TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

No. 124, 2012, 41–49

DANIEL SŁAWIŃSKI* and WITOLD ZAKRZEWSKI

Dynamic analysis of the control stage of a high speed steam turbine

The Szewalski Institute of Fluid-Flow Machinery of the Polish Academy of Sciences, Centre for Thermomechanics of Fluids, Energy Conversion Department, Fiszera 14, 80-231 Gdańsk, Poland

Abstract

This paper analyzes the possibility of resonances within a control stage of a high speed turbine. An original method for steady state operation is used for the analysis of the stiffness matrix representing: initial displacement blade (nominal shape blade obtained by nominal working point of turbine) and residual stresses following local crossing yield plastic strain. In the analysis there has been found a possibility of occurance of the resonance frequency, and mistuning by showing the safe working ranges. The calculation used the 3D modal and harmonic analysis within computational solid dynamics (CSD) and the results were compared with the 0D data from literature. After local crossing of yields stresses, on the basis of the harmonic analysis, there has been shown operation of the blade in the resonance point. Before crossing the yield stresses the blade is working in mistuning and a safe point.

Keywords: Control stage; Residual stresses; Initial displacement blade; Vibrations blade

Nomenclature

$\mathbf{A}$	–	angular velocity matrix
$\vec{b}$	–	body force
$\mathbf{D}$	–	material properties matrix
$\mathbf{G}_s$	–	coordination of geometrical matrix
$f = \omega/2\pi$	–	frequency, Hz

*E-mail address: daniel.slawinski@imp.gda.pl

$\mathbf{F}^{ext}, \mathbf{F}^{rot}$	–	load of Eq.(5), N
$\mathcal{F}^c = \{\rho\vec{v} \otimes \vec{v}\}$	–	conservative momentum flux, Pa
$\mathcal{F}^e = \{\overset{\leftrightarrow}{\sigma}\}$	–	elastic momentum flux, Pa
$\mathcal{F}^\nu = \{\overset{\leftrightarrow}{\tau}\}$	–	viscous momentum flux, Pa
$\mathbf{K}$, $\mathbf{K}^{NL}$, $\mathbf{K}^R$, $\mathbf{K}^\Omega$	–	stiffness matrices
$\mathcal{S} = \{\rho\vec{b}^{fic}\}$	–	source
$\mathbf{T}$	–	matrix of coordinates
$\vec{v}(\vec{x}, t)$	–	velocity vector
$\vec{x} = x_i\vec{e}_i$	–	$\vec{e}_i$ are reference unit vector, $(i = 1, 2, 3)$
t	–	time
$\vec{u}$	–	drift velocity vector
$\vec{w}$	–	relative velocity vector: $\vec{v} = \vec{w} + \vec{u}$

Greek symbols

$\vec{\Omega}$	–	angular velocity, rad/s
$2\vec{\Omega} \times \vec{w} = 2\,(\Omega_i w_j)\,\vec{e}_i \times \vec{e}_j$	–	Coriolis acceleration vector, m/s^2
$\vec{\Omega} \times \left(\vec{\Omega} \times \vec{x}\right) = \Omega_i^2 x_j \vec{e}_i \times \vec{e}_j \times \vec{e}_i$	–	Galileo acceleration vector, m/s^2
ρ	–	density of material, kg/m^3
$\left(\frac{d}{dt}\vec{\Omega}\right) \times \vec{x} = \left(\frac{d}{dt}\Omega_i x_j\right)\vec{e}_i \times \vec{e}_j$	–	Euler acceleration vector, m/s^2

Subscripts and subscripts

$\rightarrow$	–	vector
$\leftrightarrow$	–	tensor
$\times$	–	Cartesian product
$\otimes$	–	tensor product or dyadic product

1 Introduction

It is known that vibrations generated by nonstationary loads form a significant problem in the lifetime of first and last stages of steam turbines. Even short working times of blades in resonance due to non-stationary loads, may result in degradation of turbine stage material. Generally, the resonance caused by excitation loads, leads to very large blades displacement, and local exceeding the yield stresses in blade material. Such situation is very dangerous. Local crossing the yield stresses can significantly effect on the value of natural frequency of the blade. Blade, before local crossing of the plastic yield strain maybe working in safe point of operation in the Campbell diagram, but after local exceeding the yield strain maybe found in the resonance (see Fig. 1).

The problem of natural vibration has been discussed in [2,6], modal vibration and harmonic analysis by used the finite element method (FEM) in [3,7,8,15]. The problem of determination of nonstationary load coming from the steam flow, amplitudes of excitation loads for nonstationary flow and partial power turbine have recently been discussed in the works [4,5,10,11,14].

In this study we will be concentrated on determination of frequency for non-

Figure 1. The damage of blades after a short time of operation in the resonance point [4].

stationary load and designation of their natural frequencies. We have resolved a typical model of a control stage of high speed steam turbine. The model under consideration is the assembly of a disc and 57 blades (see Fig. 2). The blades are connected to the disc by two bolts. Start-up and shutdown of turbine has not been considered. Numerical results shall by analyzed by their comparison to the analytical results. Rotational speed for the analysis is assumed to be 10400 rpm and a value of excitation from the pressure load is 0.9333 MPa. These parameters are calculated from the thermodynamic relations and functions of frequency only [10,11].

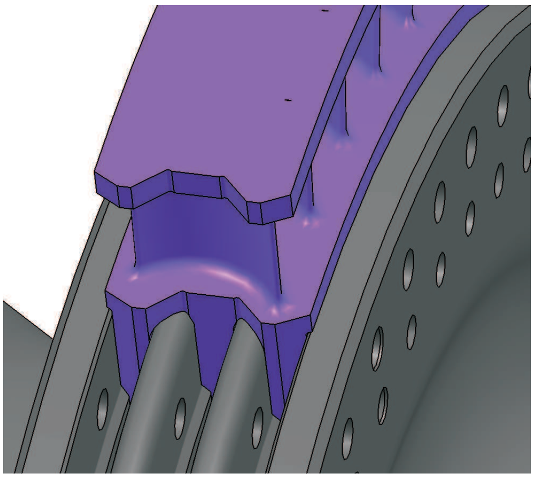

Figure 2. Control stage by high speed steam turbine.

2 Methodology

Our analysis and calculations are based on the 3D modal and harmonic analysis typical for the CSD[1] [1,15]. Results of excitation load have been verified by results obtained in the 0D method from the literature [10,12].

Let for CSD analysis be taken the following definitions:

- velocity vector

$$\vec{v}(\vec{x},t) = \vec{w} + \vec{u} = \frac{d\vec{x}}{dt} + \vec{\Omega} \times \vec{x} \,, \tag{1}$$

- law of momentum balance in a conservative form [1]

$$\frac{d}{dt}\mathcal{U} + \mathrm{div}(\mathcal{F}^c + \mathcal{F}^e) = \mathrm{div}\mathcal{F}^\nu + S \,, \tag{2}$$

- fictitious force due to rotation [1]

$$\vec{b}^{fic} = 2\vec{\Omega} \times \vec{w} + \left(\frac{d}{dt}\vec{\Omega}\right) \times \vec{x} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{x}\right) \,. \tag{3}$$

Formula leads to a balance of momentum proper for discretisation by finite element method. The general form of the equation of moments when the Coriolis effects are included, and the damping effect is not neglected reads

$$\rho\frac{d}{dt}w_i = \left(\sigma_{ij} + \tau_{ij}\right)_{,j} + \rho b_i + \rho b_i^{fic} \,, \tag{4}$$

where σ_{ij}, τ_{ij} is the elastic and viscous tensor stress of Cauchy.

The stress and strain calculated will be saved by means of second principal stresses and strain. For the modal analysis the momentum balance received a form useful to a computational applications of the finite element method [1,7,9]

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\mathbf{K} + \mathbf{K}^{NL} + \mathbf{K}^{R} - \mathbf{K}^{\Omega}\right)\mathbf{q} = \mathbf{F}^{ext} + \mathbf{F}^{rot} \,, \tag{5}$$

where 'dots' represent a time derivative, and $\mathbf{M}, \mathbf{K}, \mathbf{K}^{NL}, \mathbf{K}^{R}, \mathbf{K}^{\Omega}, \mathbf{q}, \mathbf{F}^{rot}$ are the mass matrix, linear stiffness matrix, initial displacement matrix (it is original definition of nonlinear displacement of the blade), residual matrix (is original definition representing stiffness matrix after local crossing of yields stress in the

[1] CSD is point-blank analogy to CFD (Computational Fluid Dynamics). Both methods use the same balance equations (mass, momentum and energy). Discretization method for the CSD and CFD is similar as the equations are identical. This architecture for solving equations greatly simplifies FSI the analyses (fluid solid interaction) as well as T-FSI (thermal-fluid solid interaction). The name (CSD), was developed 25 years ago of the foundation of the Energy Conversion Department IFFM PAS.

material blade) and matrix of rotational stiffness, respectively [15], defined as follows:

$$\mathbf{M} = \iiint\limits_{V_e} \mathbf{N}^T \rho \mathbf{N} dV \ , \tag{6}$$

$$\mathbf{K} = \iiint\limits_{V_e} (\mathbf{N}^T \mathbf{B}^T \mathbf{D} \mathbf{B}_{NL} \mathbf{N} + \mathbf{N}^T \mathbf{B}_{NL}^T \mathbf{D} \mathbf{B}_{NL} \mathbf{N} + \mathbf{N}^T \mathbf{B}_{NL}^T \mathbf{D} \mathbf{B} \mathbf{N}) dV \ , \tag{7}$$

$$\mathbf{K}^R = \iiint\limits_{V_e} \mathbf{N}^T \mathbf{T}^T \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ & \sigma_{yy} & \sigma_{yz} \\ sym & & \sigma_{zz} \end{bmatrix} \mathbf{T} \mathbf{N} dV \ , \tag{8}$$

$$\mathbf{K}^\Omega = \rho \iiint\limits_{V_e} \mathbf{N}^\mathrm{T} \mathbf{G}_s^\mathrm{T} \mathbf{A} \mathbf{G}_s \mathbf{N} dV \ , \tag{9}$$

where $\mathbf{B}$ and $\mathbf{B}_{NL}$ are linear and nonlinear strain-displacement transformations matrices[2], $\mathbf{D}$ is the material properties matrix, $\mathbf{T}$ is matrix of coordinates, $\mathbf{A}$ is angular velocity matrix, and $\mathbf{G}_s$ is geometrical coordination of matrix, ρ is density of blade and disk material:

$$\mathbf{A} = \begin{bmatrix} \Omega_y^2 + \Omega_z^2 & -\Omega_x \Omega_y & -\Omega_y \Omega_z \\ & \Omega_x^2 + \Omega_z^2 & -\Omega_y \Omega_z \\ sym & & \Omega_x^2 + \Omega_y^2 \end{bmatrix} , \tag{10}$$

$$\mathbf{G}_s = \begin{bmatrix} r_{xx} & r_{xy} & r_{xz} \\ & r_{yy} & r_{yz} \\ sym & & r_{zz} \end{bmatrix} , \tag{11}$$

$$\mathbf{B} = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ 0 & 0 & \partial_z \\ 0 & \partial_z & \partial_y \\ \partial_z & 0 & \partial_x \\ \partial_y & \partial_x & 0 \end{bmatrix} , \tag{12}$$

where $\Omega_x, \Omega_y, \Omega_z$ are the rotation velocity of the direction x, y, z and r_{xx}, r_{xy}, r_{xz} are the radial dimension, respectively. The expresion of the centrifugal load is

[2] Fundamental relationships are defined by formulas:

$$\mathbf{u} = \mathbf{N}\mathbf{q}, \ \ \varepsilon = \mathbf{B}\mathbf{N}\mathbf{q}, \ \ \sigma = \mathbf{D}\mathbf{B}\mathbf{N}\mathbf{q}$$

where $\mathbf{N}$ is the shape function matrix. We begin writing triple integrals denoted as $\iiint dV$ to distinguish them from single integrals $\int dV$.

defined by

$$\mathbf{F}^{rot} = \rho \iiint\limits_{V_e} \mathbf{G}_s^{\mathrm{T}} = \mathbf{A} \left\{ \begin{array}{c} x \\ y \\ z \end{array} \right\} dV \ , \tag{13}$$

where V_e is volume of the machine element. When the Coriolis and Euler effects are neglected Eq. (13) is identical to the law of the moment balance in a conservative form (4). Equation (4) is formed by the basic updated Lagrangian description, and is solved using the Newton-Raphson modified procedure [1,9,16,17].

3 Results of analysis

In Fig. 3 presented have been natural modes for the analyzed control stage of the turbine for rotational speed 10400 rpm, given by 5th, 11th, 12th and 13th mode. These modes have been excited by non-stationary load posted Campbell diagram (see circle in height 13th natural frequency $\omega 13$ calculated by the control stage) and presented in Tab. 1 (see coloring cells).

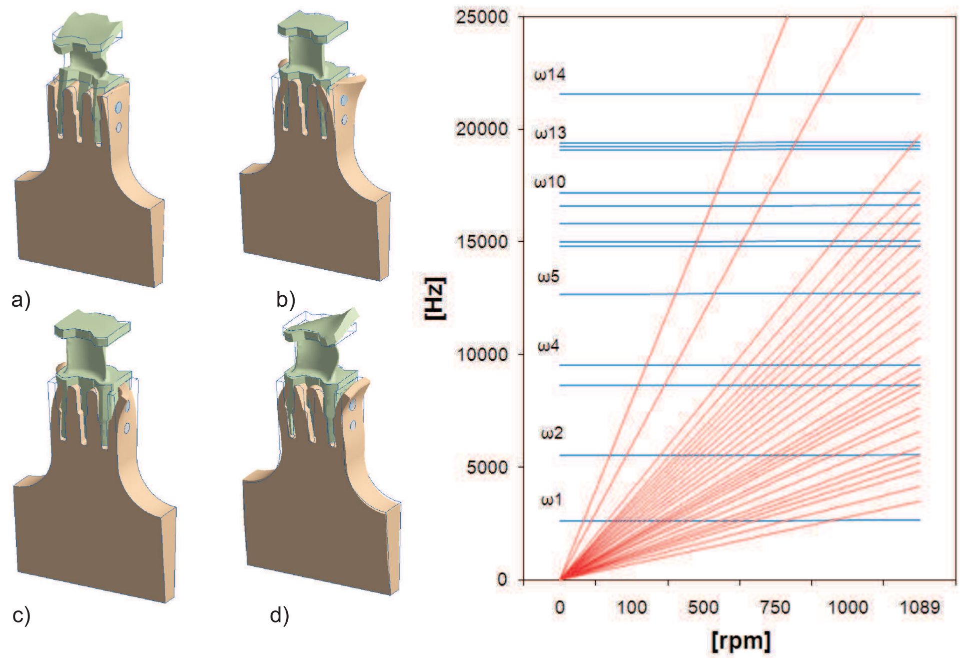

Figure 3. Normal modes for control stage vibrations of rotation speed 10400 rpm a) 5th, b) 11th, c) 12th, d) 13th, and Campbell diagram for the control stage (see in right).

Potential resonance will occur at a point by excitation load line crossing the natural frequency line of the analyzed element (see circle in the Campbell diagram). For the Computational Solid Dynamics harmonic analysis (see Fig. 5) these are two potential points of resonance which can be unduced by the non-stationary load. First, when excitation load is operating with the amplitude of 1274 Hz, and a second point, when working at 19120 Hz. The first value of excitation load potential excites the 5th natural mode of the control stage whereas the second invariant excites the 11th natural mode.

In Tab.1 have been presented natural frequencies (see the last column in table) control stage and frequencies of load excitation (see columns 2–6) potentially determining operation of the control stage by resonance. Results have been calculated using 0D method postulated by [10,12,13].

Table 1. Frequency of excitation load, potential determining operation of the control stage by resonance (calculated 0D) and natural frequency control stage, calculated by classical methods [10,12,13].

Mode No.	Frequency of excitation [Hz]					Natural frequency [Hz]
	Angular velocity [rad/s]					[rad/s]
	0	100	500	750	1089	0
1	2620	2620.99	2623.13	2625.863	2630	9440
2	5529.528	5529.644	5532.424	5536.041	5542	10540
3	8612.745	8612.754	8616.344	8621.016	8628	11420
4	9508.665	9508.649	9508.257	9507.748	9508	12080
5	12684.586	12684.9	12692.44	12702.245	12716	12740
6	14791.08	14791.212	14794.18	14798.041	14804	13620
7	15008.95	15009.152	15013.16	15018.373	15026	14940
8	15806.73	15806.869	15810.239	15814.623	15821	15600
9	16596.34	16596.494	16600.053	16604.66	16612	16260
10	17186.532	17186.539	17186.704	17186.925	17187	17580
11	19081.77	19082.25	19093.76	19108.56	19129	19120
12	19241.74	19242.92	19246.22	19251.84	19256	29740

Maximal principal stresses by 5th mode operating in resonance is 913 MPa and -74 MPa for 11th natural mode. Maximal principal stresses generated by nominal rotational speed 10400 rpm is only 274 MPa. It is consistent with the other reported articles and engineering handbook [4,5,12–15]. Extremes of stresses are in the locations close to the fixed blade. Such situation in short timescale must be leading to the catastrophe and destruction of the machine as a result of high cyclic fatigue. For the simulation which does not include of two additional stiffness matrices ($\mathbf{K}^{NL}$ representing initial displacement stiffness matrix, and $\mathbf{K}^{R}$ representing residual stiffness matrix after local crossing yields stress in the material) the blade in stage operates in safe point on the Campbell diagram.

4 Conclusions

In this article presented has been analysis devoted to location of the areas of potential resonance in the control stage for high speed turbine. The methodology of computational solid dynamics (CSD is the name created 25 yearsh ago by prof. Janusz Badur) has been used based on equation balance of mass, momentum and energy, resolved simultaneously [1,10,15].

An original method for this study uses the stiffness matrix representing the initial displacement blade (nominal shape blade obtained by nominal working point of the turbine) and residual stiffness matrix after local crossing of the plastic strain yield. For the simulation which does not include two additional stiffness matrices for the blade in stage, the operation in a safe point on the Campbell diagram is obtained.

Acknowledgement This research work was supported by the "InnoDoktorant Scholarships for PhD students, IVth edition". Project is co-financed by the European Union within the European Social Fund.

Received 14 April 2012

References

[1] Badur J.: *Five Lectures of Contemporary Fluid Thermomechanics*, 2nd Edn. IFFM PAS Gdańsk 2005 (in Polish).

[2] Gawroński W.: *Advanced Structural Dynamics and Active Control of Structures.* Springer 2004.

[3] Gawroński W. *et al.*: *The Finite Element Method in Dynamics Structures.* Arkady, Warsaw 1984.

[4] Gwo-Chung T.: *Rotating vibration behavior of the turbine blades witch different groups of blades.* J. Sound Vib. **271**(2004), 547–575.

[5] Hushmandi N.B.: *Unsteady forces of rotor blades in full and partial admission turbines.* J. Turbomach. **133**(2011), 1–12.

[6] Kaliński K.J.: *Chatter vibration surveillance by the optimal linear spindle speed control.* Mech. Syst. Sig. Proc. **25**(2011), 383–399.

[7] Kaliński K.J.: *The finite element method application to linear closed loop steady system vibration analysis*. Int. J. Mech. Sci. **39**(1997), 315–330.

[8] Kaliński K.J.: *The finite element method in the closed loop mechanical systems vibrations analysis*. PhD thesis, Gdańsk University of Technology, Gdansk 1998 (in Polish).

[9] Kleiber M.: *Handbook of Computational Solid Mechanics*. Springer 1998.

[10] Knitter D.: *Issues of inlet/outlet adaptation problem of the steam turbine to the new operation condition*. PhD thesis IFFM PAS, Gdańsk 2007 (in Polish).

[11] Lampart P.: *Aerodynamics research into blading systems used by Computational Fluid Dynamics methods*. IFFM PAS 544/1503/2006, Gdańsk 2006 (in Polish).

[12] Perycz S.: *Steam and Gas Turbines*. Gdańsk University of Technology, Gdańsk 1988 (in Polish).

[13] Rao J.S.: *Turbine Blade Vibration*. Wiley Eastern Ltd, New Delhi 1991.

[14] Richter Ch.H.: *Structural design of modern steam turbine blades using ADINATM*. Comput. Struct. **81**(2003), 919–927.

[15] Sławiński D., Zakrzewski W., Badur J.: *Strength analysis by means of CSD for the control stage of steam turbine 100 MW*. Rep. IFFM PAS 67/2012, Gdańsk 2012 (in Polish).

[16] *Theory of Reverence for the Mechanical APDL and Mechanical Applications*. Ansys Inc. Canonsburg, PA. Release 12.0, April 2009.

[17] Zienkiewicz O.C.: *Finite Element Method*, Vol. I,II,III. Elsevier 2005.