10⁴-2·10⁴	200.1-400	180.1-240	20.1-50	84.1-120	140.1-200
(6) very bad	>500	>2·10⁴	>400	>240	>50	>120	>200

Based on Main Inspectorate for Environmental Protection (http://powietrze.gios.gov.pl/pjp/current?lang=en)

3. Modelling air pollution process

The air pollution process $S_{\mathrm{POLL}}(t)$, $t \in \langle 0, +\infty)$ with the discrete air pollutant's concentration states from the set $\{s^1_{\mathrm{POLL}}, s^2_{\mathrm{POLL}}, \ldots, s^v_{\mathrm{POLL}}\}$ is defined, where POLL is the kind of pollutant. It is assumed that the pollutant's concentration in the air takes v, $v \in N$ different concentration states $s^1_{\mathrm{POLL}}, s^2_{\mathrm{POLL}}, \ldots, s^v_{\mathrm{POLL}}$, that have an influence on the air pollution. Next, a semi-Markov model of the air pollution process $S_{\mathrm{POLL}}(t)$, $t \in \langle 0, +\infty)$ is assumed. Its random conditional sojourn time at the air pollutant's concentration state s^k_{POLL} while the next transition will be done to the state s^l_{POLL}, $k, l = 1,2, \ldots, v$, $k \neq l$ is denoted by $\theta^{kl}_{\mathrm{POLL}}$. Thus the air pollution process $S_{\mathrm{POLL}}(t)$, $t \in \langle 0, +\infty)$ is described by the following parameters that can be evaluated by expert or identified statistically using the methods (Bogalecka, 2020, 2021; Grabski, 2015; Iosifescu, 1980; Kołowrocki, 2014; Limnios & Oprisian, 2005; Smith, 1955):

- the matrix of probabilities $[p^{kl}_{\mathrm{POLL}}]_{vxv}$ of the air pollution process $S_{\mathrm{POLL}}(t)$, $t \in \langle 0, +\infty)$ transitions between the air pollutant's concentration states s^k_{POLL} and s^l_{POLL},

$$p^{kl}_{\mathrm{POLL}},\ k, l = 1,2, \ldots, v,\ k \neq l \tag{1}$$

where $\forall k = 1,2, \ldots, v$, $\theta^{kk}_{\mathrm{POLL}} = 0$,

- the matrix of mean values $[M^{kl}_{\mathrm{POLL}}]_{vxv}$ of the air pollution process $S_{\mathrm{POLL}}(t)$, $t \in \langle 0, +\infty)$ conditional sojourn times $\theta^{kl}_{\mathrm{POLL}}$ at the air pollutant's concentration state s^k_{POLL} while its next transition will be done to the state s^l_{POLL}, $k, l = 1,2, \ldots, v$, $k \neq l$,

$$M^{kl}_{\mathrm{POLL}} = E[\theta^{kl}_{\mathrm{POLL}}] = \int_0^\infty t dH^{kl}_{\mathrm{POLL}}(t)$$

$$= \int_0^\infty t dh^{kl}(t),\ k, l = 1,2, \ldots, v,\ k \neq l, \tag{2}$$

where $\forall k = 1,2, \ldots, v$, $M^{kk}_{\mathrm{POLL}} = 0$, and where

$$H^{kl}_{\mathrm{POLL}}(t) = P(\theta^{kl}_{\mathrm{POLL}} < t),\ t \in \langle 0, +\infty), \tag{3}$$

for

$$k, l = 1,2, \ldots, v,\ k \neq l,$$

are the conditional distribution functions of the air pollution process $S_{\mathrm{POLL}}(t)$, $t \in \langle 0, +\infty)$ conditional sojourn times $\theta^{kl}_{\mathrm{POLL}}$, $k, l = 1,2, \ldots, v$, $k \neq l$, at the states corresponding to conditional density functions

$$h^{kl}_{\mathrm{POLL}}(t) = \frac{dH^{kl}_{\mathrm{POLL}}(t)}{dt},\ t \in \langle 0, +\infty), \tag{4}$$

for

$$k, l = 1,2, \ldots, v,\ k \neq l,$$

- the vector of mean values $[M^k_{\mathrm{POLL}}]_{1xv}$ of the air pollution process $S_{\mathrm{POLL}}(t)$, $t \in \langle 0, +\infty)$ unconditional sojourn times θ^k_{POLL}, $k = 1,2, \ldots, v$, at the air pollutant's concentration states

$$M^k_{\mathrm{POLL}} = E[\theta^k_{\mathrm{POLL}}] = \sum_{l=1}^{v} p^{kl}_{\mathrm{POLL}} M^{kl}_{\mathrm{POLL}}, \tag{5}$$

for

$$k = 1,2, \ldots, v,$$

where p^{kl}_{POLL} and M^{kl}_{POLL} are defined by (1) and (2) respectively,

- the vector $[p^k_{\mathrm{POLL}}]_{1xv}$ of limit values of transient probabilities

$$p^k_{\mathrm{POLL}}(t) = P(S_{\mathrm{POLL}}(t) = s^k_{\mathrm{POLL}}), \tag{6}$$

for

$t \in \langle 0, +\infty), k = 1,2, \dots, \upsilon,$

of the air pollution process $S_{\text{POLL}}(t)$, $t \in \langle 0, +\infty)$ at the particular states s_{POLL}^{k}, $k = 1,2, \dots, \upsilon$, where

$$p_{\text{POLL}}^{k} = \lim_{t \to \infty} p_{\text{POLL}}^{k}(t) = \frac{\pi_{\text{POLL}}^{k} M_{\text{POLL}}^{k}}{\sum_{l=1}^{\upsilon} \pi_{\text{POLL}}^{l} M_{\text{POLL}}^{l}} \qquad (7)$$

for

$k = 1,2, \dots, \upsilon,$

where M_{POLL}^{k}, $k = 1,2, \dots, \upsilon$, are given by (5), and the probabilities π_{POLL}^{k}, $k = 1,2, \dots, \upsilon$, satisfy the system of equations (Bogalecka 2021)

$$\begin{cases} [\pi_{\text{POLL}}^{k}] = [\pi_{\text{POLL}}^{k}][p_{\text{POLL}}^{kl}] \\ \sum_{l=1}^{\upsilon} \pi_{\text{POLL}}^{l} = 1 \end{cases} \qquad (8)$$

where

$[\pi_{\text{POLL}}^{k}] = [\pi_{\text{POLL}}^{1}, \pi_{\text{POLL}}^{2}, \dots, \pi_{\text{POLL}}^{\upsilon}],$

and $[p_{\text{POLL}}^{kl}]$ is given by (1),

- the vector $[\hat{M}_{\text{POLL}}^{k}]_{1x\upsilon}$ of the mean values of the total sojourn times $\hat{\theta}_{\text{POLL}}^{k}$, $k = 1,2, \dots, \upsilon$,

$$\hat{M}_{\text{POLL}}^{k} = E[\hat{\theta}_{\text{POLL}}^{k}] \cong p_{\text{POLL}}^{k} \theta, \qquad (9)$$

at the particular states s_{POLL}^{k}, $k = 1,2, \dots, \upsilon$ of the air pollution process $S_{\text{POLL}}(t)$, $t \in \langle 0, +\infty)$ in the fixed time interval $\langle 0, \theta \rangle$, $\theta > 0$, where p_{POLL}^{k} are given by (7).

4. Application of air pollution process

4.1. Air pollution in Gdańsk agglomeration in 2019 – state of art

The experiment is performed in Gdańsk that belongs to Tri-City (Gdynia, Sopot and Gdańsk) agglomeration in Poland. This agglomeration is situated in Pomerania – the north and seaside part of Poland and has a population of over 1 million people.

The experiment area is affected by the pollution coming from transport as well as industrial sectors and domestic sources. The air pollutants' concentration data come from three monitoring stations located in Gdańsk, at a distance of no more than 10 km from each other (Fig. 1):

- station *S1* measures concentration of SO_2, CO, NO_2, PM_{10},
- station *S2* measures concentration of O_3, $PM_{2.5}$,
- station *S3* measures concentration of C_6H_6.

Figure 1. Location of air pollution monitoring stations.

This stations belong to the regional air quality monitoring network where concentrations of pollutants are continuous measured (counted every hour). The data coming from the monitoring system are free accessible through the internet (http://powietrze.gios.gov.pl/pjp/archives).

The variations of pollutants' concentration in 2019 recorded at monitoring stations in Gdańsk are presented in Figures 2–8 (colours of fields in diagrams correspond to colours of fields of the pollution levels given in Table 1).

Based on data coming from the sampling points of the above mentioned monitoring station in Gdańsk, only CO and NO_2 concentrations are classified as *very good* or *good* (less than 100 $\mu g/dm^3$ for CO and less than $6 \cdot 10^3$ $\mu g/dm^3$ for NO_2) in 2019.

The recorded concentrations of SO_2, O_3, C_6H_6, $PM_{2.5}$ are usually classified as *very good* or *good* (less than 100 $\mu g/dm^3$, 120 $\mu g/dm^3$, 10 $\mu g/dm^3$ and 36 $\mu g/dm^3$ for particular pollutants respectively), but there are some records for these pollutants classified as *moderate* or *sufficient* in 2019. Namely, the *moderate* pollution level has been maintained for no more than 44 hours per year 2019 for SO_2, and 22 hours per year 2019 for O_3

and *moderate* or *sufficient* pollution level only for no more than 3 hours per year 2019 for C_6H_6 whereas *moderate* pollution level – 16.7 days per the second half of year 2019 and *sufficient* pollution level – 31 hours per the second half of year 2019 for $PM_{2.5}$.

Only the PM_{10} concentration exceeded the *bad* pollution level: 21 hours per year 2019, and the *very bad* pollution level: 1 hour per year 2019.

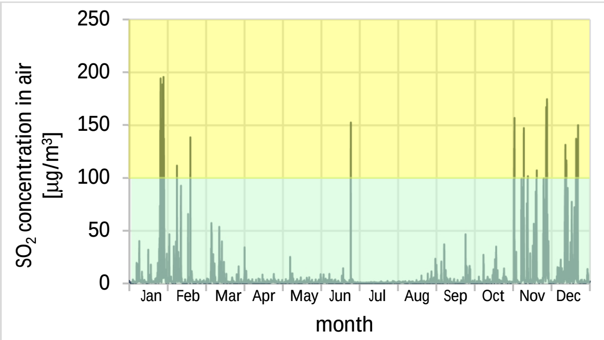

Figure 2. Variations of SO_2 concentration in 2019 recorded at *S1* monitoring station in Gdańsk (own work based on data of Main Inspectorate for Environmental Protection).

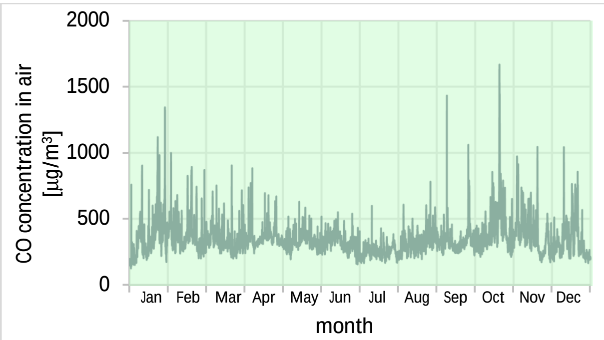

Figure 3. Variations of CO concentration in 2019 recorded at *S1* monitoring station in Gdańsk (own work based on data of Main Inspectorate for Environmental Protection).

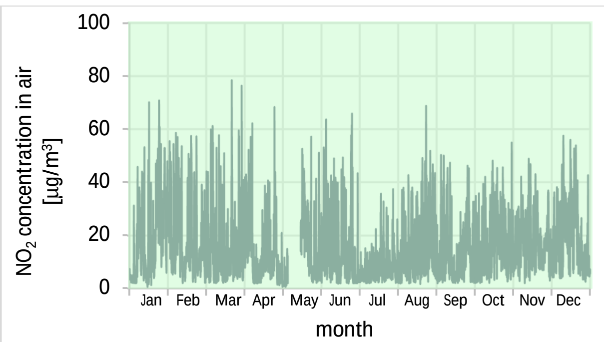

Figure 4. Variations of NO_2 concentration in 2019 recorded at *S1* monitoring station in Gdańsk (own work based on data of Main Inspectorate for Environmental Protection).

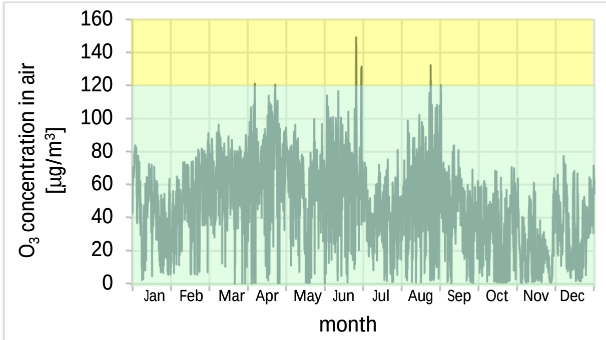

Figure 5. Variations of O_3 concentration in 2019 recorded at *S2* monitoring station in Gdańsk (own work based on data of Main Inspectorate for Environmental Protection).

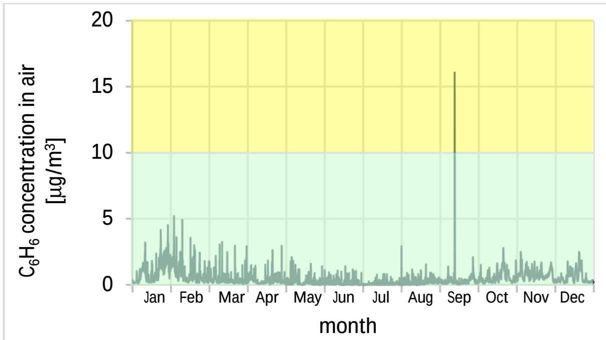

Figure 6. Variations of C_6H_6 concentration in 2019 recorded at *S3* monitoring station in Gdańsk (own work based on data of Main Inspectorate for Environmental Protection).

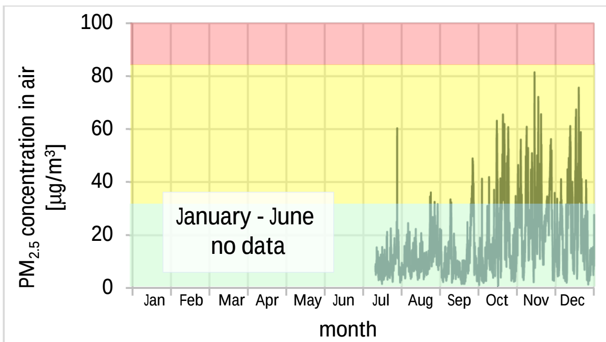

Figure 7. Variations of $PM_{2.5}$ concentration in 2019 recorded at *S2* monitoring station in Gdańsk (own work based on data of Main Inspectorate for Environmental Protection).

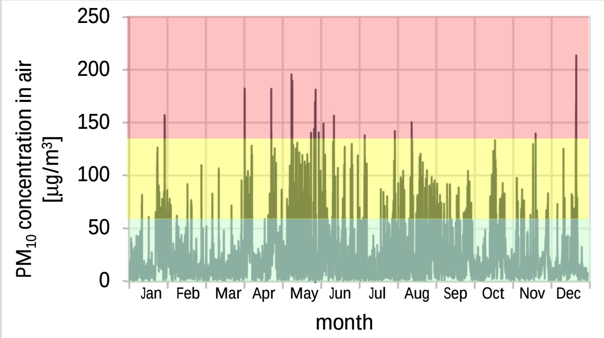

Figure 8. Variations of PM_{10} concentration in 2019 recorded at *S1* monitoring station in Gdańsk (own work based on data of Main Inspectorate for Environmental Protection).

4.2. Modelling air pollution process for Gdańsk agglomeration

Under the assumption that the air pollutant's concentration changing in time and taking into account data from Table 1, the following $\upsilon = 9$ particular air pollutants' concentration states s_{POLL}^{k}, $k = 1,2,\dots,9$ of the air pollution process $S_{\mathrm{POLL}}(t)$, $t \in \langle 0, +\infty)$ are distinguished and presented in Table 2 (to get more palpable tangible data, the level 1 from Table 1 is divided into four additional sublevels expressed with state s_{POLL}^{1}, s_{POLL}^{2}, s_{POLL}^{3} and s_{POLL}^{4} respectively).

Table 2. Air quality according to pollutants' concentration

Pollutant's concentration state	Pollutant's concentration [µg/dm³]						
	SO_2	CO	NO_2	O_3	C_6H_6	$PM_{2.5}$	PM_{10}
s_{POLL}^{1}	0-3.5	$0\text{-}1.4\cdot 10^2$	0-2.8	0-4.9	0-0.35	0-0.84	0-1.4
s_{POLL}^{2}	3.6-17.5	$1.41\cdot 10^2\text{-}7\cdot 10^2$	2.9-14	5.0-24.5	0.36-1.75	0.85-4.2	1.5-7
s_{POLL}^{3}	17.6-35	$7.1\cdot 10^2\text{-}1.4\cdot 10^3$	14.1-28	24.6-49	1.76-3.5	4.3-8.4	7.1-14
s_{POLL}^{4}	35.1-50	$1.4\cdot 10^3\text{-}2\cdot 10^3$	28.1-40	49.1-70	3.6-5	8.5-12	14.1-20
s_{POLL}^{5}	50.1-100	$2.1\cdot 10^3\text{-}6\cdot 10^3$	40.1-100	70.1-120	5.1-10	12.1-36	20.1-60
s_{POLL}^{6}	100.1-200	$6.1\cdot 10^3\text{-}1\cdot 10^4$	100.1-150	120.1-150	10.1-15	36.1-60	60.1-100
s_{POLL}^{7}	200.1-350	$1.1\cdot 10^4\text{-}1.4\cdot 10^4$	150.1-200	150.1-180	15.1-20	60.1-84	100.1-140
s_{POLL}^{8}	350.1-500	$1.41\cdot 10^4\text{-}2\cdot 10^4$	200.1-400	180.1-240	20.1-50	84.1-120	140.1-200
s_{POLL}^{9}	>500	$>2\cdot 10^4$	>400	>240	>50	>120	>200

Next, on the basis of statistical data coming from the mentioned above monitoring stations and collected in 2019, the probabilities p_{POLL}^{kl}, $k, l = 1,2,\dots,9$, $k \neq l$, of the air pollution process transitions between the air pollutant's concentration states s_{POLL}^{k} and s_{POLL}^{l}, $k, l = 1,2,\dots,9$, $k \neq l$, are evaluated according to the formula

$$p_{\mathrm{POLL}}^{kl} = \frac{n_{\mathrm{POLL}}^{kl}}{n_{\mathrm{POLL}}^{k}} \qquad (10)$$

for

$k, l = 1,2,\dots,9$, $k \neq l$,

where and n_{POLL}^{kl}, $k, l = 1,2,\dots,9$, $k \neq l$, is the realization of the air pollution process transitions between the air pollutant's concentration states s_{POLL}^{k} and s_{POLL}^{l}, and n_{POLL}^{k}, $k = 1,2,\dots,9$, is the realization of the total number of the air pollution process departures from the air pollutant's concentration state s_{POLL}^{k} during the experimental time.

The matrices of probabilities of the air pollution process transitions between the air pollutant's concentration states s_{POLL}^{k} and s_{POLL}^{l}, $k, l = 1,2,\dots,9$, $k \neq l$, for particular kinds of pollutants take the following forms:

- for SO_2

$$[p_{SO_2}^{kl}]_{9x9} =$$

$$\begin{bmatrix} 0 & 0.95 & 0.04 & 0 & 0.01 & 0 & 0 & 0 & 0 \\ 0.76 & 0 & 0.15 & 0.03 & 0.05 & 0.01 & 0 & 0 & 0 \\ 0.03 & 0.64 & 0 & 0.15 & 0.14 & 0.04 & 0 & 0 & 0 \\ 0 & 0.31 & 0.47 & 0 & 0.18 & 0.04 & 0 & 0 & 0 \\ 0 & 0.26 & 0.28 & 0.25 & 0 & 0.21 & 0 & 0 & 0 \\ 0 & 0 & 0.08 & 0.04 & 0.88 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (11)$$

- for CO

$$\left[p_{\mathrm{CO}}^{kl}\right]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.02 & 0 & 0.96 & 0.02 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.96 & 0 & 0.04 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.33 & 0.67 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

- for NO_2

$$\left[p_{\mathrm{NO}_2}^{kl}\right]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 0.99 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.23 & 0 & 0.71 & 0.05 & 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0.33 & 0 & 0.61 & 0.06 & 0 & 0 & 0 & 0 \\ 0 & 0.08 & 0.65 & 0 & 0.27 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0.21 & 0.78 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

- for O_3

$$\left[p_{\mathrm{O}_3}^{kl}\right]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 0.95 & 0.04 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0.28 & 0 & 0.68 & 0.04 & 0 & 0 & 0 & 0 & 0 \\ 0.02 & 0.40 & 0 & 0.56 & 0.02 & 0 & 0 & 0 & 0 \\ 0 & 0.03 & 0.56 & 0 & 0.41 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.04 & 0.92 & 0 & 0.04 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

- for C_6H_6

$$\left[p_{\mathrm{C}_6\mathrm{H}_6}^{kl}\right]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 0.98 & 0.01 & 0 & 0 & 0.01 & 0 & 0 & 0 \\ 0.84 & 0 & 0.15 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.94 & 0 & 0.06 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.13 & 0.74 & 0 & 0.13 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.50 & 0 & 0.50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{15}$$

- for $PM_{2.5}$

$$\left[p_{\mathrm{PM}_{2.5}}^{kl}\right]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.02 & 0 & 0.95 & 0.02 & 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0.34 & 0 & 0.57 & 0.04 & 0.05 & 0 & 0 & 0 \\ 0 & 0.01 & 0.49 & 0 & 0.50 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0.06 & 0.67 & 0 & 0.26 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.02 & 0.79 & 0 & 0.19 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

- for PM_{10}

$$\left[p_{\mathrm{PM}_{10}}^{kl}\right]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 0.67 & 0.19 & 0.08 & 0.06 & 0 & 0 & 0 & 0 \\ 0.06 & 0 & 0.62 & 0.16 & 0.14 & 0.02 & 0 & 0 & 0 \\ 0.02 & 0.44 & 0 & 0.30 & 0.22 & 0.02 & 0 & 0 & 0 \\ 0.01 & 0.15 & 0.42 & 0 & 0.39 & 0.02 & 0.01 & 0 & 0 \\ 0.01 & 0.14 & 0.26 & 0.32 & 0 & 0.24 & 0.03 & 0 & 0 \\ 0 & 0.03 & 0.07 & 0.06 & 0.72 & 0 & 0.10 & 0.02 & 0 \\ 0 & 0.07 & 0.06 & 0.03 & 0.26 & 0.49 & 0 & 0.08 & 0.01 \\ 0 & 0 & 0.05 & 0 & 0.11 & 0.42 & 0.42 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \tag{17}$$

Further, on the basis of statistical data coming from the mentioned above monitoring stations and collected in 2019, the matrices of the mean values of the air pollution process conditional sojourn times $\theta_{\mathrm{POLL}}^{kl}$, $k, l = 1,2,\ldots,9$, $k \neq l$ at the air pollutant's concentration states for particular kinds of pollutants take the following forms:

- for SO_2

$$\left[M_{\mathrm{SO}_2}^{kl}\right]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 25.6 & 15.5 & 0 & 14.5 & 0 & 0 & 0 & 0 \\ 3.0 & 0 & 3.0 & 2.4 & 3.5 & 1.4 & 0 & 0 & 0 \\ 2.7 & 1.4 & 0 & 1.7 & 1.3 & 1.0 & 0 & 0 & 0 \\ 0 & 1.3 & 1.3 & 0 & 1.8 & 1.0 & 0 & 0 & 0 \\ 0 & 1.3 & 1.2 & 2.1 & 0 & 1.8 & 0 & 0 & 0 \\ 0 & 0 & 2.0 & 1.0 & 1.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

- for CO

$$[M_{\mathrm{CO}}^{kl}]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 6.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 24.0 & 0 & 163.9 & 312.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.7 & 0 & 2.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 2.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

- for NO_2

$$[M_{\mathrm{NO_2}}^{kl}]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 3.3 & 1.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7.8 & 0 & 6.6 & 3.9 & 3.0 & 0 & 0 & 0 & 0 \\ 0 & 2.7 & 0 & 3.2 & 2.0 & 0 & 0 & 0 & 0 \\ 0 & 2.0 & 2.5 & 0 & 1.8 & 0 & 0 & 0 & 0 \\ 0 & 1.5 & 2.3 & 2.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (20)$$

- for O_3

$$[M_{\mathrm{O_3}}^{kl}]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 4.2 & 2.4 & 3.0 & 0 & 0 & 0 & 0 & 0 \\ 4.5 & 0 & 3.5 & 2.1 & 0 & 0 & 0 & 0 & 0 \\ 3.3 & 4.5 & 0 & 4.6 & 3.0 & 0 & 0 & 0 & 0 \\ 0 & 2.9 & 4.7 & 0 & 4.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7.9 & 6.2 & 0 & 4.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

- for C_6H_6

$$[M_{\mathrm{C_6H_6}}^{kl}]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 14.3 & 21.0 & 0 & 0 & 5.0 & 0 & 0 & 0 \\ 8.0 & 0 & 14.4 & 10.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.9 & 0 & 11.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 2.0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

- for $PM_{2.5}$

$$[M_{\mathrm{PM_{2.5}}}^{kl}]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3.0 & 0 & 3.9 & 2.0 & 6.0 & 0 & 0 & 0 & 0 \\ 0 & 4.2 & 0 & 4.7 & 3.8 & 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 2.9 & 0 & 3.0 & 0 & 0 & 0 & 0 \\ 0 & 5.5 & 6.1 & 7.1 & 0 & 13.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 5.8 & 0 & 9.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (23)$$

- for PM_{10}

$$[M_{\mathrm{PM_{10}}}^{kl}]_{9\mathrm{x}9} = \begin{bmatrix} 0 & 1.1 & 1.1 & 1.0 & 1.0 & 0 & 0 & 0 & 0 \\ 2.8 & 0 & 2.1 & 2.1 & 1.6 & 1.4 & 0 & 0 & 0 \\ 2.3 & 1.8 & 0 & 1.9 & 1.6 & 1.3 & 0 & 0 & 0 \\ 1.3 & 1.3 & 1.5 & 0 & 1.3 & 1.3 & 1.8 & 0 & 0 \\ 2.0 & 2.1 & 2.3 & 2.7 & 0 & 3.5 & 2.5 & 0 & 0 \\ 0 & 1.8 & 1.4 & 1.3 & 1.7 & 0 & 1.5 & 1.3 & 0 \\ 0 & 1.6 & 1.3 & 1.0 & 1.2 & 1.3 & 0 & 1.2 & 1.0 \\ 0 & 0 & 1.0 & 0 & 1.0 & 1.1 & 1.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \end{bmatrix} \quad (24)$$

This way the air pollution processes for particular kinds of pollutant are defined and their main characteristics can be predicted.

Namely, applying (5) and considering (11) and (18) the approximate mean values of unconditional sojourn times of variables $\theta_{\mathrm{SO_2}}^{k}$, $k = 1,2,\dots,9$ can be evaluated for SO_2 pollutant. The values that are not equal to 0 are presented only, and they as follows:

$$\begin{aligned} M_{\mathrm{SO_2}}^{1} &= p_{\mathrm{SO_2}}^{12} M_{\mathrm{SO_2}}^{12} + p_{\mathrm{SO_2}}^{13} M_{\mathrm{SO_2}}^{13} + p_{\mathrm{SO_2}}^{15} M_{\mathrm{SO_2}}^{15} \\ &= 0.95 \cdot 25.6 + 0.04 \cdot 15.5 + 0.01 \cdot 14.5 \\ &= 24.32 + 0.62 + 0.15 = 25.09, \end{aligned} \quad (25)$$

$$\begin{aligned} M_{\mathrm{SO_2}}^{2} &= p_{\mathrm{SO_2}}^{21} M_{\mathrm{SO_2}}^{21} + p_{\mathrm{SO_2}}^{23} M_{\mathrm{SO_2}}^{23} + p_{\mathrm{SO_2}}^{24} M_{\mathrm{SO_2}}^{24} \\ &\quad + p_{\mathrm{SO_2}}^{25} M_{\mathrm{SO_2}}^{25} + p_{\mathrm{SO_2}}^{26} M_{\mathrm{SO_2}}^{26} \\ &= 0.76 \cdot 3.0 + 0.15 \cdot 3.0 + 0.03 \cdot 2.4 \\ &\quad + 0.05 \cdot 3.5 + 0.01 \cdot 1.4 \\ &= 2.28 + 0.45 + 0.07 + 0.18 + 0.01 \\ &= 2.99, \end{aligned} \quad (26)$$

$$M_{SO_2}^3 = p_{SO_2}^{31}M_{SO_2}^{31} + p_{SO_2}^{32}M_{SO_2}^{32} + p_{SO_2}^{34}M_{SO_2}^{24} + p_{SO_2}^{35}M_{SO_2}^{35} + p_{SO_2}^{36}M_{SO_2}^{36} = 0.03 \cdot 2.7 + 0.64 \cdot 1.4 + 0.15 \cdot 1.7 + 0.14 \cdot 1.3 + 0.04 \cdot 1.0 = 0.08 + 0.90 + 0.26 + 0.20 + 0.04 = 1.46, \quad (27)$$

$$M_{SO_2}^4 = p_{SO_2}^{42}M_{SO_2}^{42} + p_{SO_2}^{43}M_{SO_2}^{43} + p_{SO_2}^{45}M_{SO_2}^{45} + p_{SO_2}^{46}M_{SO_2}^{46} = 0.31 \cdot 1.3 + 0.47 \cdot 1.3 + 0.18 \cdot 1.8 + 0.04 \cdot 1.0 = 0.40 + 0.61 + 0.32 + 0.04 = 1.37, \quad (28)$$

$$M_{SO_2}^5 = p_{SO_2}^{52}M_{SO_2}^{52} + p_{SO_2}^{53}M_{SO_2}^{53} + p_{SO_2}^{54}M_{SO_2}^{54} + p_{SO_2}^{56}M_{SO_2}^{56} = 0.26 \cdot 1.3 + 0.28 \cdot 1.2 + 0.25 \cdot 2.1 + 0.21 \cdot 1.8 = 0.34 + 0.34 + 0.52 + 0.38 = 1.58, \quad (29)$$

$$M_{SO_2}^6 = p_{SO_2}^{63}M_{SO_2}^{63} + p_{SO_2}^{64}M_{SO_2}^{64} + p_{SO_2}^{65}M_{SO_2}^{65} = 0.08 \cdot 2.0 + 0.04 \cdot 1.0 + 0.88 \cdot 1.6 = 0.16 + 0.04 + 1.41 = 1.61. \quad (30)$$

The results (25)–(30) can be presented as the vector $[M_{SO_2}^k]_{1x9}$ of mean values of the air pollution process unconditional sojourn times $\theta_{SO_2}^k$, $k = 1,2,\dots,9$, at the particular SO_2 concentration states

$$[M_{SO_2}^k]_{1x9} = [25.09, 2.99, 1.46, 1.37, 1.58, 1.61, 0, 0, 0]. \quad (31)$$

In the similarly way, applying (5) and considering (12)–(17) and (19)–(24) the vectors of mean values of the air pollution process unconditional sojourn times θ_{POLL}^k, $k = 1,2,\dots,9$, at the air pollutant's concentration states for other kinds of pollutants are obtained and take the following forms:

- for CO

$$[M_{CO}^k]_{1x9} = [6.00, 164.06, 2.69, 1.67, 0, 0, 0, 0, 0], \quad (32)$$

- for NO_2

$$[M_{NO_2}^k]_{1x9} = [3.28, 6.71, 2.96, 2.27, 2.53, 0, 0, 0, 0], \quad (33)$$

- for O_3

$$[M_{O_3}^k]_{1x9} = [4.12, 3.72, 4.50, 4.48, 6.20, 2.20, 0, 0, 0], \quad (34)$$

- for C_6H_6

$$[M_{C_6H_6}^k]_{1x9} = [14.27, 8.99, 4.33, 1.74, 2.00, 1.00, 1.00, 0, 0], \quad (35)$$

- for $PM_{2.5}$

$$[M_{PM_{2.5}}^k]_{1x9} = [1.00, 3.88, 4.31, 2.93, 8.56, 6.36, 2.60, 0, 0], \quad (36)$$

- for PM_{10}

$$[M_{PM_{10}}^k]_{1x9} = [1.09, 2.06, 1.79, 1.39, 2.69, 1.45, 1.28, 1.08, 1.00]. \quad (37)$$

To find the limit values of the transient probabilities p_{POLL}^k, $k = 1,2,\dots,9$ at particular states of the process $S_{POLL}(t)$, $t \in \langle 0, +\infty)$ the system of equations (8) has to be solved that for particular pollutants, considering (11)–(17) it takes the following forms:

- for SO_2

$$\begin{cases} \pi_{SO_2}^1 = 0.76\pi_{SO_2}^2 + 0.03\pi_{SO_2}^3 \\ \pi_{SO_2}^2 = 0.95\pi_{SO_2}^1 + 0.64\pi_{SO_2}^3 + 0.31\pi_{SO_2}^4 + 0.26\pi_{SO_2}^5 \\ \pi_{SO_2}^3 = 0.04\pi_{SO_2}^1 + 0.15\pi_{SO_2}^2 + 0.47\pi_{SO_2}^4 + 0.28\pi_{SO_2}^5 + 0.08\pi_{SO_2}^6 \\ \pi_{SO_2}^4 = 0.03\pi_{SO_2}^2 + 0.15\pi_{SO_2}^3 + 0.25\pi_{SO_2}^5 + 0.04\pi_{SO_2}^6 \\ \pi_{SO_2}^5 = 0.01\pi_{SO_2}^1 + 0.05\pi_{SO_2}^2 + 0.14\pi_{SO_2}^3 + 0.18\pi_{SO_2}^4 + 0.88\pi_{SO_2}^6 \\ \pi_{SO_2}^6 = 0.01\pi_{SO_2}^2 + 0.04\pi_{SO_2}^3 + 0.04\pi_{SO_2}^4 + 0.21\pi_{SO_2}^5 \\ \pi_{SO_2}^1 + \pi_{SO_2}^2 + \dots + \pi_{SO_2}^6 = 1 \end{cases}$$

whereas its solution is

$$\pi_{SO_2}^1 = 0.3177, \pi_{SO_2}^2 = 0.4132, \pi_{SO_2}^3 = 0.1205, \pi_{SO_2}^4 = 0.0497,$$

$$\pi_{SO_2}^5 = 0.0727, \pi_{SO_2}^6 = 0.0262, \tag{38}$$

- for CO

$$\begin{cases} \pi_{CO}^1 = 0.02\pi_{CO}^2 \\ \pi_{CO}^2 = \pi_{CO}^1 + 0.96\pi_{CO}^3 + 0.33\pi_{CO}^4 \\ \pi_{CO}^3 = 0.96\pi_{CO}^2 + 0.67\pi_{CO}^4 \\ \pi_{CO}^4 = 0.02\pi_{CO}^2 + 0.04\pi_{CO}^3 \\ \pi_{CO}^1 + \pi_{CO}^2 + \pi_{CO}^3 + \pi_{CO}^4 = 1 \end{cases}$$

whereas its solution is

$$\pi_{CO}^1 = 0.0096, \pi_{CO}^2 = 0.4807,$$
$$\pi_{CO}^3 = 0.4808, \pi_{CO}^4 = 0.0289, \tag{39}$$

- for NO_2

$$\begin{cases} \pi_{NO_2}^1 = 0.23\pi_{NO_2}^2 \\ \pi_{NO_2}^2 = 0.99\pi_{NO_2}^1 + 0.33\pi_{NO_2}^3 + 0.08\pi_{NO_2}^4 \\ \qquad + 0.01\pi_{NO_2}^5 \\ \pi_{NO_2}^3 = 0.01\pi_{NO_2}^1 + 0.71\pi_{NO_2}^2 + 0.65\pi_{NO_2}^4 \\ \qquad + 0.21\pi_{NO_2}^5 \\ \pi_{NO_2}^4 = 0.05\pi_{NO_2}^2 + 0.61\pi_{NO_2}^3 + 0.78\pi_{NO_2}^5 \\ \pi_{NO_2}^5 = 0.01\pi_{NO_2}^2 + 0.06\pi_{NO_2}^3 + 0.27\pi_{NO_2}^4 \\ \pi_{NO_2}^1 + \pi_{NO_2}^2 + \cdots + \pi_{NO_2}^5 = 1 \end{cases}$$

whereas its solution is

$$\pi_{NO_2}^1 = 0.0426, \pi_{NO_2}^2 = 0.1854,$$
$$\pi_{NO_2}^3 = 0.3557, \pi_{NO_2}^4 = 0.3095,$$
$$\pi_{NO_2}^5 = 0.1068, \tag{40}$$

- for O_3

$$\begin{cases} \pi_{O_3}^1 = 0.28\pi_{O_3}^2 + 0.02\pi_{O_3}^3 \\ \pi_{O_3}^2 = 0.95\pi_{O_3}^1 + 0.40\pi_{O_3}^3 + 0.03\pi_{O_3}^4 \\ \pi_{O_3}^3 = 0.04\pi_{O_3}^1 + 0.68\pi_{O_3}^2 + 0.56\pi_{O_3}^4 \\ \qquad + 0.04\pi_{O_3}^5 \\ \pi_{O_3}^4 = 0.01\pi_{O_3}^1 + 0.04\pi_{O_3}^2 + 0.56\pi_{O_3}^3 \\ \qquad + 0.92\pi_{O_3}^5 \\ \pi_{O_3}^5 = 0.02\pi_{O_3}^3 + 0.41\pi_{O_3}^4 + \pi_{O_3}^6 \\ \pi_{O_3}^6 = 0.04\pi_{O_3}^5 \\ \pi_{O_3}^1 + \pi_{O_3}^2 + \cdots + \pi_{O_3}^6 = 1 \end{cases}$$

whereas its solution is

$$\pi_{O_3}^1 = 0.0586, \pi_{O_3}^2 = 0.1873,$$
$$\pi_{O_3}^3 = 0.3063, \pi_{O_3}^4 = 0.3055,$$
$$\pi_{O_3}^5 = 0.1368, \pi_{O_3}^6 = 0.0055, \tag{41}$$

- for C_6H_6

$$\begin{cases} \pi_{C_6H_6}^1 = 0.84\pi_{C_6H_6}^2 \\ \pi_{C_6H_6}^2 = 0.98\pi_{C_6H_6}^1 + 0.94\pi_{C_6H_6}^3 \\ \qquad + 0.13\pi_{C_6H_6}^4 \\ \pi_{C_6H_6}^3 = 0.01\pi_{C_6H_6}^1 + 0.15\pi_{C_6H_6}^2 \\ \qquad + 0.74\pi_{C_6H_6}^4 \\ \pi_{C_6H_6}^4 = 0.01\pi_{C_6H_6}^2 + 0.06\pi_{C_6H_6}^3 + \pi_{C_6H_6}^5 \\ \pi_{C_6H_6}^5 = 0.13\pi_{C_6H_6}^4 + 0.50\pi_{C_6H_6}^6 \\ \pi_{C_6H_6}^6 = 0.01\pi_{C_6H_6}^1 + \pi_{C_6H_6}^7 \\ \pi_{C_6H_6}^7 = 0.50\pi_{C_6H_6}^6 \\ \pi_{C_6H_6}^1 + \pi_{C_6H_6}^2 + \cdots + \pi_{C_6H_6}^7 = 1 \end{cases}$$

whereas its solution is

$$\pi_{C_6H_6}^1 = 0.4009, \pi_{C_6H_6}^2 = 0.4773,$$
$$\pi_{C_6H_6}^3 = 0.0876, \pi_{C_6H_6}^4 = 0.0161,$$
$$\pi_{C_6H_6}^5 = 0.0061, \pi_{C_6H_6}^6 = 0.0080,$$
$$\pi_{C_6H_6}^7 = 0.0040, \tag{42}$$

- for $PM_{2.5}$

$$\begin{cases} \pi_{PM_{2.5}}^1 = 0.02\pi_{PM_{2.5}}^2 \\ \pi_{PM_{2.5}}^2 = \pi_{PM_{2.5}}^1 + 0.34\pi_{PM_{2.5}}^3 + 0.01\pi_{PM_{2.5}}^4 \\ \qquad + 0.01\pi_{PM_{2.5}}^5 \\ \pi_{PM_{2.5}}^3 = 0.95\pi_{PM_{2.5}}^2 + 0.49\pi_{PM_{2.5}}^4 \\ \qquad + 0.06\pi_{PM_{2.5}}^5 \\ \pi_{PM_{2.5}}^4 = 0.02\pi_{PM_{2.5}}^2 + 0.57\pi_{PM_{2.5}}^3 \\ \qquad + 0.67\pi_{PM_{2.5}}^5 + 0.02\pi_{PM_{2.5}}^6 \\ \pi_{PM_{2.5}}^5 = 0.01\pi_{PM_{2.5}}^2 + 0.04\pi_{PM_{2.5}}^3 \\ \qquad + 0.50\pi_{PM_{2.5}}^4 + 0.79\pi_{PM_{2.5}}^6 \\ \pi_{PM_{2.5}}^6 = 0.05\pi_{PM_{2.5}}^3 + 0.26\pi_{PM_{2.5}}^5 + \pi_{PM_{2.5}}^7 \\ \pi_{PM_{2.5}}^7 = 0.19\pi_{PM_{2.5}}^6 \\ \pi_{PM_{2.5}}^1 + \pi_{PM_{2.5}}^2 + \cdots + \pi_{PM_{2.5}}^7 = 1 \end{cases}$$

whereas its solution is

$$\pi^1_{PM_{2.5}} = 0.0019, \pi^2_{PM_{2.5}} = 0.0933,$$
$$\pi^3_{PM_{2.5}} = 0.2529, \pi^4_{PM_{2.5}} = 0.3064,$$
$$\pi^5_{PM_{2.5}} = 0.2366, \pi^6_{PM_{2.5}} = 0.0915,$$
$$\pi^7_{PM_{2.5}} = 0.0174, \tag{43}$$

- for PM_{10}

$$\begin{cases} \pi^1_{PM_{10}} = 0.06\pi^2_{PM_{10}} + 0.02\pi^3_{PM_{10}} \\ \qquad + 0.01\pi^4_{PM_{10}} + 0.01\pi^5_{PM_{10}} \\ \pi^2_{PM_{10}} = 0.67\pi^1_{PM_{10}} + 0.44\pi^3_{PM_{10}} \\ \qquad + 0.15\pi^4_{PM_{10}} + 0.14\pi^5_{PM_{10}} \\ \qquad + 0.03\pi^6_{PM_{10}} + 0.07\pi^7_{PM_{10}} \\ \pi^3_{PM_{10}} = 0.19\pi^1_{PM_{10}} + 0.62\pi^2_{PM_{10}} \\ \qquad + 0.42\pi^4_{PM_{10}} + 0.26\pi^5_{PM_{10}} \\ \qquad + 0.07\pi^6_{PM_{10}} + 0.06\pi^7_{PM_{10}} \\ \qquad + 0.05\pi^8_{PM_{10}} \\ \pi^4_{PM_{10}} = 0.08\pi^1_{PM_{10}} + 0.16\pi^2_{PM_{10}} \\ \qquad + 0.30\pi^3_{PM_{10}} + 0.32\pi^5_{PM_{10}} \\ \qquad + 0.06\pi^6_{PM_{10}} + 0.03\pi^7_{PM_{10}} \\ \pi^5_{PM_{10}} = 0.06\pi^1_{PM_{10}} + 0.14\pi^2_{PM_{10}} \\ \qquad + 0.22\pi^3_{PM_{10}} + 0.39\pi^4_{PM_{10}} \\ \qquad + 0.72\pi^6_{PM_{10}} + 0.26\pi^7_{PM_{10}} \\ \qquad + 0.11\pi^8_{PM_{10}} \\ \pi^6_{PM_{10}} = 0.02\pi^2_{PM_{10}} + 0.02\pi^3_{PM_{10}} \\ \qquad + 0.02\pi^4_{PM_{10}} + 0.24\pi^5_{PM_{10}} \\ \qquad + 0.49\pi^7_{PM_{10}} + 0.42\pi^8_{PM_{10}} + \pi^9_{PM_{10}} \\ \pi^7_{PM_{10}} = 0.01\pi^4_{PM_{10}} + 0.03\pi^5_{PM_{10}} \\ \qquad + 0.10\pi^6_{PM_{10}} + 0.42\pi^8_{PM_{10}} \\ \pi^8_{PM_{10}} = 0.02\pi^6_{PM_{10}} + 0.08\pi^7_{PM_{10}} \\ \pi^9_{PM_{10}} = 0.01\pi^7_{PM_{10}} \\ \pi^1_{PM_{10}} + \pi^2_{PM_{10}} + \cdots + \pi^9_{PM_{10}} = 1 \end{cases}$$

whereas its solution is

$$\pi^1_{PM_{10}} = 0.0214, \pi^2_{PM_{10}} = 0.1968,$$
$$\pi^3_{PM_{10}} = 0.2708, \pi^4_{PM_{10}} = 0.1909,$$
$$\pi^5_{PM_{10}} = 0.2229, \pi^6_{PM_{10}} = 0.0766,$$
$$\pi^7_{PM_{10}} = 0.0175, \pi^8_{PM_{10}} = 0.0029,$$
$$\pi^9_{PM_{10}} = 0.0002. \tag{44}$$

Next, according to (7) and considering (31)–(37) and (38)–(44) respectively, the vectors $[p^k_{POLL}]_{1x9}$ of approximate limit values of the transient probabilities p^k_{POLL}, $k = 1,2,\ldots,9$ at the particular states s^k_{POLL} of the process $S_{POLL}(t)$, $t \in \langle 0, +\infty)$ for particular pollutants are as follows:

- for SO_2

$$[p^k_{SO_2}]_{1x9} = [0.8297, 0.1286, 0.0183, 0.0071, 0.0119, 0.0044, 0, 0, 0], \tag{45}$$

- for CO

$$[p^k_{CO}]_{1x9} = [0.0007, 0.9826, 0.0161, 0.0006, 0, 0, 0, 0], \tag{46}$$

- for NO_2

$$[p^k_{NO_2}]_{1x9} = [0.0410, 0.3649, 0.3088, 0.2061, 0.0792, 0, 0, 0, 0], \tag{47}$$

- for O_3

$$[p^k_{O_3}]_{1x9} = [0.0531, 0.1533, 0.3032, 0.3011, 0.1866, 0.0027, 0, 0, 0], \tag{48}$$

- for C_6H_6

$$[p^k_{C_6H_6}]_{1x9} = [0.5478, 0.4109, 0.0363, 0.0027, 0.0012, 0.0007, 0.0004, 0, 0], \tag{49}$$

- for $PM_{2.5}$

$$[p^k_{PM_{2.5}}]_{1x9} = [0.0004, 0.0723, 0.2178, 0.1794, 0.4047, 0.1163, 0.0091, 0, 0], \tag{50}$$

- for PM_{10}

$$[p^k_{PM_{10}}]_{1x9} = [0.0122, 0.2117, 0.2531, 0.1385, 0.3131, 0.0580, 0.0117, 0.0016, 0.0001]. \tag{51}$$

Finally, by (9) and considering (45)–(51) respectively, the vectors $[\hat{M}^k_{POLL}]_{1x9}$ of approximate mean values of the sojourn total time $\hat{\theta}^k_{POLL}$ of the process $S(t)$ in the fixed time interval $\theta = 1$ year (365 days) at the particular states s^k_{POLL}, $k = 1,2,\ldots,9$ for particular pollutants, expressed in days, are as follows (Fig. 9):

- for SO_2

$$[\hat{M}^k_{SO_2}]_{1x9} = [302.84, 46.94, 6.68, 2.59, 4.34, 1.61, \ldots, 0] \quad (52)$$

- for CO

$$[\hat{M}^k_{CO}]_{1x9} = [0.25, 358.65, 5.88, 0.22, \ldots, 0] \quad (53)$$

- for NO_2

$$[\hat{M}^k_{NO_2}]_{1x9} = [14.96, 133.19, 112.71, 75.23, 28.91, \ldots, 0] \quad (54)$$

- for O_3

$$[\hat{M}^k_{O_3}]_{1x9} = [19.38, 55.95, 110.67, 109.90, 68.11, 0.99, \ldots, 0] \quad (55)$$

- for C_6H_6

$$[\hat{M}^k_{C_6H_6}]_{1x9} = [199.95, 149.98, 13.25, 0.98, 0.44, 0.25, 0.15, 0, 0] \quad (56)$$

- for $PM_{2.5}$

$$[\hat{M}^k_{PM_{2.5}}]_{1x9} = [0.15, 26.39, 79.50, 65.48, 147.71, 42.45, 3.32, 0, 0] \quad (57)$$

- for PM_{10}

$$[\hat{M}^k_{PM_{10}}]_{1x9} = [4.45, 77.27, 92.38, 50.55, 114.28, 21.17, 4.27, 0.59, 0.04]. \quad (58)$$

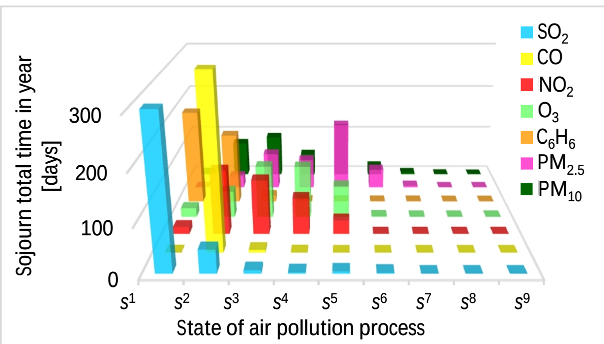

Figure 9. Approximate mean values of the sojourn total time of air environmental process at particular states for particular pollutants in 1 year interval.

The air pollution of Gdańsk agglomeration, based on the above obtained results, can be generally classified as *very good* and *good* as the highest approximate mean values of the sojourn total time of air environmental process at the particular states for particular pollutants concern the states of lower pollutants' concentration (Fig. 9).

Despite this it is suggested not to use only one kind of pollutant to assess the air quality as is the case in some countries or cities. It is very possible that the concentration of one pollutant is low whereas the concentration of another one is high in the same time (as an example of this, compare Figures 3–4 with Figure 8).

Moreover, the results (45)–(51) can play a fundamental and practically important role in the minimization of air pollution and its consequences mitigation through looking corresponding optimal values of limit transient probabilities p^k_{POLL}, $k = 1,2, \ldots, 9$ at the particular states s^k of the process $S_{POLL}(t)$ for particular pollutants to minimize the mean values of the sojourn total time at these states.

5. Conclusion

The semi-Markov model of the environmental pollution process as a novel approach to assess the air quality is presented in the chapter. The proposed methods provides to obtain the limit values of transient probabilities as well as the approximate mean values of the sojourn total times staying at the established pollutant's concentration states. These variables predicted using the semi-Markov model are different than those ones directly estimated from real data. This fact justifies sensibility of considering the semi-markovian approach to modelling air pollution, especially when distributions of the air pollution conditional sojourn times are different than exponential. Thanks to this the prediction of the air pollution process' characteristics is more precise. These results can be also applied to the air pollution and its consequences optimization based on the linear programming.

The obtained results can be essential for some authorities responsible for carrying out the air quality assessment and environment protection. Moreover, the proposed model of the environmental pollution process is a universal tool that can be used to assess the quality of other ecosystems threatened by pollutants.

Acknowledgment

The chapter presents results developed in the scope of the research project “Ecological problems in terms of quality management of polymers and sea transport”, supported by Gdynia Maritime University (project grant no. WZNJ/2021/PZ/02).

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