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Pinning synchronization of two general complex networks with periodically intermittent control

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In this paper, the method of periodically pinning intermittent control is introduced to solve the problem of outer synchronization between two complex networks. Based on the Lyapunov stability theory, differential inequality method and adaptive technique, some simple synchronous criteria have been derived analytically. At last, both the theoretical and numerical analysis illustrate the effectiveness of the proposed control methodology. This method not only reduces the conservatism of control gain but also saves the cost of production.These advantages make this method having a large application scope in the real production process.

Key words: complex networks synchronization, pinning control, intermittent control.

1. Introduction

Nowadays, the development of society is becoming more and more complicated and networked, thus complex networks have emerged and aroused much attention [1-4]. A complex network is a large set of interconnected nodes, in which the nodes and connections can denote everything. So, complex networks are useful tools to describe and represent complex systems, such as food webs, communication networks, social networks, power grids, cellular networks, metabolic systems, disease transmission networks, etc.

Over the past few years, as a kind of basic and important behavior of dynamics, synchronization has become a hot topic in various fields of science and engineering. Usually, synchronization can be divided into "inner synchronization" and "outer synchronization". "Inner synchronization" denotes the behavior of all the nodes among one complex networks become synchronous [5, 6, 7]. Up to now, many different criteria for inner synchronization have been investigated [8]–[15].

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"Outer synchronization" refers to the synchronization occurring between two or more coupled complex networks regardless of happening of inner synchronization [16, 17, 18, 19]. Since the time of Li first proposed the concept of outer synchronization in 2007 [16], it has quickly become a focus point of researching. In our lives, outer synchronization exists everywhere. From the angle of sociology, the countries can be divided into developed countries and developing countries which consisted of the developed networks and developing networks. With the gradual increase of international exchange, the two networks will be synchronized. In the animal world, if no external intervention appears, the synchronization phenomena will exist universally. For example, the number of prey and that of predators are usually invariant under no outside invasion[16, 20]. All these challenging topics show the great importance of outer synchronization between coupled networks. In some cases, some networks have the character of achieving outer synchronization by themselves. However, there is still the situation that the complex networks can not achieve synchronization by themselves, thus some control techniques become useful [21]–[25].

Because of the complexity of the complex networks, it is difficult or even unfeasible to add controllers to every node in the complex networks. Pinning control is a method of adding controllers to partial nodes in the network. In 1999, the concept of pinning control was first proposed by Hu [26], and several good results have been obtained from then on [23, 27, 28]. In the view of the complexity of complex networks and the characters of the pinning control, the method not only simplifies the coupling structure, but also saves the production costs [23, 27]. Therefore, pinning control has large application scope in the real production process.

In the process of signal transmission, the signal will become weaker and weaker due to the character of the system's dissipativeness, then some external controllers are needed for making the signal back to an upper level. Consider the cost of controlling, the external controllers can be removed when the signals are in upper level and this method is a kind of discontinuous control. In many control techniques, both impulsive control and intermittent control are discontinuous controls and the intermittent control has attracted more interest due to its wide applications in engineering fields [29]–[32]. In the period of intermittent control, the time can be divide into two parts, one is "working time", and the other is "rest time" (see Fig.1) [33]. Because the pinning control and the intermittent control both could save product cost, if we combine two kinds of control together tactfully, the production cost will be greatly saved. This method should been paid much more attention, especially in the engineering applications.

In this paper, we introduced the periodically pinning intermittent control technique to outer synchronization between two general complex networks for the first time. By utilizing mathematical induction method and the adaptive analysis technique, some novel synchronization criteria are derived. This method not only reduces the conservatism of control gain but also saves the cost of production. All these advantages lead this method to a larger application scope in the real production process.

The rest of this paper is organized as follows. In Section 2, some general driver and response complex dynamical network models are introduced, and some necessary

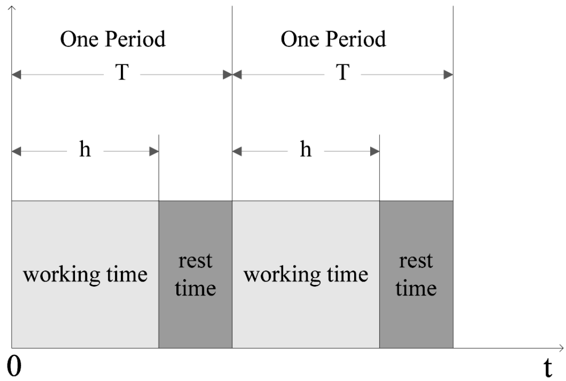

Figure 1: Sketch map of intermittent control.

preliminaries are given. In Section 3, based on the Lyapunov stability theorem and mathematical induction, some periodically pinning intermittent controllers are designed to ensure the driver and response systems achieve outer synchronization. In Section 4, some numerical simulations are given to verify the effectiveness of proposed theoretical results and the Section 5 is the conclusion of this paper.

2. Model and preliminaries

Consider a complex network consisting of N identical linearly and diffusively coupled nodes. Every node in the network is an m dimensional dynamical unit. Then the network model can be denoted as:

$$\dot{x}_i(t) = f(t, x_i(t)) + c\sum_{j=1}^{N} g_{ij} A x_j(t), \quad i = 1, 2, \cdots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{im}(t))^T \in R^m$ is the state vector of the *ith* node, $f : R \times R^m$ is a continuous vector-valued function. $A \in R^{m\times m}$ is the inner connecting matrix, $c > 0$ is the coupling strength, $G = (g_{ij})_{N\times N}$ is the coupling configuration matrix which represent the topological structure of the whole network. In terms of the physical interpretation of whole networks, the elements g_{ij} of matrix G are usually been chosen as positive constant and denote the relationship among every nodes. Thus, the matrix element $g_{ij} \in R$ are defined as follows : if there is a coupling from node i to node j $(i \neq j)$, $g_{ij} > 0$; otherwise, $g_{ij} = 0$. In general, we assume that the coupling configuration matrix G satisfies the following properties: $\sum_{j=1}^{N} g_{ij} = 0$.

Compared with the response system mentioned, the drive complex network is denoted as:

$$\dot{y}_i(t) = f(t, y_i(t)) + c\sum_{j=1}^{N} g_{ij} A y_j(t), \quad i = 1, 2, \cdots, N, \tag{2}$$

where $y_i(t)=(y_{i1}(t),y_{i2}(t),\ldots,y_{im}(t))^T \in R^m$ still represents the state variable of the ith node in the response system, and other parameters are the same with the corresponding parameters in the drive system (1).

Remark 1 In the drive and response systems, the outer coupling configuration matrix G does not need to be symmetric or irreducible.

From the expression of the drive system and response system, error variables between the two complex networks are denoted as

$$e_i(t)=x_i(t)-y_i(t), \quad i=1,2,\ldots,N. \tag{3}$$

In this paper, we assume that $\|A\|=\alpha>0$, and $\rho_{\min}$ denotes the minimum eigenvalue of matrix $(A+A^T)/2$. $\hat{G}$ is a modifying matrix of G via replacing the diagonal elements g_{ii} by $(\rho_{\min}/\alpha)g_{ii}$. Choose the matrix $\hat{G}^s=(\hat{G}+\hat{G}^T)/2$ and its eigenvalues are expressed as $\lambda_1\geq\lambda_2\geq\ldots\lambda_N$.

Notations: Throughout this paper, we let $\|\cdot\|$ be Euclidean norm. I_m is an $m\times m$ identity matrix. If A is a vector or matrix, its transpose is denoted by A^T.

3. Main results

In this part, the outer synchronization between the drive network (1) and the response network (2) will be investigated. With the purpose of reducing the cost of production, some extra controllers are required to add on the partial nodes of the drive system. Without loss of generality, we choose the first l nodes in the drive system as the pinning nodes. Then the response network with suitable controllers are described as:

$$\begin{cases} \dot{x}_i(t)=f(t,x_i(t))+c\sum_{j=1}^{N}g_{ij}Ax_j(t)+u_i(t), & i=1,2,\cdots,l, \\ \dot{x}_i(t)=f(t,x_i(t))+c\sum_{j=1}^{N}g_{ij}Ax_j(t), & i=l+1,\cdots,N. \end{cases} \tag{4}$$

Usually, it is not allowed the control gain k to be much larger than the real need. So, it is necessary to adopt the adaptive control method to obtain a proper control gain. In this part, we will design some adaptive periodically pinning intermittent controllers to make the drive system (1) and the response system (2) achieve synchronization.

Based on the expression of (4), the adaptive periodically pinning intermittent controllers are designed as

$$u_i(t)=\begin{cases} -k_i(t)e_i(t), & 1\leq i\leq l, & t\in[nT,nT+h), \\ 0, & l+1\leq i\leq N, & t\in[nT,nT+h), \\ 0, & 1\leq i\leq N, & t\in[nT+h,(n+1)T), \end{cases} \tag{5}$$

and the updating laws

$$\dot{k}_i(t) = \begin{cases} \alpha_i \exp(a_1 t) \| e_i(t) \|_2^2, & t \in (nT, nT+h], \\ 0, & t \in (nT+h, (n+1)T], \end{cases} \tag{6}$$

where $\alpha_i (i = 1,2,\ldots,N)$ and a_1 are positive constants, $k_i(0) > 0 (i = 1,2,\ldots,N)$ are initial values and $k_i((n+1)T) = k_i(nT+h)$, $n = 0,1,2\ldots$ $T > 0$ denotes the control period, $h \in (0,T)$ denotes the working time. Then, the error system (3) can be rewritten as

$$\begin{cases} \dot{e}_i(t) = f(t, x_i(t)) - f(t, y_i(t)) + c\sum_{j=1}^{N} g_{ij} A e_j(t) - k_i(t) e_i(t), \\ \qquad i = 1,2,\cdots,l, t \in [nT, nT+h), \\ \dot{e}_i(t) = f(t, x_i(t)) - f(t, y_i(t)) + c\sum_{j=1}^{N} g_{ij} A e_j(t), \\ i = l+1, l+2,\cdots, N, t \in [nT, nT+h), ort \in [nT+h, (n+1)T). \end{cases} \tag{7}$$

According to the expressions mentioned above, it is obvious that the objective of control is to find an appropriate controllers to make the solutions of the controlled network (4) synchronize with the solution of system (2), in the sense that

$$\lim_{t\to\infty} \| x_i(t) - y_i(t) \| = 0, \quad i = 1,2,\ldots,N. \tag{8}$$

In the following, lemma and assumption will be presented.

Lemma 1. (Schur complement) [34]. *The following linear matrix inequality (LMI)*

$$\begin{pmatrix} \mathcal{A}(x) & \mathcal{B}(x) \\ (\mathcal{B}(x))^T & \mathcal{C}(x) \\ & \end{pmatrix} > 0,$$

where $\mathcal{A}(x) = A(x))^T, \mathcal{C}(x) = C(x))^T$ *is equivalent to one of the following conditions:*

(a) $\mathcal{A}(x) > 0$ *and* $\mathcal{C}(x) - \mathcal{B}(x)^T \mathcal{A}(x)^{-1} \mathcal{B}(x) > 0$;

(b) $\mathcal{C}(x) > 0$ *and* $\mathcal{A}(x) - \mathcal{B}(x)\mathcal{C}(x)^{-1}\mathcal{B}(x)^T > 0$.

Assumption 1 *The nonlinear function* $f(\cdot)$ *satisfies the following Lipschitz condition:*

$$\left\| f(t, x_i(t)) - f(t, y_i(t)) \right\| \le L \left\| (x_i(t) - y_i(t)) \right\|, \ \forall \ x_i(t), \ y_i(t) \in R^m,$$

where L is a known positive constant.

Next, we will give the main results as below.

Based on the Assumption 1, the synchronization criteria of system (1) and (2) by adaptive periodically pinning controllers will be deduced as follows.

Theorem 1 *Under Assumption* 1, *if there exist positive constants* ε, a_1, a_2, *such that*

$$\left(L+\frac{1}{2}a_1\right)I_N + c\alpha\bar{\hat{G}}^s \le 0,$$

$$(L-\frac{1}{2}(a_2-a_1))I_N + c\alpha\hat{G}^s \le 0, \tag{9}$$

$$\varepsilon = a_1 - a_2(1-\frac{h}{T}) > 0.$$

then the drive system (1) *and the response system* (2) *could achieve synchronization under the adaptive periodically pinning intermittent controllers* (5) *and the updating law* (6).

Proof Define the Lyapunov function as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i},$$

where k' is a undetermined sufficiently large positive constant. The derivative of $V(t)$ with respect to time t along with the solutions of (7) can be calculated as follows.

When $t \in [nT, nT+h)$, for $n = 0,1,2\ldots$.

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{l}\exp(-a_1 t)\frac{k_i(t)-k'}{\alpha_1}\dot{k}_i(t) \\ &\quad -\frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i} \\ &= \sum_{i=1}^{N} e_i^T(t)[f(t,x_i(t)) - f(t,y_i(t))] + c\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)g_{ij}Ae_j(t) \\ &\quad -\sum_{i=1}^{l} k_i(t)e_i^T(t)e_i(t) + \sum_{i=1}^{l}(k_i(t)-k')e_i^T(t)e_i(t) \\ &\quad -\frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i} \\ &\le L\sum_{i=1}^{N} e_i^T(t)e_i(t) + c\sum_{i=1}^{N}\sum_{j=1, j\ne i}^{N} \alpha g_{ij}\|e_i(t)\|\|e_j(t)\|\end{aligned}$$

$$+c\sum_{i=1}^{N}g_{ii}\rho_{\min}e_i^T(t)e_i(t)-k'\sum_{i=1}^{l}e_i^T(t)e_i(t)$$

$$-\frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1t)\frac{(k_i(t)-k')^2}{\alpha_i}$$

$$=L\sum_{i=1}^{N}e_i^T(t)e_i(t)+c\alpha\hat{G}-k'\sum_{i=1}^{l}e_i^T(t)e_i(t)$$

$$-\frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1t)\frac{(k_i(t)-k')^2}{\alpha_i}$$

$$=e^T(t)((L+\frac{a_1}{2})I_N+c\alpha\hat{G}^s-K')e(t)-\frac{a_1}{2}e^T(t)e(t).$$

$$-\frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1t)\frac{(k_i(t)-k')^2}{\alpha_i} \tag{10}$$

where

$$K'=diag(\underbrace{k',\quad \ldots,\quad k'}_{l},\underbrace{0,\quad \ldots,\quad 0}_{N-l}).$$

since k' is an undetermined sufficiently large positive constant.

Here we let $Q=(L+\frac{a_1}{2})I_N+c\alpha\hat{G}^s$, and $M=(L+\frac{a_1}{2})I_N+c\alpha\hat{G}^s-K'$.

Rewrite the matrix M as $M=(L+\frac{1}{2})I_N+c\alpha\hat{G}^s-K'=\begin{pmatrix} E-\bar{K}' & B \\ B^T & \bar{Q}\end{pmatrix}$ where

$$\bar{K}=diag(\underbrace{k',\quad \ldots,\quad k'}_{l})$$

and $\bar{Q}$ is obtained by removing the first $1,2,...,l$ row-column pairs of matrix M. Thus $\bar{Q}=(L+\frac{a_1}{2})I_{N-l}+c\alpha\bar{\hat{G}}^s$, and $\bar{\hat{G}}^s=\hat{G}_{l+i,l+j}$, for $i=1,2,...N-l$. The matrix E and B are matrix with appropriate dimensions.

Then according to the lemma 1 and the first condition of Theorem 1, we obtain the result that as long as $k'>\lambda_{\max}(E-B\bar{Q}^{-1}B^T)$ and $\bar{Q}<0$, the matrix $M=(L+\frac{1}{2})I_N+c\alpha\hat{G}^s-K'=\begin{pmatrix} E-\bar{K}' & B \\ B^T & \bar{Q}\end{pmatrix}<0$ will hold. Thus there is

$$\dot{V}(t)\le -a_1V(t). \tag{11}$$

Similarly, when $t \in [nT+h,(n+1)T)$, we have

$$\dot{V}(t) = \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{l} \exp(-a_1 t)\frac{k_i(t)-k'}{\alpha_i}\dot{k}_i(t)$$

$$-\frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i}$$

$$= \sum_{i=1}^{N} e_i^T(t)[f(x_i(t)) - f(y_i(t))] + c\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t) g_{ij} A e_j(t)$$

$$-\frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i}$$

$$\le L\sum_{i=1}^{N} e_i^T(t)e_i(t) + c\sum_{i=1}^{N}\sum_{j=1, j\ne i}^{N} \alpha g_{ij} \| e_i(t) \| \| e_j(t) \|$$

$$+c\sum_{i=1}^{N} g_{ii}\rho_{\min} e_i^T(t)e_i(t) - \frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i}$$

$$= L\sum_{i=1}^{N} e_i^T(t)e_i(t) + c\alpha\hat{G} - \frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i}$$

$$\le e^T(t)((L - \frac{1}{2}(a_2 - a_1))I_N + c\alpha\hat{G}^s)e(t) + \frac{1}{2}(a_2 - a_1)e^T(t)e(t)$$

$$-\frac{a_1}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i} + \frac{a_2}{2}\sum_{i=1}^{l}\exp(-a_1 t)\frac{(k_i(t)-k')^2}{\alpha_i}$$

$$\le (a_2 - a_1)V(t). \tag{12}$$

Choose $\hat{Q} = V(0)$ and $\hat{P}(t) = \exp(a_1 t)V(t)$. Let $\Phi(t) = \hat{P}(t) - \beta\hat{Q}$, where $\beta > 1$ is a constant. One gets

$$\Phi(t) < 0, \tag{13}$$

for all $t = 0$.

In the follows, we will use the reduction to absurdity to proof $\Phi(t) < 0$, for all $t \in [0,h)$.

Since $\Phi(0) < 0$, we assume that there exists $t_0 \in [0,h)$, such that

$$\Phi(t_0) = 0, \dot{\Phi}(t_0) > 0, \tag{14}$$

$$\Phi(t) < 0, 0 \le t < t_0. \tag{15}$$

Calculating the value of $\dot{\Phi}(t)$ at t_0, we have

$$\begin{aligned}\dot{\Phi}(t_0) &= \dot{\hat{P}}(t_0)\\ &= a_1 \exp(a_1 t_0) V(t_0) + \exp(a_1 t_0)\dot{V}(t_0)\\ &\le a_1 \hat{P}(t_0) - a_1 \exp(a_1 t_0) V(t_0)\\ &= (a_1 - a_1)\hat{P}(t_0)\\ &= 0\end{aligned}$$

and this result is contradict with the assumption of $\dot{\Phi}(t_0) > 0$ in (14). Thus, $\Phi(t) < 0$, for all $t \in [0, h)$.

Next, we will adopt the similar method to proof that $\Psi(t) = \hat{P}(t) - \beta\hat{Q}\exp(a_2(t-h)) < 0$, for $t \in [h, T)$. Assume that there exists $t_1 \in [h, T)$, such that

$$\Psi(t_1) = 0, \dot{\Psi}(t_1) > 0, \tag{16}$$

$$\Psi(t) < 0,\ h \le t < t_1. \tag{17}$$

Calculating the value of $\dot{\Psi}(t)$ at t_1, we have

$$\begin{aligned}\dot{\Psi}(t_1) &= \dot{\hat{P}}(t_1) - a_2\beta\hat{Q}\exp(a_2(t_1 - h))\\ &= a_1 \exp(a_1 t_1) V(t_1) + \exp(a_1 t_1)\dot{V}(t) - a_2\beta\hat{Q}\exp(a_2(t_1 - h))\\ &\le a_1\hat{P}(t_1) + \exp(a_1 t_1)(a_2 - a_1)V(t_1) - a_2\beta\hat{Q}\exp(a_2(t_1 - h))\\ &= a_1\hat{P}(t_1) + a_2\hat{P}(t_1) - a_1\hat{P}(t_1) - a_2\beta\hat{Q}\exp(a_2(t_1 - h))\\ &= a_2(\hat{P}(t_1) - \beta\hat{Q}\exp(a_2(t_1 - h))).\end{aligned} \tag{18}$$

According to the assumption of $\Psi(t_1) = 0$ in (16), we know $\hat{P}(t_1) = \beta\hat{Q}\exp(a_2(t_1 - h))$.

Then the inequation (18) can be simplified as

$$\dot{\Psi}(t_1) \le 0$$

and this result is contradict with $\dot{\Psi}(t_1) > 0$ in (16).

Hence, for $t \in [h, T)$, $\Psi(t) = \hat{P}(t) - \beta\hat{Q}\exp(a_2(t-h)) < 0$ holds, that is, for $t \in [h, T)$, we have

$$\hat{P}(t) < \beta\hat{Q}\exp(a_2(t-h)) \le \beta\hat{Q}\exp(a_2(T-h)). \tag{19}$$

On the other hand, from $\Phi(t)<0,$ for $t\in[0,h)$, we have

$$\hat{P}(t)<\beta\hat{Q}<\beta\hat{Q}\exp(a_2(T-h)). \tag{20}$$

Thus, combine (19) and (20), there is

$$\hat{P}(t)<\beta\hat{Q}\exp(a_2(T-h),$$

for all $t\in[0,T)$.

Similarly, we can prove that for $t\in[T,h+T)$,

$$\hat{P}(t)<\beta\hat{Q}<\beta\hat{Q}\exp(a_2(t-h)).$$

and for $\in[T+h,2T)$,

$$\hat{P}(t)<\beta\hat{Q}<\beta\hat{Q}\exp(a_2(t-2h)).$$

Through the mathematical induction, we can estimate the value of $\hat{P}(t)$ for any integer n. That is for $t\in[nT,nT+h),n=0,1,2,\ldots$.

$$\hat{P}(t)<\beta\hat{Q}\exp(a_2n(T-h))\leq\beta\hat{Q}\exp(a_2(1-\frac{h}{T})t).$$

And for $t\in[nT+h,(n+1)T),n=0,1,2,\ldots$.

$$\hat{P}(t)<\beta\hat{Q}\exp(a_2(t-(n+1)h))\leq\beta\hat{Q}\exp(a_2(1-\frac{h}{T})t).$$

Here, we let $\beta\to 1$ and from the definition of $\hat{P}(t)$, we have

$$V(t)\leq\hat{Q}\exp(-(a_1-a_2(1-\frac{h}{T})t))$$

$$=V(0)\exp(-\varepsilon t),\quad t\geq 0.$$

In conclusion, according to the third condition in the Theorem, the drive system (1) and response system (2) can achieve synchronization with adaptive periodically pinning intermittent controllers when $t\to\infty$. The proof has been completed. □

From the matrix characteristics and the theory of Schur complement, we can obtain the following corollary,

Corollary 1 *If there exist positive constants* a_1 *and* a_2, *such that*

$$L+\frac{1}{2}a_1+c\alpha\lambda_{\max}\bar{\bar{G}}^s<0,$$

$$L - \frac{1}{2}(a_2 - a_1) + c\alpha\lambda_1 \le 0, \tag{21}$$

$$\varepsilon = a_1 - a_2(1 - \frac{h}{T}) > 0.$$

then the drive system (1) *and the response system* (2) *could achieve synchronization under the adaptive periodically pinning intermittent controllers* (5) *and the updating law* (6).

4. Numerical Simulations

In this section, we will take a concrete example to illustrate the effectiveness of the proposed method. In the computer world, all educators computers form education networks while all researchers computers compose research networks. These two networks are coupled via the Internet. If all educators and researchers explore the same Internet source, the congestion will appears, which is harmful and should be avoided [16]. Thus, it is necessary to study the outer synchronization between the two coupled networks. In this example, we choose ten computers as the nodes in education networks and researcher networks respectively. The Chen system is selected to describe the dynamical behavior of every computer and its dynamical equation are described as follows,

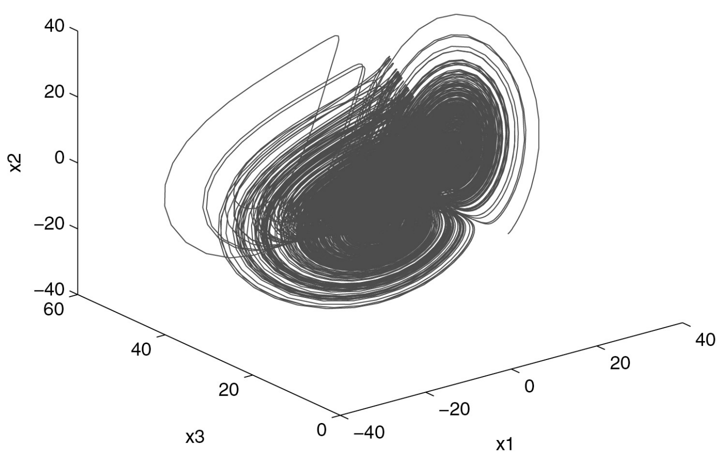

Figure 2: Chaotic behavior of Chen attractors.

$$\begin{cases} \dot{x}_{i1}(t) &= 35(x_{i2}(t) - x_{i1}(t)), \\ \dot{x}_{i2}(t) &= -7x_{i1}(t) - x_{i1}(t)x_{i3}(t) + 28x_{i2}(t), \\ \dot{x}_{i3}(t) &= x_{i1}(t)x_{i2}(t) - 3x_{i3}(t). \end{cases} \tag{22}$$

From the chaotic behavior of Chen system shown in Fig.1, it is found that the chaotic attractor is bounded in a certain region. Our theoretical and numerical analysis show that there exist constants $M_1 = 23, M_2 = 32$, and $M_3 = 61$ satisfying $\| x_{ij} \| \le M_j$ for $1 \le i \le 6$ and $1 \le j \le 3$. Thus one gets

$$\| f(t, x_i(t)) - f(t, y_i(t)) \|$$

$$= \sqrt{[-x_{i1}x_{i3} - (-y_{i1}y_{i3})]^2 + [x_{i1}x_{i2} - y_{i1}y_{i2}]^2}$$

$$= \sqrt{[-x_{i3}e_{i1} - y_{i1}e_{i3}]^2 + [x_{i2}e_{i1} + y_{i1}e_{i2}]]}$$

$$\le 2M \| e_i(t) \| . \quad (23)$$

Thus, the Chen system satisfies the Assumption 1, and choose $L = 117.6291$.

In this example, we assume the connection of computers in the education and research networks obey the scale-free distribution of the BA model. The parameters of the BA model are given by $m_0 = m = 3, N = 10$. The inner connecting matrix A are chosen as $A = diag(1,1.2,1)$. In that case, we choose 6 computers from education network as the control object. Thus, $\lambda_1 = 0.6790$, the Maximum eigenvalue of matrix $\hat{G}^s$ is $\lambda_{\max}\hat{G}^s = -1.4999$. Other suitable parameters are chosen as $T = 0.2, h = 0.178$, $c = 80$, $a_1 = 52$, $a_2 = 418$. So it is easy to verify that Eq.(21) in Corollary 1 is satisfied. The initial conditions of the simulations are chosen randomly in $(0,1)$. Then, the synchronization errors $e_i(t)$ are illustrated in Figs.3–5, and the adaptive control gain are shown in Fig.6. From the simulations, we know that the controllers in education networks needn't to work all the time, the education networks and the research networks can also achieve outer synchronization.

5. Conclusion

In this paper, we introduced the periodically pinning intermittent control method to resolve the problem of outer synchronization between two complex networks. Based on the Lyapunov stability theory and differential inequality method, some simple synchronous criteria have been derived analytically. At last, both the theoretical and numerical analysis illustrate the effectiveness of the proposed control methodology. Considering the complexity of the complex networks, this method not only reduce the constraint of configuration matrix, such as symmetry or reducibility, but also save the cost of production. Especially, we adopt a adaptive technique to keep the control gain *k* more suitable, which could avoid the control gain *k* much larger than the actual need. So, in the real production process, this method has important significance.

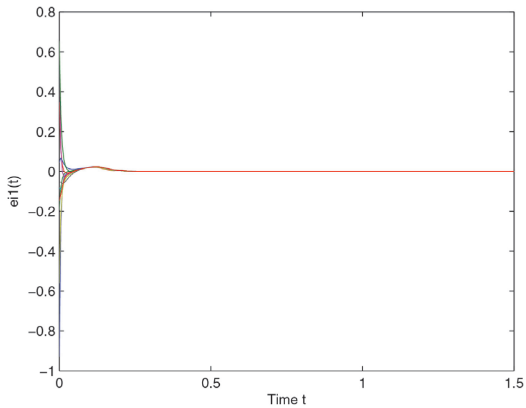

Figure 3: Synchronization errors $e_{i1}(t), i = 1, 2, ...10$ .between the drive and response systems for $0 \leq t \leq 1.5$.

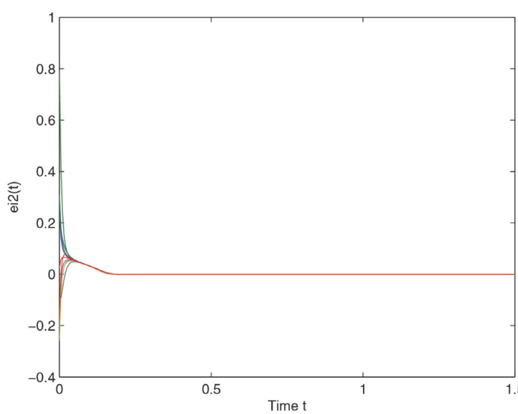

Figure 4: Synchronization errors $e_{i2}(t), i = 1, 2, ...10$.between the drive and response systems for $0 \leq t \leq 1.5$.

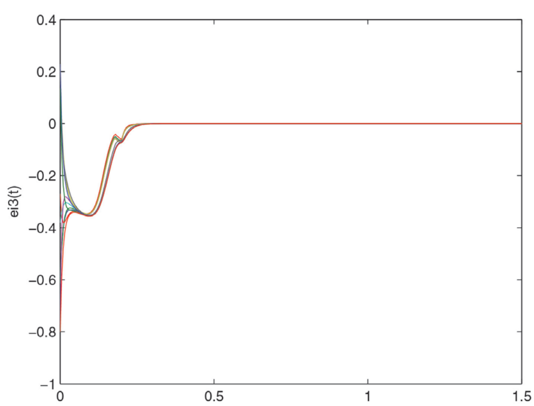

Figure 5: Synchronization errors $e_{i3}(t), i = 1, 2, ...10$ .between the drive and response systems for $0 \leq t \leq 1.5$.

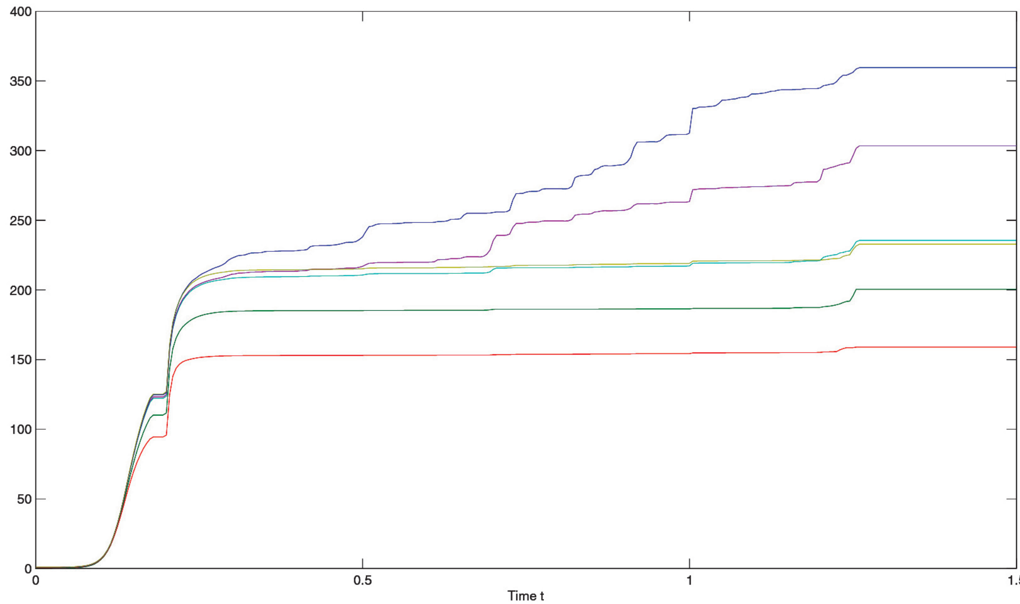

Figure 6: Adaptive control gain of controllers.

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