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MODELLING OF COMPOSITE ELEMENTS OF POWER TRANSMISSION SYSTEMS CONSIDERING NONLINEAR MATERIAL CHARACTERISTICS

Key words: construction of drive shaft, carbon-epoxy composite, dynamic model, critical velocity.

Abstract: Engineering calculations of elements of the power transmission systems made of high-strength carbon-epoxy and carbon-aramid composites are very difficult due to the lack of reliable data concerning non-linear features of these materials. This paper presents a proposal of building and identifying a dynamic model of bi-supported drive shaft. A properly identified mathematical model allowed predicting dynamic behaviours at various velocities of start-up and braking.

Modelowanie kompozytowych elementów przeniesienia mocy z uwzględnieniem nieliniowych charakterystyk materiałowych

Słowa kluczowe: konstrukcja wału maszynowego, kompozyt węglowy, model dynamiczny, prędkości krytyczne.

Streszczenie: Inżynierskie obliczenia elementów układów przeniesienia mocy wykonanych z wysokowytrzymałych kompozytów węglowych i węglowo-aramidowych napotykają na znaczne trudności z powodu wiarygodnych danych dotyczących nieliniowych cech materiałowych. W pracy przedstawiono propozycję budowy i identyfikacji modelu dynamicznego dwupodporowego wału maszynowego. Dobrze zidentyfikowany model matematyczny pozwolił na predykcję zachowań dynamicznych przy różnych prędkościach rozruchu (hamowania).

Introduction

Widely understood plastics currently constitute an essential group of materials applied in engineering techniques. Composite materials are especially popular in machine building. This happens due to their material features that are often impossible to obtain in metallic elements. The main, extremely attractive, from the engineering point of view, property of these materials is the possibility of achieving high strength with low mass. Thus, the natural application domain of composite materials is in aviation and naval engineering (and more precisely yachting, in which composites dominated other materials). However, these are not the only application domains. A tendency of making machine parts, including power transmission systems, of high-strength carbon-epoxy composites is currently observed. This fact creates new engineering challenges, since the actual knowledge, allowing for a comfortable designing of bearing elements contains essential gaps within the range of dynamic behaviours of high-speed systems, where applications of ultra-light materials seems attractive. It should be noticed that composite materials are anisotropic, which indicate non-linear features in elastic strains, are characterised by hysteresis at large strains and that they depend – to a large degree – on the applied technology and the shape and size of the element. This causes a lack of the well-defined similarity theory, and thus difficulties in transferring the laboratory results on ready elements of different shapes.

The design engineer needs to know which data should be used use in designing such elements. Available information can be divided into two groups. Firstly, the theory of composite structures, attempting to describe the problem starting from the micro-scale and leading to complicated dependencies (partial differential equations) [1–2], was lately developed. For the time being, this theory, which is undoubtedly increasing our knowledge, is difficult for engineering applications. Secondly, relatively numerous laboratory tests are published [3]; however because of the scale effect, they cannot be directly applied in practice. Information on how nonlinear disturbances of composite elements vibrations (often very weak) can be measured for the technical diagnostics [4] can be found much easier in the available references than analysis of the system dynamics under conditions of normal operations [5–6]. Thus, the designer can see patterns of existing structures, but the secrecy clause is valid there. The engineer who is supposed to construct a high-strength element to be subjected to dynamic forces can obtain only approximate material data, which is the most often linearized, e.g., an equivalent Young modulus [7].

Thus, every project task requires, apart preliminary approximate calculations, performing special investigation tests. The authors are of the opinion that it would be useful to have enough verified (well-identified) dynamic models to allow for a relatively easy simulation of expected behaviours under variable operation conditions.

1. Investigated object

The above problems will be discussed on the example of the high-speed rotor shaft, i.e. the main element of power transmission systems. The rotor shaft made of carbon-epoxy composite with a relatively significant mass, placed in the middle of the shaft length, was the investigated object. The assumed geometrical dimensions (length: app. 1 m, diameter: 26 mm) are large enough in relation to the composite structure that the scale problem, which is one of difficulties at transferring the generalised laboratory results on real objects, is no longer a problem. The shaft was preliminarily ‘calculated’ by the simple engineering technique for averaged linear values (equivalent Young modulus Ez = 100 GPa) as the equivalent of a steel shaft of the same indicator of the cross-section bending strength. The questions were as follows:

1. Is it possible to apply in practice, with a certain assumed safety coefficient, the method of calculating the strength and stiffness simplified to the linear form?
2. Will the linear modelling allow for the accurate estimation the object behaviour in critical states?
3. Are the widely-understood non-linear effects accompanying the shaft movement significant enough that they have an essential influence on the structure? How should these effects be modelled with the sufficient accuracy to obtain a model useful in the designing process allowing one to predict dynamic behaviours?

The broad experimental program was performed on the research set-up (Fig. 1) in the Vibroacoustics Laboratory IPBM PW. The system was accelerated and decelerated at various rates within the range exceeding the first critical velocity, and the deflection in the central point of the shaft was recorded.

First the examples of the results presented in Figure 2 will be discussed. On the left side, the process of the shaft acceleration with the velocity applied in practice at the ‘quiet’ start-up is presented. On the right side, the acceleration is maximally slowed down to obtain the result similar to the ‘static’ amplitude-frequency characteristic.

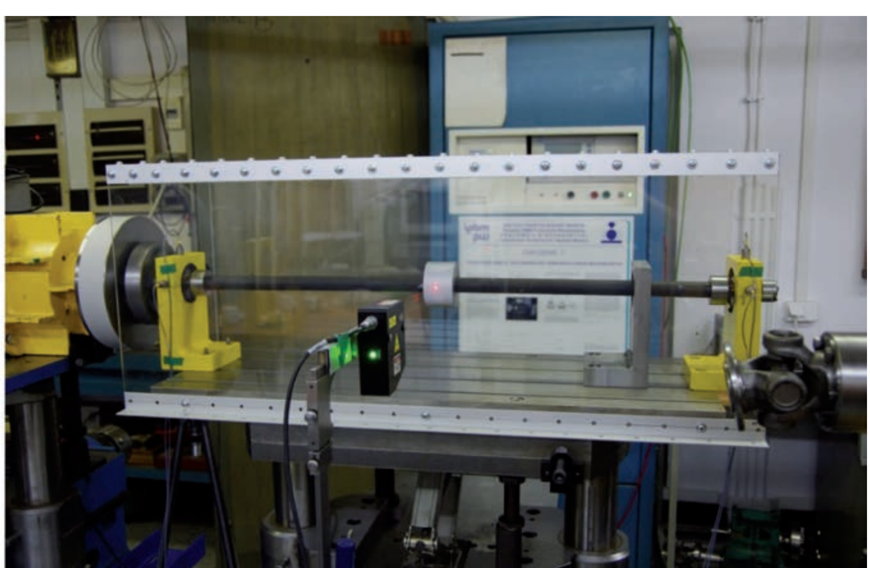


Fig. 1. Research set-up for testing critical velocities of shafts

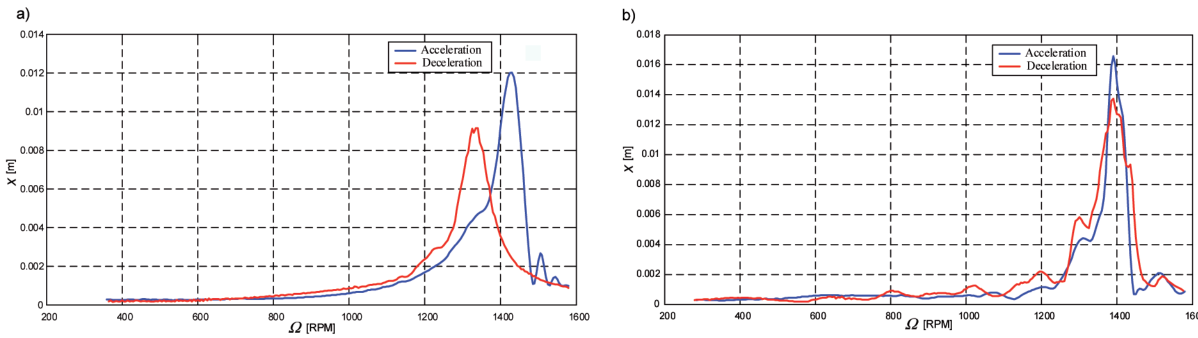


Fig. 2. Experimentally determined characteristic of the acceleration and braking of the composite shaft

As known from the vibration theory, the frequency difference at which the critical state occurs at acceleration and braking is caused by the acceleration (braking) rate, while the difference in amplitudes is mainly caused by unstable zones on the resonance curve of non-linear systems. The situation is pictorially shown in Figure 3*.

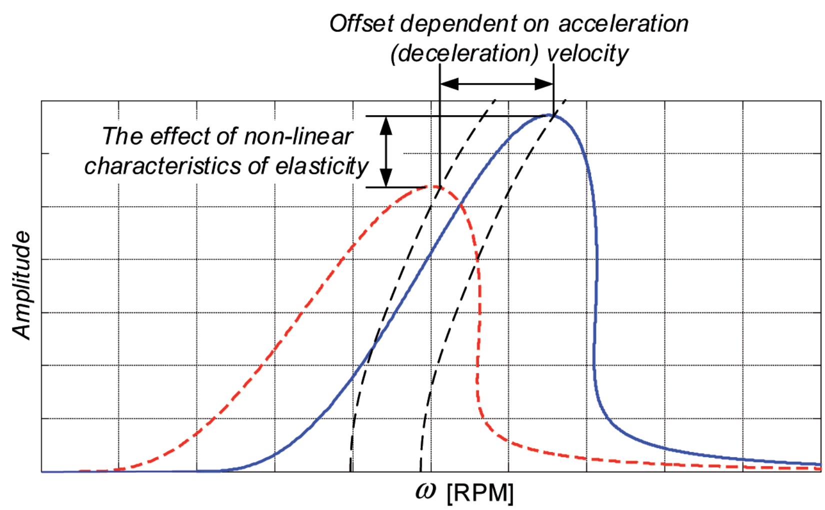


Fig. 3. Interpretation of various amplitude-frequency characteristics at accelerations and braking

Thus, the considered machine shaft has undoubtedly non-linear features [5–6, 8] and the model which will take these features sufficiently into account should be looked for.

2. Proposition of the dynamic model

The light and elastic shaft (m = ~ 0.13 kg) is loaded by the heavy drive wheel (M = 1.5 kg) in the tested system. The difference in masses causes that it is possible to apply the discrete model (concentrated mass and weightless elastic element). As it is known, at the assumption of the linear elastic characteristic and omitting the torsional rigidity, the effect can be described by the system of equations:

$$\begin{gathered} m\ddot{h} + kh = me \cdot (\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) \\ m\ddot{v} + kv = me \cdot (-\ddot{\varphi}\cos\varphi + \dot{\varphi}^2\sin\varphi) - mg \\ (J + me^2)\ddot{\varphi} = me \cdot \left(\ddot{h}\sin\varphi - \ddot{v}\cos\varphi\right) + M_1(\omega) + M_2(t) \end{gathered} \tag{1}$$

where h and v – horizontal and vertical coordinate, φ – rotation angle of the shaft.

For the need of determining the critical frequency, the constant rotational frequency $\ddot{\varphi} = 0$ is usually

* For the sake of accuracy, it should be added that this happens for relatively small acceleration (braking) rates, considered in this example. For very high velocities the amplitudes are significantly decreasing, due to the system inertia.

assumed, and the task is reduced to a rather trivial problem of equation solving:

$$m\ddot{x} + kx = me\omega^2 \sin \Omega t \tag{2}$$

where Ω = const.

In the considered case, the assumption of the equivalent Young modulus and the resulting coefficient of elasticity allows one to only ascertain the critical velocity with an error from 12% to even 50%.

As shown in paper [7], among others, the application of known rheological models does not allow for the accurate identification of pathways presented in Figure 2. Thus, it should be assumed that the non-linear characteristic of elasticity and damping are given by a certain function developing into power series as follows:

$$f(x,\dot{x}) = a_1 x + a_2 \dot{x} + a_3 x^2 + a_4 \dot{x}^2 + a_5 x\dot{x} + \cdots$$

and that the angular velocity $\dot{\varphi} = \varphi(t) + \psi(t)$,

where $\phi(t)$ means the main linear component of the angular velocity, constant in a steady motion and variable during acceleration (braking), $\psi(t)$ – is the velocity disturbance caused by an unbalance.

Taking into account these dependences in the above equations (1) will cause huge complications. Even in the simplest possible case, when the expansion will be limited to the first four terms (with omitting rectangle terms and dividing functions concerning damping and elasticity), i.e. to the form: $F_s = kx + kx^3$ and $C_x = c\dot{x} + c\dot{x}^2$ and assuming $\phi(t) = \Omega t + \psi(t)$, Ω = const the equations will be of the following form:

$$\begin{gathered}
m\ddot{h}_1 + \frac{1}{2}\Big(3k_1h_1^2 + 4k_1h_1e\cos\big(\Omega t+\psi(t)\big) + k_1e^2\cos^2\big(\Omega t+\psi(t)\big) + \\
+5k_2h_1^4 + 8k_2h_1^3e\cos\big(\Omega t+\psi(t)\big) + 3k_2h_1^2\,e^2\cos^2\big(\Omega t+\psi(t)\big)\Big) + \\
+\frac{1}{2}\Big(3c_1\dot{h}_1^2 + 4c_1\dot{h}_1e\left(\Omega+\dot{\psi}(t)\right)\sin\big(\Omega t+\psi(t)\big) + c_1e^2\left(\Omega+\dot{\psi}(t)\right)^2\sin^2\big(\Omega t+\psi(t)\big) + \\
+4c_2\dot{h}_1^3 + 6c_2\dot{h}_1^2e\left(\Omega+\dot{\psi}(t)\right)\sin\big(\Omega t+\psi(t)\big) + 2c_2\dot{h}_1e^2\left(\Omega+\dot{\psi}(t)\right)^2\sin^2\big(\Omega t+\psi(t)\big)\Big) = 0
\end{gathered}$$

$$\begin{gathered}
m\ddot{v}_1 + \tfrac{1}{2}\Big(3k_1v_1^2 + 4k_1v_1e\sin\big(\Omega t+\psi(t)\big) + k_1e^2\sin^2\big(\Omega t+\psi(t)\big) + \\
+5k_2v_1^4 + 8k_2v_1^3e\sin\big(\Omega t+\psi(t)\big) + 3k_2v_1^2\,e^2\sin^2\big(\Omega t+\psi(t)\big)\Big) + \\
+\tfrac{1}{2}\Big(3c_1\dot{v}_1^2 - 4c_1\dot{v}_1e\left(\Omega+\dot{\psi}(t)\right)\cos\big(\Omega t+\psi(t)\big) + c_1e^2\left(\Omega+\dot{\psi}(t)\right)^2\cos^2\big(\Omega t+\psi(t)\big) + \\
+4c_2\dot{v}_1^3 - 2c_2\dot{v}_1^2e\left(\Omega+\dot{\psi}(t)\right)\cos\big(\Omega t+\psi(t)\big) + c_2\dot{v}_1e^2\left(\Omega+\dot{\psi}(t)\right)^2\cos^2\big(\Omega t+\psi(t)\big)\Big) = 0
\end{gathered} \tag{3}$$

$$\begin{gathered}
J\ddot{\psi}(t) + k_s\psi(t) + c_s\dot{\psi}(t) + \\
+\tfrac{1}{2}\Big(-2k_1h_1^2e\sin\big(\Omega t+\psi(t)\big) - 2k_1h_1e^2\cos\big(\Omega t+\psi(t)\big)\sin\big(\Omega t+\psi(t)\big) + \\
-2k_2h_1^4e\sin\big(\Omega t+\psi(t)\big) - 2k_2h_1^3e^2\cos\big(\Omega t+\psi(t)\big)\sin\big(\Omega t+\psi(t)\big)\Big) + \\
+\tfrac{1}{2}\Big(2k_1v_1^2e\cos\big(\Omega t+\psi(t)\big) + 2k_1v_1e^2\cos\big(\Omega t+\psi(t)\big)\sin\big(\Omega t+\psi(t)\big) + \\
+2k_2v_1^4e\cos\big(\Omega t+\psi(t)\big) + 2k_2v_1^3e^2\cos\big(\Omega t+\psi(t)\big)\sin\big(\Omega t+\psi(t)\big)\Big) + \\
+\tfrac{1}{2}\Big(2c_1\dot{h}_1^2e\sin\big(\Omega t+\psi(t)\big) + 2c_1\dot{h}_1e^2\,\dot{\psi}(t)\sin^2\big(\Omega t+\psi(t)\big) + \\
+2c_2\dot{h}_1^3e\sin\big(\Omega t+\psi(t)\big) + 2c_2\dot{h}_1^2e^2\,\dot{\psi}(t)\sin^2\big(\Omega t+\psi(t)\big)\Big) + \\
+\tfrac{1}{2}\Big(-2c_1\dot{v}_1^2e\cos\big(\Omega t+\psi(t)\big) + 2c_1\dot{v}_1e^2\,\dot{\psi}(t)\cos^2\big(\Omega t+\psi(t)\big) + \\
-2c_2\dot{v}_1^3e\cos\big(\Omega t+\psi(t)\big) + 2c_2\dot{v}_1^2e^2\,\dot{\psi}(t)\cos^2\big(\Omega t+\psi(t)\big)\Big) = 0
\end{gathered}$$

The identification task for this form consists of finding parameters k_i..., c_i and function $\psi(t)$, after introducing the proper metric allowing to compare pathways presented in figure 2 and solving this equation. The complexity of the solution procedure of this task was the reason for making an attempt to do sequential identification, which means finding functions describing elasticity and damping for special cases describable by much simpler dependencies and using them later in the 'full' simulation model.

3. Task of the dynamic model identification

The theoretical task of the parametric identification with a possibility of having a significant number of investigation results for various work conditions (e.g., various acceleration rates) is solvable; however, a large number of parameters looked for (especially when further approximations are reached) can cause identification ambiguity and calculation difficulties.

Thus, the following reasoning will be performed. Let it be assumed that the acceleration time does not have an essential influence on the elasticity and damping characteristic, then an attempt of the preliminary identification of the simplified model with the assumption $\ddot{\varphi}(t) = 0$ is made. The empirically determined elastic characteristic for static deformation is assumed as the first approximation, and the viscous damping coefficient is calculated on the basis of damped vibrations (Fig. 4).

a)

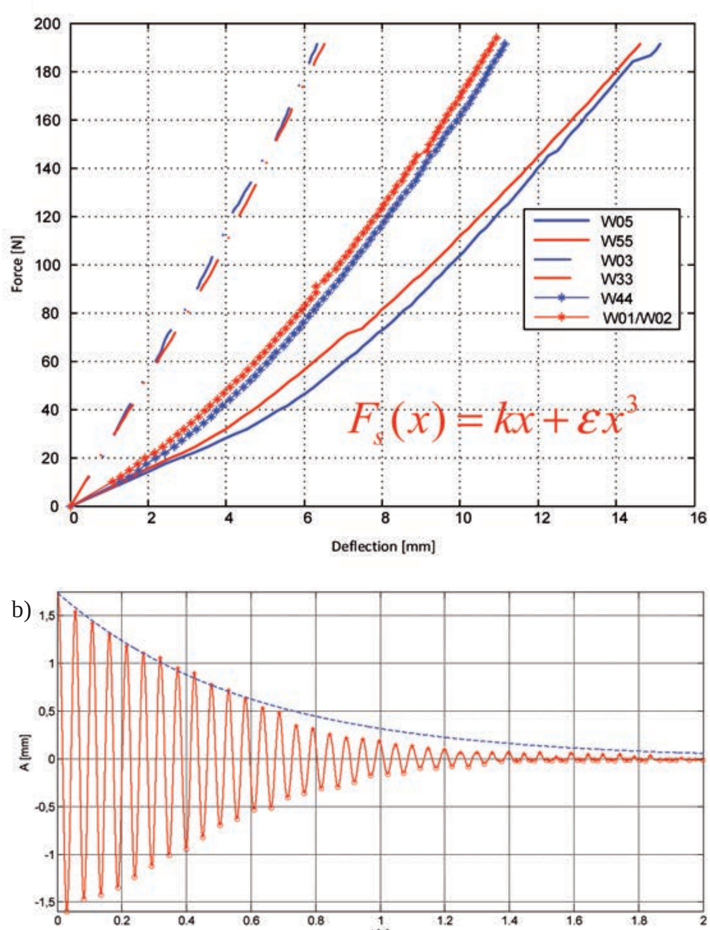


Fig. 4. Determination of preliminary parameters of the model: a) Elastic characteristic of static deformations of exemplary profiles (profile W03/W33 was used in investigations), b) Determination of damping coefficient on the basis of damped vibrations

As can be seen, the elastic characteristic can be easily and with a high accuracy approximated to the odd-degree parabola, and viscous damping values are sufficiently accurate to be described by the pathways from Figure 4b, according the dependence

$$\Lambda = \ln \frac{A_n}{A_{n+1}} = hT_w$$

and

$$f_w = \frac{1}{T_w}, \quad h = \frac{c}{2m} \tag{4}$$

$$\Rightarrow c = 2mf_w\Lambda$$

where Λ – logarithmic damping decrement, T_w – period of damped vibrations, c – damping coefficient.

When the elasticity and damping characteristic obtained in such way are introduced into the equations (1) and the simulation in the Matlab®-Simulink program is performed, the non-linear model is obtained. However, in defiance of expectations, it cannot be tuned to pathways from Figure 2 without essential changes of the remaining parameters, i.e. mass and eccentric. Thereby the model is not identifiable in the permissible values collection. Since both values are determined with a high accuracy, the conclusion is drawn that another phenomenon that was not taken into account must be occurring in the proposed description of the investigated object (material) characteristic.

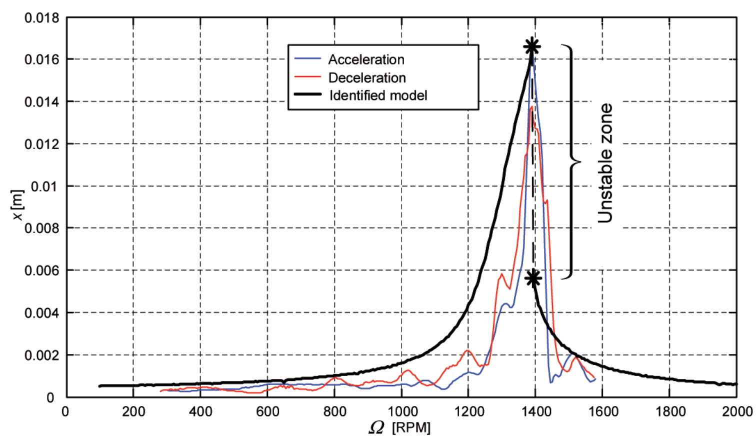


Fig. 5. Effect of the simplified model identification for a very small acceleration rate

According to the authors [5–7], this phenomenon is the dependence of the elasticity force on the deformation rate. Thus, let it be assumed that the elasticity force in the first approximation is written in the following form:

$$F_s\big(x(t),\dot{x}(t)\big) = kx(t) + \varepsilon_1 x^3(t) + \varepsilon_2 \dot{x}(t) \tag{5}$$

Then, introducing Equation (5) into the previous equations (1) and looking for coefficient ε_2 from the condition of the distance minimization between the solution and observed pathways, the sufficient closeness can be obtained, and it occurs that this simple model is identifiable (Fig. 5). The elastic characteristic of the shaft obtained in such way is the spatial characteristic (plate of continuous and differentiable surface in domain $(x, \dot{x})$, and its cross-sections have visible inflection points, which means that it constitutes the degrading-progressive characteristic. Therefore, a'priori identification is difficult (Fig. 6).

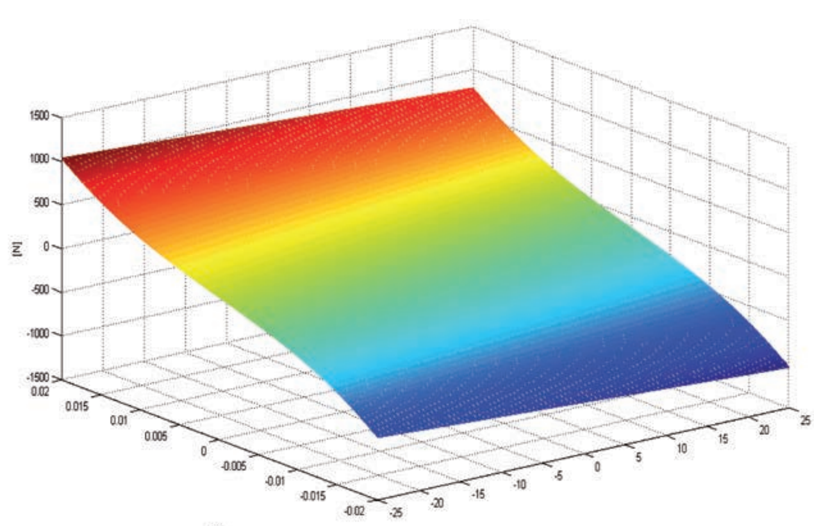
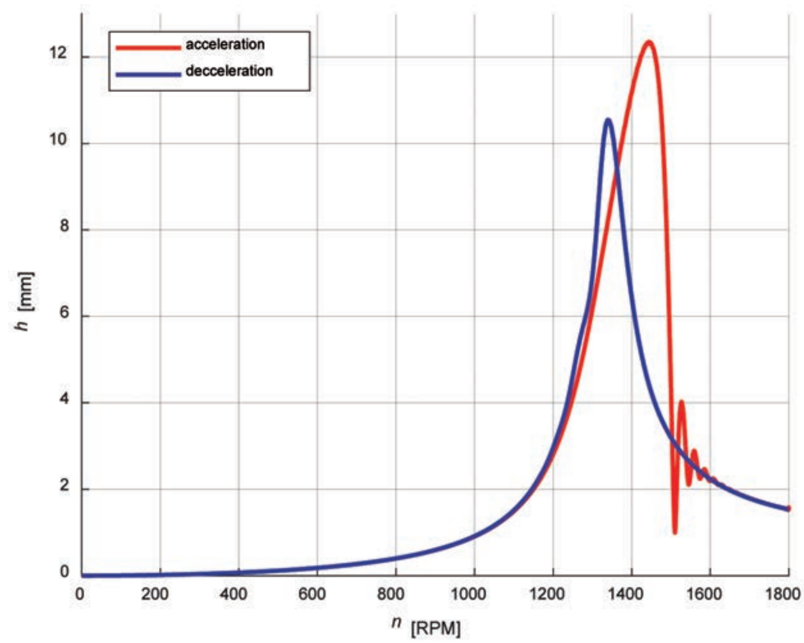


Fig. 6. Characteristic of elasticity as a function of the shaft displacement and deformation rate

Now, it is assumed that the elastic influence and the damping force are not changing as the function of the acceleration rate of the system, which seems logical since both functions have the character of material features. The obtained values are introduced into the full spatial model ($\ddot{\varphi} \neq 0$).

It occurs that the model requires only insignificant tuning (mainly the eccentric parameter, difficult for measuring). The effect of the simulation in the Matlab®-Simulink program, in the form of the amplitude-frequency characteristic obtained from the model, corresponding to previously shown empirical investigations (Fig. 2a), is presented in Figure 7 as an example. Thus, it can be stated that, due to investigations, a well-identified dynamic model that allows for system analysis when critical states were taking place with various velocities was obtained and that the elastic characteristic that is adequate for the applied composite structure was found.



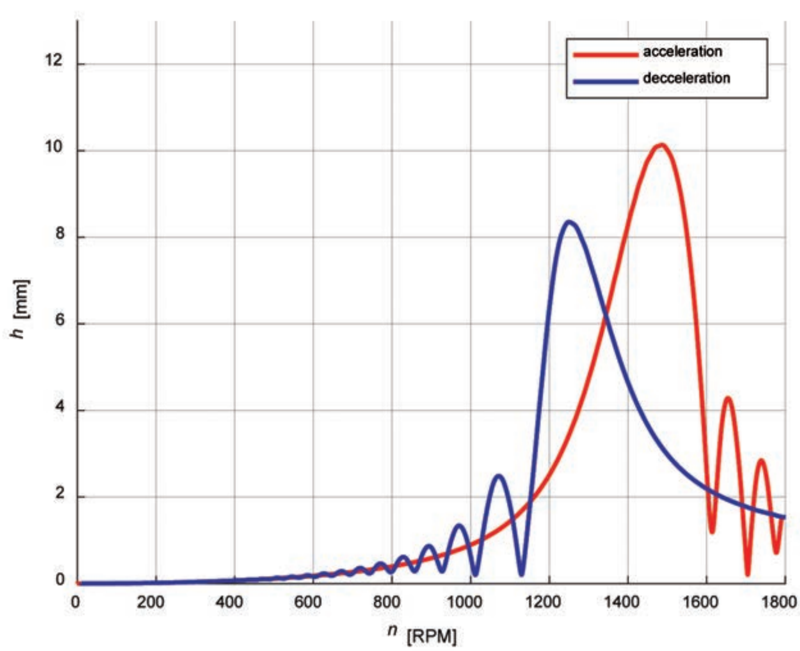


Fig. 7. Simulation result by means of the identified dynamic model

Fig. 8. Simulation investigations of effects of passing the critical state at high acceleration and braking rates

The model can be used for investigations of various acceleration modes and the system behaviour in passing the critical state zone. An example of such a simulation, which is very useful in designing the power transmission systems, is shown in Fig. 8.

Conclusions

The tested machine shaft loaded by one concentrated mass occurred to be a non-linear system of an interesting elastic characteristic. It is worth drawing attention to one detail. The elastic force is presented in the applied description as the sum of two functions (one displacement dependent, another velocity dependent). This second member, from a formal point of view, has the same structure as the function concerning viscous damping. Thus, it would be possible to identify the model selecting an abstract damping coefficient being outside the permissible zone. However, in such model, it would be very difficult to interpret ‘physically’ the results, especially to extract the ‘adequate’ elasticity or damping from the description of visco-elastic features, which is necessary, e.g., when taking into account the stiffness of supports or when designing a vibration damper.

Of course, the accurate numerical values cannot be easily directly transferred to another shaft structure; however, in the qualitative respect, these results have a general meaning concerning the shape of the spatial characteristic of the elastic force as well as concerning the proposed method of the model identification.

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