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DETECTION AND LOCALIZATION OF DELAMINATIONS IN COMPOSITE BEAMS USING FRACTIONAL B-SPLINE WAVELETS WITH OPTIMIZED PARAMETERS

DETEKCJA I LOKALIZACJA ROZWARSTWIEŃ W KOMPOZYTOWYCH BELKACH Z WYKORZYSTANIEM B-SPLAJNOWYCH FALEK UŁAMKOWYCH Z OPTYMALIZOWANYMI PARAMETRAMI*

In this paper the method of detection and localization of delaminations in composite layered beams using an algorithm based on the wavelet transform of bending modal shapes of beams was presented. For the basis functions the fractional B-spline wavelets with single- and multi-objective optimization of the values of their parameters were selected. The analysis was carried out basing on the results of numerical simulations. Several cases of the delaminations occurrence with respect to their location on the thickness of the beams, different sizes and geometrical features, were analyzed. Results of the conducted analyzes show the high effectiveness of a method in the task of detection of delaminations and a possibility of its application in industrial conditions.

Keywords: *detection and localization of delaminations, layered composites, fractional B-spline wavelets, optimization.*

W pracy przedstawiono metodę detekcji i lokalizacji rozwarstwień w kompozytowych belkach warstwowych z wykorzystaniem algorytmu opartego na transformacji falkowej giętnych postaci własnych drgań belek. Jako funkcje bazowe zastosowano ułamkowe falki B-splajnowe z jedno- i wielokryterialną optymalizacją wartości ich parametrów. Analizę przeprowadzono na wynikach obliczeń numerycznych. Przeanalizowano przypadki występowania rozwarstwień w różnej lokalizacji na grubości płyt oraz przypadki z rozwarstwieniami o różnych rozmiarach i postaciach geometrycznych. Wyniki przeprowadzonych analiz wykazały wysoką skuteczność metody w wykrywaniu rozwarstwień i możliwość jej zastosowania w warunkach przemysłowych.

Słowa kluczowe: *detekcja i lokalizacja rozwarstwień, kompozyty warstwowe, ułamkowe falki B-splajnowe, optymalizacja.*

1. Introduction

Application of layered composites as a constructional material in the elements of engineering constructions became currently common and popular thanks to a possibility of significant reduction of the mass of elements with simultaneous retaining of strength properties, great elasticity in design of such constructions, the possibility of integration of control elements and actuators into the structure, etc. For this reason there occurred a necessity of development of diagnostic methods dedicated for these materials. Some of the methods and techniques of structural diagnostics dedicated for the homogeneous materials also found an application in the case of composite materials, however for the composites diagnosis these methods need to be developed considering the possibility of diagnosing of new types of damages, which have not occurred in the homogeneous materials, e.g. delaminations, cracks of armed fibers, interphase decohesion, etc. Furthermore, these methods should be non-invasive, they should be highly sensitive to the damages, resistant to the external influences, low-cost of carrying out the research and easiness of their application in the industrial conditions.

Among the methods applied in the structural diagnostics of polymeric composites the next groups should be mentioned: interferometric, radiologic, thermographic methods and others, which require however the usage of advanced measurement devices for carrying out the research, which often limits their applicability to the laboratory conditions. Ones of the dynamically developed methods are the methods based on the analysis of modal shapes of vibration or deflection profiles with using of advanced signal processing techniques. One of such techniques, actively developed in the problems of structural diagnostics, is the wavelet transform. This technique gains a great popularity thanks to the high sensitivity to abrupt changes in the signal, and furthermore the possibility of selection of basic and scaling functions depending on the type of detected changes in the analyzed signal.

The methods of cracks detection in beams using the wavelet transform have been analyzed by many researchers. An attention should be paid to the studies [3, 14], which describe an algorithm of cracks detection in beams based on modal shapes with use of continuous wavelet transform (CWT) and symlets applied as the basis functions. The authors of [24] also used CWT and symlets of order 4 for detection of cracks in simply-supported beams. In the case of application of the wavelet transform in the problems of structural diagnostics the crucial importance has the selection of wavelet, which is used for the transform. The authors of [19] presented the problem of an identification of cracks using CWT and carried out a comparative analysis of wavelets, which shows that the most efficient wavelets in the problems of structural diagnostics are the Gabor wavelets. Another approach was presented in [16, 20], where the wavelet analysis was carried out basing on the static deflection profiles of beams. An alternative method developed by the first author of this paper [8-10] is the method based on the discrete wavelet transform (DWT), which allows to signifi-

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** cantly reduce the computation time and is characterized by the highest accuracy with respect to other types of wavelet transforms, which was conducted during the comparative study presented in [12]. The application of DWT imposes several limitations on wavelets, which could be applied during the analysis. These wavelets should consist a compact support and fulfill the conditions of orthogonality, which eliminates a possibility of application of Gabor wavelets described as the most effective in the considered problems [19]. However, an application of B-spline wavelets gives a possibility of obtaining of good results as well due to their convergence to the Gabor wavelets, especially in the case of higher order B-spline wavelets [22].

Detection and localization of delaminations in composite beams constitute the more difficult problem in comparison with cracks detection and localization. It is caused by the occurrence of non-monotonicities only on the boundaries of delaminations, which makes difficult the process of identification of these damages. Only the few studies could be found in the literature, which describe research concerned with detection and localization of delaminations in composite structures. In [18] the authors described a method of delaminations localization basing on analysis of vibration signals. The authors of a study [7] presented results of the application of the Haar wavelet for detection of delaminations in composite beams. However, the vibration-based methods are characterized by poor detectability of such damages, which confirmed the authors of [25].

Another important aspect, on which the attention should be paid, is that the diagnostic methods based on the wavelet transform require an appropriate selection of the parameters of applied wavelets. Several interesting studies, which describe this problem should be highlighted. The authors of [4] proposed a method of multi-objective optimization with application of the evolutionary algorithm for searching the parameters of wavelets applied in the analysis of ECG signals in order to increase an accuracy of the signal description. Rafiee et al. [17] show the method of application of artificial neural networks and genetic algorithms for the selection of the best wavelet functions in order to improve the results of diagnostics of the gearbox faults. This problem also was a topic of interesting studies in the area of selection of wavelets parameters applied for filtering of signals [6] and image processing [5]. From the other hand, the lack of research studies in the area of application of optimization methods for selection of parameters of fractional B-spline wavelets applied in the problem of detection and localization of delaminations in composite beams is noticeable.

In the presented paper the authors tried to solve a problem concerned with detectability of delamination due to the application of an algorithm based on DWT and the application of fractional B-spline wavelets developed by the authors of [23]. Previous results presented in [11] demonstrate the increase of the effectiveness of the method during detection of damages using fractional B-spline wavelets with respect to the B-spline wavelets of an integer order. For increasing the effectiveness of delaminations detection the algorithm was extended by adding the single- and multi-objective optimization of the parameters of fractional B-spline wavelets, which allows for automatic selection of an optimal wavelet for the considered case of a damage. Four groups of cases were analyzed in this study, which differ with respect to their shapes and location on the thickness of a beam. In particular, the influence of the shape and size of delaminations on their detectability and the variability of parameters of optimally selected wavelet was analyzed. The results of presented studies pointed out to the fact that the proposed method allows for detection and precise localization of delamination areas in composite beams.

2. Description of the method

In this section the mathematical fundamentals of the B-spline wavelets of fractional order as well as the method of detection and localization of delaminations with their application and a method of selection of parameters of the B-spline wavelets of fractional order with use of single- and multi-objective optimization will be presented.

2.1. B-spline scaling and wavelet functions of fractional order

The fundamental approach of DWT is the application of the multiresolution analysis proposed by Mallat in [15], where the B-spline scaling functions $\beta(x)$ constitute the space of square-integrable functions $L^2(\mathbf{R})$ and create the sequence of functional spaces in the form:

$$\{0\} \subset \ldots V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset L^2(\mathbf{R}). \tag{1}$$

The general form of B-spline scaling function of fractional order could be presented by the following equation [1]:

$$\beta_\tau^\alpha(x) = \sum_{k=0}^{\infty} (-1)^k \left|\begin{matrix}\alpha+1 \\ k-\tau\end{matrix}\right| \rho_\tau^\alpha(x-k), \tag{2}$$

where $\alpha \in \mathbf{R}$ is an order of a scaling function, $\tau \in \mathbf{R}$ is a shift parameter and ρ_τ^α is a function in the form:

$$\rho_\tau^\alpha(x) = -\frac{\cos \pi\tau}{2\Gamma(\alpha+1)\sin(\pi\alpha/2)}|x|^\alpha - \frac{\sin \pi\tau}{2\Gamma(\alpha+1)\cos(\pi\alpha/2)}|x|^\alpha \operatorname{sgn}(x) \tag{3}$$

where $\Gamma(\alpha+1)$ is the Euler's gamma function, which allows for fractional factorization. It should be noticed that when $\alpha \in \mathbf{Z}$ and $\tau = (\alpha+1)/2$ from the equations (2) and (3) the classic B-spline scaling functions (i.e. with an integer order) could be obtained.

Basing on the Mallat's algorithm the scaling function (2) fulfills the two-scale relation:

$$\beta_\tau^\alpha(x) = 2^{-\alpha} \sum_{k \in \mathbf{Z}} \left|\begin{matrix}\alpha+1 \\ k-\tau\end{matrix}\right| \beta_\tau^\alpha(2x-k). \tag{4}$$

The general form of the B-spline wavelets of fractional order was defined by Unser and Blu in [23]:

$$\psi_\tau^\alpha\left(\frac{x}{2}\right) = \sum_{k \in \mathbf{Z}} \frac{(-1)^k}{2^\alpha} \sum_{l \in \mathbf{Z}} \binom{\alpha+1}{l} \beta_0^{2\alpha+1}(l+k-1)\beta_\tau^\alpha(x-k). \tag{5}$$

B-spline scaling and wavelet functions of fractional order fulfill the most of properties of their analogs of an integer order. The only property, which is not fulfilled for the purposes of DWT is the existence of the non-compact support of these scaling and wavelet functions for $\alpha \notin \mathbf{Z}$. However, the algorithm of fractional discrete wavelet transform (FrDWT) proposed in [23] is based on the Fourier series and allows to avoid this problem.

2.2. Method of detection and localization of delaminations

As it is known, the process of wavelet transform could be presented as filtering with use of a set of high-pass and low-pass filters. The presented method is based on single-level decomposition (filtering) of the signals of displacements of modal shapes of beams with use of a set of filters with fractional order. Considering the two-scale relation of the B-spline scaling function of fractional order (4) the high-pass filter (scaling filter) could be defined following its pulse response in the form [1]:

$$H_\tau^\alpha\left(e^{j\omega}\right)=2^{-\alpha}\left(1+e^{j\omega}\right)^{\frac{1}{2}(\alpha+1)-\tau}\left(1+e^{-j\omega}\right)^{\frac{1}{2}(\alpha+1)+\tau}, \qquad (6)$$

the low-pass filter (wavelet filter) could be presented in the similar manner as:

$$G_\tau^\alpha\left(e^{j\omega}\right)=-e^{-j\omega}H_\tau^\alpha\left(-e^{-j\omega}\right)A^\alpha\left(-e^{j\omega}\right), \qquad (7)$$

where:

$$A^\alpha\left(e^{j\omega}\right)=\sum_{k\in Z}\beta_\tau^{2\alpha+1}(k)e^{-j\omega k}. \qquad (8)$$

The proposed algorithm is based on single-level decomposition of displacements signal of modal shapes s_n, by using (6) and (7), and then the downsampling procedure. During these operations the sets of approximation coefficients a_n and details coefficients d_n could be obtained, the length of realization of both sets is reduced twice with respect to s_n due to the downsampling operation. The information about eventual damages (non-monotonicities of a signal) is stored in the set of details coefficients. In the case of occurrence of the damages the coefficients in the location of damages gain much greater or much lower values with respect to other coefficients in the given set.

During the previous analyzes [8–10, 12] it was noted that the values of coefficients d_n are in the great dependence to the displacements values in modal shapes, i.e. in the case, when the damage is located in the node of a given modal shape its detection becomes impossible. In order to avoid of such a situation the multiple modal shapes should be considered during the analysis. Additionally, in the case when $\alpha < 2$ the details coefficients consisted of a trend, with is connected with insufficient filtering of the modal shape from a signal. In order to obtain correct detection and localization of delaminations it is necessary to apply an approximation of each of the obtained sets of details coefficients for the considered modal shapes. For increasing the detectability of the damages it is suitable to add up the absolute values of details coefficients, which eliminates the differences in signs and expose the extrema of these coefficients in the locations of damages. The scheme of the method was presented in Fig. 1.

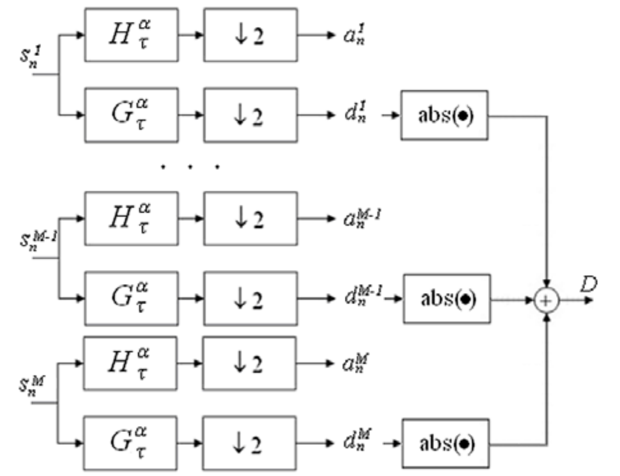

Fig.1. Scheme of the method of delaminations detection and localization

2.3. Selection of optimal parameters of wavelets

Results obtained in [13] show that the detection and localization of delaminations boundaries could be detected basing on non-monotonicities of deflection functions. As it was mentioned before, the symptoms of such a damage are the extrema of details coefficients, which occurred on the boundaries of the damaged areas. An accuracy of the proposed method of detection and localization of delaminations is strongly dependent on the appropriate selection of the B-spline scaling (2) and wavelet (5) functions of fractional order. Due to this reason it is necessary to determine optimal values of parameters of these functions. The optimization problem in this case could be reduced to the searching problem of such values of parameters α, τ, for which the multi-objective fitness function **F** in the general case reaches the minimum value. Assuming that the criteria of the fitness function are not in conflict each other the optimization problem could be formulated as follows:

$$\min_{\alpha,\tau}\mathbf{F}(\alpha,\tau)=\left[f_1(\alpha,\tau)\quad f_2(\alpha,\tau)\right]^T, \qquad (9)$$

where $\alpha, \tau \in R$, $0 < \alpha < \alpha_c$, $1 < \tau < \tau_c$, α_c and τ_c are the upper boundaries of the parameters. It should be noticed that the lower boundary value of a parameter α should be greater than -0.5 [23]. However, as the introductory studies [11] show the range of $-0.5 < \alpha \le 0$ is not suitable for the investigated problem.

Considering the above-presented assumptions, the first optimization criterion could be defined as a proportional value to the inverse of sum of maximal values of the details coefficients:

$$f_1=\left[1+\sum_{k=1}^{M}\sum_{i=1}^{2H}\max\left(D_i^k\right)\right]^{-1}, \qquad (10)$$

where H is the number of delaminations, M is the number of considered modal shapes, the details coefficients in a set $D_1^k = D^k$ and $D_i^k = D_{i-1}^k \setminus \hat{d}_{i-1}^k, i \neq 1$ for $\hat{d}_{i-1}^k$ is a point in the set of details values D_{i-1}^k, for which the maximal value of the details coefficient was obtained. The second criterion could be defined basing on a measure defined for the rest of values of details coefficients in a set D^k. Such a criterion could be presented as follows:

$$f_2=\sum_{k=1}^{M}\sum_{i=1}^{\left|\hat{D}^k\right|}d_i^k, \qquad (11)$$

where the following relation occurs: $\hat{D}^k = D^k \setminus \hat{d}_1^k \setminus \hat{d}_2^k \cdots \setminus \hat{d}_{2H}^k$.

In the general case the problems of multi-objective optimization do not reach a single global solution. Therefore, it is justified to consider a set of solutions, which fulfill the boundary conditions and optimization criteria. In the following study the problem of multi-objective optimization was solved by searching an optimum of the fitness function in the Pareto sense. The solution is Pareto-optimal, if there is no another solution, which could improve at least one of the criteria and simultaneously will not make worst the other criteria. Such a solution is considered as non-dominated one.

Often in the practical applications the problem of multi-objective optimization is reduced to the one-dimensional problem. In this study such a cases was also considered. For this purpose the scalar fitness function in the form of metacriterion created basing on weighted product of the elementary criteria:

$$U = \left[1 + \sum_{k=1}^{M} \sum_{i=1}^{2H} \max\left(D_i^k\right) \right]^{-w_1} \cdot \left(\sum_{k=1}^{M} \sum_{i=1}^{\left|\hat{D}^k\right|} d_i^k \right)^{w_2}, \qquad (12)$$

where $w_{\{1,2\}} \in \langle 0,1 \rangle$ denote the weights of the particular criteria, which values could be defined basing on experts' knowledge or by application of systematic search method.

During this study the scalar fitness function, defined based on the criterion presented in [11], was also used:

$$U_\mu = \left[1 + \sum_{k=1}^{M} \sum_{i=1}^{2H} \max\left(D_i^k\right) \right]^{-w_1} \cdot \sum_{k=1}^{M} \mu_{1/2}^{w_2}\left(\hat{D}^k\right), \qquad (13)$$

where $\mu_{1/2}\left(\hat{D}^k\right)$ denotes median achieved for a set of the rest of details coefficients. Moreover, for the fitness function in the form (12) an additional decision variable was introduced, which represents an information about the modal shapes considered during the determination of its value.

In the following study the searching for the optimal solution (minimization of the fitness function) was realized by application of evolutionary algorithms [2, 21]. The classic optimization methods (e.g. gradient-based methods) could not be adapted in this context mainly because of the form of the criterion (10), which leads to the discontinuity of the fitness function. For this reason the multi-objective optimization process was carried out using evolutionary algorithm with sorting of non-dominated solutions [2]. Whereas in the case of the scalar fitness function the classic evolutionary algorithm was applied [21].

3. Procedure and results of detection and localization of delaminations

The analysis of detection and localization of delaminations in composite beams was carried out on the simulation data obtained from numerical models prepared with use of the finite element method. Four groups of cases were considered: delamination on the large area along the length and through-the-width of a beam (symbol 'l'), delamination on the large area along the length and on the limited area along the width (symbol 'il') and the same cases for the delaminations on the small area along the length (symbols 'sl' and 'isl'). Each of the considered groups of delaminations consisted of 11 cases: the delamination was modeled between the layers of 12-layered laminate. The schemes of considered groups were presented in Fig. 2.

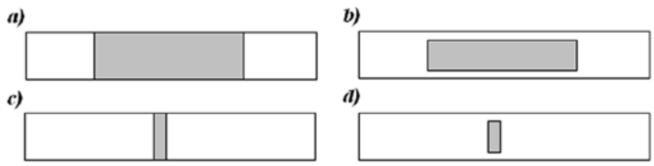

Fig. 2. Considered groups of delamination cases

3.1. Simulation data preparation

The numerical models of composite cantilever beams were prepared in the commercial software MSC Marc/Mentat. In order to simulate delaminations correctly, the beams were modeled as a three-dimensional ones with the following dimensions: length x – 200 mm, width w – 10 mm and a thickness h – 2.4 mm. The laminate consisted of 12 orthotropic layers with an equal thickness made of epoxy resin reinforced by a carbon cloth with the following material properties: Young moduli – $E_{11} = 82$ GPa, $E_{22} = 82$ GPa, $E_{33} = 8.5$ GPa; Kirchhoff moduli – $G_{12} = 5.2$ GPa, $G_{23} = 3.05$ GPa, $G_{31} = 3.47$ GPa and Poisson ratios – $\nu_{12} = 0.312$, $\nu_{23} = 0.29$, $\nu_{31} = 0.27$. The applied lay-up of the laminate is defined by the following structural formula: $[0/60/-60]_{2S}$. For the geometric models of beams the mesh of finite elements was defined. There were used hexagonal 8-node elements with the following dimensions in particular directions: 127 elements along the length – in order to fulfill the dyadic criterion of FrWT (see [23]), 5 elements along the width and 12 elements in the thickness direction. The ideal contact constraints were defined between the layers.

The delaminations were modeled by deactivation of the contact constraints in the selected regions. The locations of delaminations in the considered cases are as follows: for the groups 'l' and 'il': $x_1^0 = 85$ mm, $x_2^0 = 137$ mm and for thee groups 'sl' and 'isl': $x_1^0 = 95$ mm, $x_2^0 = 105$ mm. The numerical analysis was carried out for determination of the modal shapes of the beams. First five bending modal shapes were selected for further analysis. The displacements of these modal shapes were quantified along the length and in the half-width of the beams.

3.2. Selection of parameters of optimization algorithms

The application of an evolutionary algorithm for searching the optimal parameters of wavelets is connected with a necessity of definition of its fundamental properties. In the following study the process of single- and multi-objective optimization was carried out using the MATLAB® environment and the Genetic Algorithm Toolbox. The selection of algorithm parameters was realized following the requirements suggested in the literature [21]. The fitness function was declared basing on the criteria (10) and (11) for the case of multi-objective optimization and basing on metacriterion (12) or (13) for the one-dimensional case. Considering the results obtained in the previous study [10], the upper boundaries of the variability of the wavelet's parameters were defined as $\alpha_c = \tau_c = 18$. For the both cases the real-number coding of the individuals in a population was assumed, where the genes in particular chromosomes represented the wavelets parameters. The initial population was selected randomly (with the uniform distribution) with taking into consideration the assumed boundaries. The ranking method was applied for scaling the fitness function whereas the uniform stochastic selection method was employed for selection of the parents, which create the new individuals for the next initial population. The reproduction method was carried out basing on the elite succession (the number of individuals was 2) and the operators of crossover and mutation. The crossover function was realized using the heuristic method, where the descendant individual is created as a linear combination of genes of the parental individuals (the multiplication factor of the individual of the better fitness value was $\lambda = 1.2$). It was assumed that the crossover operation will be realized with the probability p_c. Remained parental individuals were processed using the adaptive mutation method, where the diminishing of a probability of genes mutation in chromosomes is dependent on the event, if in the last epoch the improvement of the fitness function took place.

In order to determine appropriate values of the crossover probability p_c and the number of individuals in a population N the results of research in the range of convergence of the evolutionary algorithm presented in [13] were used. The study was based on the systematic search of the combinations of values of these parameters (p_c = {0.4, 0.5, …, 1} and N = {5, 10, 20, 30}). Finally, the following values were assumed: $p_c = 0.6$ i $N = 30$, for which the lowest averaged values of

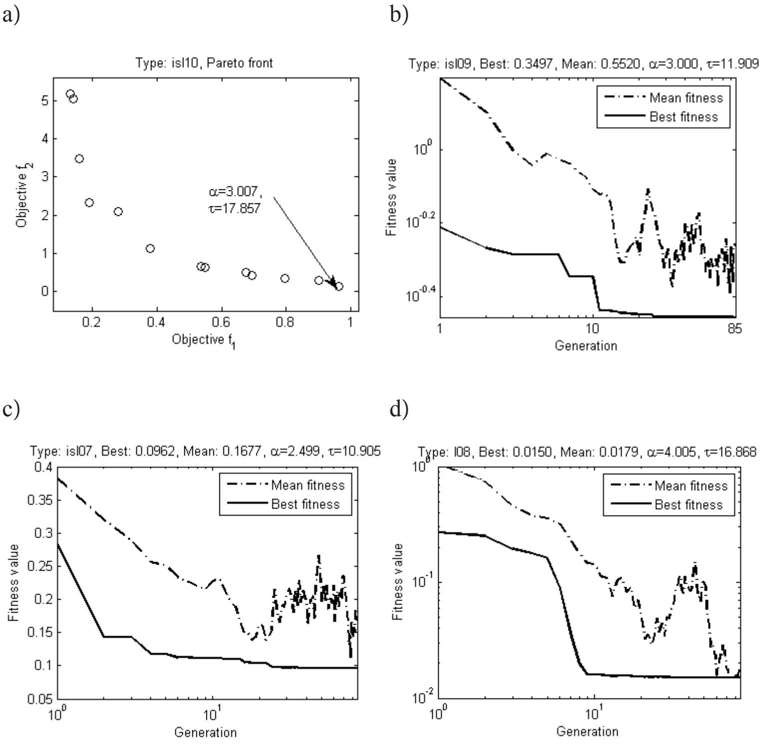

Fig.3. Exemplary results of multi-objective (a) and single-objective (b-d) optimization

the fitness function with the minimal standard deviation of the results were obtained.

Fig. 3 presents the selected results of optimization, which was performed following various strategies. The first case (Fig. 3a) shows the optimal solution in the Pareto sense. The optimal parameters of a wavelet ($\alpha = 3.007$, $\tau = 17.857$) were selected for a case, for which the minimal value of the noise and simultaneously the maximal possible value of the magnitude of peaks (coefficients) on the boundaries of the delamination were obtained. Fig. 3b-d present the results of single-objective optimization which was carried out for the fitness function: b) in the form of metacriterion (12) with the weights of $w_1 = w_2 = 0.5$; c) similarly as in the previous case, but only for the second modal shape; d) in the form of metacriterion (13). In the further part of the paper the detailed comments concerned these cases will be presented.

3.3. Analysis of results of localization of delaminations

The analysis of effectiveness of the delamination localization was carried out with use of the decomposition algorithm described in the section 2.2 and using the algorithm of optimal selection of parameters of B-spline wavelets of fractional order presented in the section 2.3. During the analysis the strategies of multi-objective optimization following the criteria (10) and (12) and single-objective optimization following the metacriterion (12) were considered. Moreover, the comparative studies for the metacriterion (13), which also was a topic of research in [11], were carried out. The results of detection and localization of delaminations for the cases of the single- and multi-objective optimization were comparable. Considering a great capability of single-objective optimization in the practical application further the results obtained for this case were described in detail.

The evaluation of results of detection and localization of delaminations was realized basing on the two measures described as follows:

$$m_{diff} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{abs}\left(Diff_i\right) = \frac{1}{n}\sum_{i=1}^{n} \mathrm{abs}\left[\left(x_2^0 - x_1^0\right) - \left(x_2^{opt} - x_1^{opt}\right)\right]_i, \quad (9)$$

$$m_{cd} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{abs}\left(Err_i\right) = \frac{1}{n}\sum_{i=1}^{n} \mathrm{abs}\left[\frac{1}{2}\left(x_2^0 - x_1^0\right) - \frac{1}{2}\left(x_2^{opt} - x_1^{opt}\right)\right]_i, \quad (10)$$

where m_{diff} describes the mean absolute value from the difference of real positions of the boundaries of the delamination x_1^0 i x_2^0 and the locations of the boundaries of a delamination detected by the proposed algorithm x_1^{opt} i x_2^{opt}; m_{cd} describes in the same way the mean value of the deviation of geometric center of the real and detected delamination. Obtained results for the four considered groups of damages were stored in Table 1. In this table the data for the locations of delaminations between various layers with enumeration from the bottom of beams was presented, i.e. the case 1 denotes the delamination between the layers 1 and 2, the case 2 – between the layers 2 and 3. In order to visualize the results from each group two representative cases were selected. They were presented in Fig. 4.

As it could be noticed, the results of delaminations localization presented in Fig. 4 in many cases are close to the real locations of the delaminations. In many considered cases, especially in the groups ‘sl’ and ‘isl’, the order of a wavelet is integer, however there is no characteristic relation between α and τ for the B-spline wavelets of the integer order: $\tau = (\alpha + 1)/2$. The tabulation of the values of wavelets parameters for the considered cases was presented in Table 2.

Basing on the obtained results one could conclude that the optimal order of the wavelet is consisted in the range of $\alpha \in [2,4]$. This fact is confirmed by the results of previous studies presented in [11]. The occurrence of the α value in this range is justified by the lowest number of moments of wavelets and their shortest support, which have an influence on the quicker decay of energy of a wavelet from the center to the boundaries of the support. This allows for minimization

Table 1. Averaged measures of sensitivity obtained during optimization following the metactriterion (12) for the considered locations of delaminations

Symbol of a group	‘l’		‘il’		‘sl’		‘isl’	
No. of a case	m_{diff}, mm	m_{cd}, mm	m_{diff}, mm	m_{cd}, mm	m_{diff}, mm	m_{cd}, mm	m_{diff}, mm	m_{cd}, mm
1	20.0	10.0	38.6	19.3	25.2	12.6	4.5	2.2
2	21.5	10.7	23.5	11.7	25.7	12.9	20.0	10.0
3	6.2	3.1	18.9	9.5	26.2	13.1	30.6	15.3
4	23.1	11.5	18.1	9.0	24.9	12.4	30.5	15.2
5	27.7	11.8	29.7	14.9	23.2	11.6	32.2	16.1
6	19.3	9.7	31.5	15.8	27.4	13.7	33.4	16.7
7	18.8	9.4	24.7	12.3	26.1	13.0	31.4	15.7
8	17.5	8.7	28.8	14.4	24.1	12.0	35.4	17.7
9	18.9	9.4	12.4	6.2	25.7	12.9	36.1	18.1
10	29.5	14.8	16.3	8.1	26.6	13.3	38.1	19.0
11	19.1	9.5	13.6	6.8	30.1	15.0	31.1	15.5

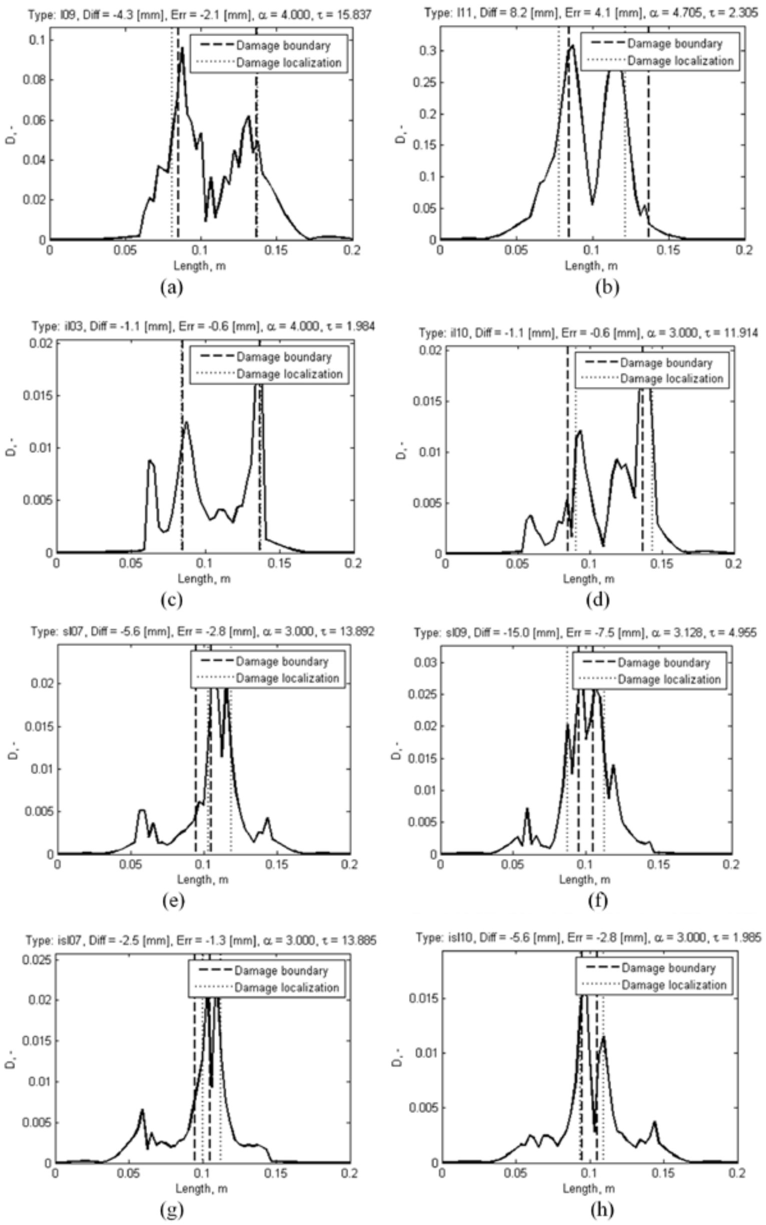

Fig. 4. Visualization of localization of delaminations for the considered groups of the cases

Table 2. Tabulation of the values of parameters of B-spline wavelets obtained during the optimization following the metacriterion (12)

Symbol of a group	'l'		'il'		'sl'		'isl'	
No. of a case	α, -	τ, -	α, -	τ, -	α, -	τ, -	α, -	τ, -
1	6.345	17.704	4.000	16.871	3.000	9.918	3.000	3.965
2	6.408	17.787	5.130	14.829	3.000	17.851	3.000	14.878
3	4.863	10.907	4.000	1.984	3.000	7.940	3.000	17.854
4	4.893	5.404	2.161	14.873	3.000	11.906	3.000	15.870
5	5.461	2.472	4.293	16.855	3.000	3.965	3.000	8.924
6	3.188	15.879	3.000	2.983	3.000	17.850	3.000	15.870
7	4.962	11.781	4.647	15.867	3.000	13.892	3.000	13.885
8	5.123	9.805	3.073	1.991	3.000	1.981	3.000	9.917
9	4.000	15.837	2.000	16.921	3.128	4.955	3.000	11.909
10	4.186	2.299	3.000	11.914	3.000	15.874	3.000	1.985
11	4.705	2.305	2.000	16.920	3.000	17.857	3.000	8.930

of disturbances near the detected boundaries of the delamination (cf. e.g. Fig. 4a and Fig. 4h).

For some of the considered cases of delaminations the incorrectly detected boundaries of the delaminations were observed. Exemplary results for these cases were presented in Fig. 5. There are two reasons which have an influence on the incorrect detection of the delaminations boundaries. As it could be observed, for the cases presented in Fig. 5a and Fig. 5b the peak values for the tight band of delamination are consisted in the range of [0.12, 0.145] of the length of a beam, however the width of this range was not sufficient for defining both of the boundaries on it and the optimization algorithm was tended to finding the additional peak value in order to fulfill the defined criterion. In the rest of the considered cases (Fig. 5c, d) in the sets of details coefficients the peak values were occurred, which exceeded the values of details coefficients of real locations of the delaminations boundaries. It was caused by the generation of the random initial population in the evolutionary algorithm, which resulted in incorrect selection of optimal parameters of the scaling and wavelet functions applied to the problem of detection and localization of delamination boundaries. It should be mentioned that the values τ for these cases achieve the highest values from the considered cases (see Tab. 2). These values cause the braking of the symmetry of a wavelet, which influences unfavorably on the obtained details coefficients [10]. In order to illustrate this phenomenon two extreme values of τ from the presented results were selected: the wavelet, whose parameters are close to the B-spline wavelet of an integer order: $\alpha = 3$, $\tau = 1.985$ (Fig. 4h) (for the wavelet of an integer order the value of τ will be equal 2) and the wavelet used in the case presented in the Fig. 5c: $\alpha = 3$, $\tau = 17.851$. The comparison of these wavelets was presented in Fig. 6.

As it was mentioned, the selection of single objective optimization, which was preformed following the metacriterion (12) was conditioned by obtaining quantitatively and qualitatively better results of detection and localization of the delaminations boundaries. Furthermore, the selection of such a method for optimization of wavelets parameters was justified by the greatest potential of this method in the further practical applications with respect to the solutions based on the multi-objective optimization. In order to compare these approaches the studies with usage of the metacriterion (13) were carried out, which was the problem of interest in [11]. During this stage of research the great attention should be paid to the most difficult detectable group of delaminations described as 'isl'. The cases 'isl7' (Fig. 4g) and 'sil10' (Fig. 4h) were selected. Obtained results of detection and localization with application of optimization following the metacriterion (13) were presented in Fig. 7.

Results presented in Fig. 7 reveal that the metacriterion (13) is not adjusted to the considered problem and justified the selection of the metacriterion (12) in the performed analyzes.

3.4. Influence of the number of considered modal shapes on the effectiveness of detection and localization of delaminations

In the presented algorithm of detection and localization of delamination boundaries first five bending modal shapes were considered. However, the selection of the modal shapes, which contained the maximal quantity of the diagnostic information about the damages will allow for increasing of a sensitivity of the method for occurred damages by

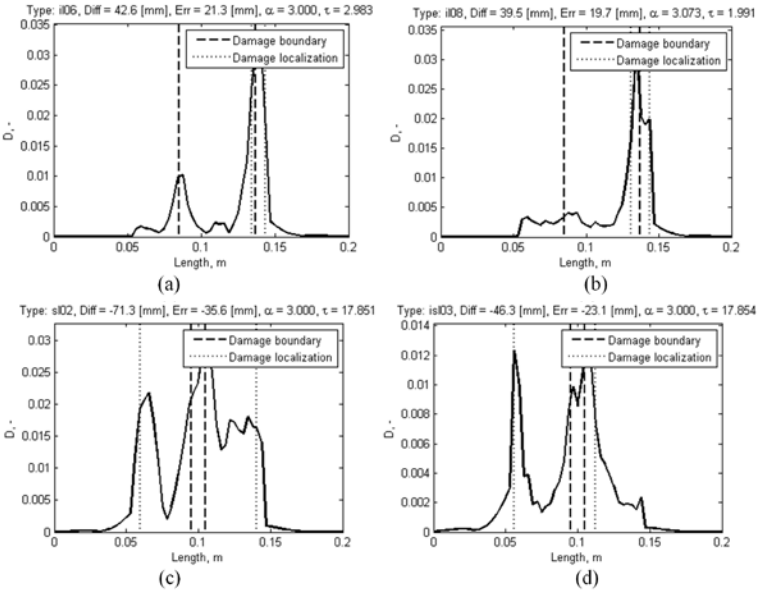

Fig. 5. Examples of incorrectly localized delaminations

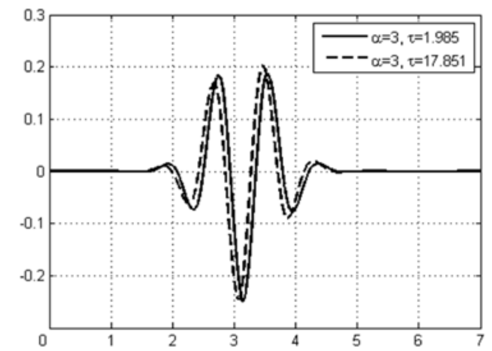

Fig. 6. Comparison of the B-spline wavelets of fractional order with different values of the shift parameter

omitting the modal shapes, which do not bring new diagnostic information and simultaneously to be the source of noise added to the resulted coefficients D. Additionally, it will allow for reduction of quantity of processed data in the case of considering of the lower number of modal shapes, which could accelerate the processing of the detection and localization algorithm.

In order to investigate the influence of considered modal shapes in the analysis the additional optimization problem following the metacriterion (12) was formulated with the additional decision variable, which represented the number and identifiers of the considered modal shapes. The weights of minimization of the noise level and searching for the maximal values of the details coefficients described as equal, i.e. $w_1 = w_2 = 0.5$. The results of analyzes for the selected cases (one for each considered group of damages – Fig. 2) were presented in Fig. 8.

Besides the defined metacriterion, which assumes the consideration of variance from all of the five modal shapes, each time only one of them was selected. The reason of such behaviour of an algorithm is the equal values of the weights of criteria concerned with minimization of the noise and searching for the maximal values of the details coefficients.

Basing on the results presented in Fig. 8 it could be observed that in the case of a delamination from the group 'l' and considering only the second modal shape the boundaries of a delamination were detected with high precision, whereas for the case from the group 'il' one of the boundaries of a delamination was detected incorrectly. In this case it was reasoned by low values of magnitude of displacements in the first modal shape selected by an optimization algorithm. In the other two cases (Fig. 8c,d) the delaminations were localized incorrectly, in each case the third modal shape was selected.

In order to obtain the cases, where the multiple modal shapes were considered it is suitable to change the ratio of weights w_1 i w_2 in a metacriterion (12), where w_1 is the weight responsible for maximization of the peak values of details coefficients on the boundaries of a delamination, while w_2 is responsible for the minimization of noise in the signal. Additionally six cases of weights ratios were considered: $w_1 = 0.8$, $w_2 = 0.2$; $w_1 = 0.2$, $w_2 = 0.8$; $w_1 = 0.6$, $w_2 = 0.4$; $w_1 = 0.55$, $w_2 = 0.45$ and two extreme cases $w_1 = 0.95$, $w_2 = 0.05$; $w_1 = 0.05$, $w_2 = 0.95$. In order to compare all of the considered cases the numbers of cases with correctly localized boundaries of delaminations m and the ranges of considered number of layers M for each of the considered groups were prepared (see Table 3). The cases, where the difference between the real and detected boundaries of delaminations was lower than 15 mm were assumed as cases with correct localization.

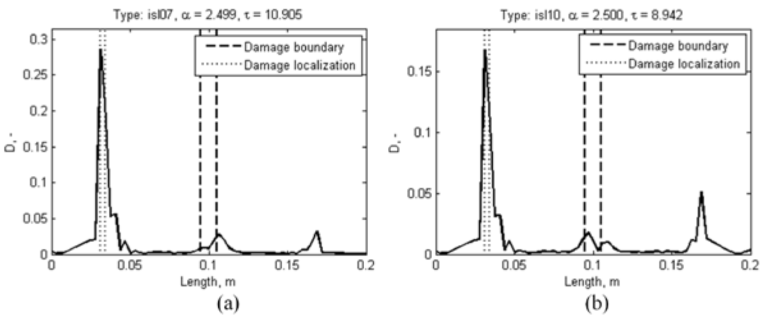

Fig. 7. Results of localization of delaminations boundaries obtained during optimization following the metacriterion (13)

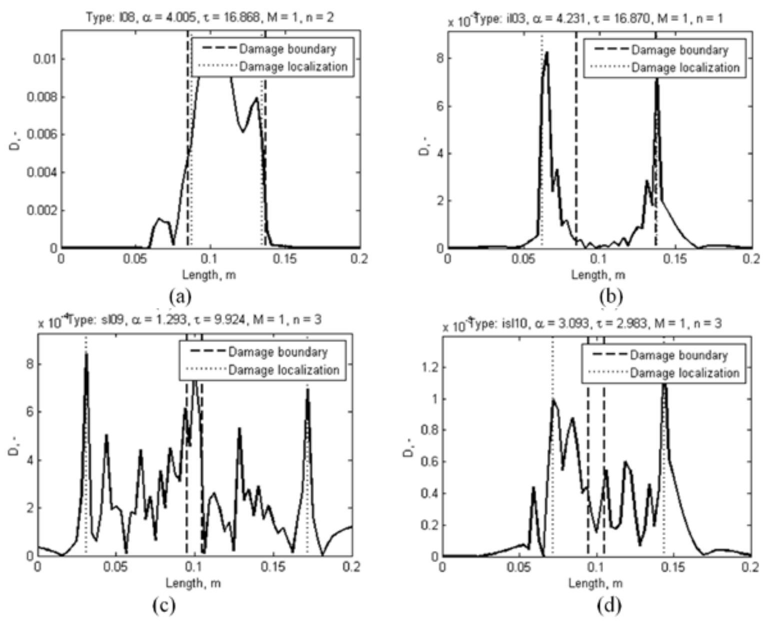

Fig. 8. Results of localization of delaminations boundaries with additional optimization of the number and enumeration of considered modal shapes

Table 3. Comparison of the effectiveness of delamination detection with respect to optimization of number of considered modal shapes and various combinations of weights in the metacriterion (12)

No. of a case	w_1, -	w_2, -	The number of cases with correct localization of delaminations and the number of considered modal shapes							
			'l'		'il'		'sl'		'isl'	
			m, -	*M*, -	*m*, -	*M*, -	*m*, -	*M*, -	*m*, -	*M*, -
1	0.5	0.5	5	1	4	1	0	1	2	1
2	0.8	0.2	4	1÷4	0	3÷5	0	3÷5	0	3÷5
3	0.2	0.8	7	1	1	1	0	1	2	1÷2
4	0.6	0.4	8	1	5	1÷2	2	1÷2	1	1÷2
5	0.55	0.45	8	1	6	1	2	1÷2	3	1÷2
6	0.95	0.05	2	3÷5	0	3÷5	0	3÷5	0	4÷5
7	0.05	0.95	9	1	6	1	1	1÷2	2	1÷2
Without optimization of the number of considered modal shapes			10	5	4	5	10	5	8	5

The results of performed analyzes show that the weight w_2, which was responsible for the noise minimization, has the much greater influence on the correct localization of the boundaries of delamination. During this process the lower number of modal shapes is considered, which was affected by an input of additional disturbances during adding each additional modal shapes considered in the analysis. Additionally, the results presented in Table 3 pointed to the difficulties of an algorithm in the detection of delaminations in the areas, which do not reach the boundaries of the specimens along the width (the groups of cases 'il' and 'isl'). The reason of such errors in the localization of the delamination boundaries for these cases is the much lower displacements in the areas of delaminations during resonant vibrations.

4. Conclusions

In the paper the new method of structural diagnostics of composite beams oriented to the detection and localization of delaminations boundaries was presented. The delaminations are the one of the most difficult damages to detect by applying the methods based on the modal analysis. The results of the studies reveal a potential of a method in the detection of such damages thanks to the application of evolutionary algorithms for single- and multi-objective optimization of parameters of B-spline wavelets of fractional order. The research results show that the method is characterized by the high effectiveness in the detection and localization of through-the-width delaminations. In the case of the internal delaminations the method reveals the lower effectiveness and the recognition of the delamination is about 50%. It was observed that the order of applied wavelets in the considered cases was relatively low and usually was located in the range of $\alpha \in [2,4]$. This is because that the optimization algorithm selected the wavelets with short supports, which in consequence generated the lower disturbances during the application of the wavelet transform. Moreover, the best results were achieved for such cases, where the value of a shift parameter τ was selected by an optimization algorithm in such a way that the wavelet tended to the symmetry with respect to the center of its support.

The discrepancies between the groups of damages in delaminations localization presented in the paper could be reduced by the development of separate optimization criteria for various shapes of delaminations, which is planned in the further studies. Furthermore, the increase of accuracy of damage detection with the use of the proposed algorithm could be achieved due to the creation of an initial population from the cases, for which the damages were detected and localized correctly in the presented analyzes (see Table 2). After performing analyzes on the simulation data it is planned to verify the method experimentally.

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