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GRAPH MODELS OF AUTOMOBILE GEARS - KINEMATICS

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In the present paper, kinematical analysis of an automotive gear is described. Versatile graph based methods have been utilized for this purpose. An application of mixed, contour and bond graphs gives the same results. It allows the detection of possible mistakes as well as a deeper insight into the designed artifact. The graphs can also be used for further analyses which will be published in a separate document.

Key words: kinematical analysis, ratios, mixed graphs, contour graphs, bond graphs.

1. Introduction

In the present paper, graph-theoretical models of automobile gears are discussed. The methods of modeling of gears are versatile (Drewniak *et al*., 2014; Drewniak and Zawiślak, 2010; 2010; Rajasri *et al*., 2014; Zanasi and Grossi, 2009). In particular, graph-based models can be really different, however, they depend on the goal of modeling. Within recent years, these models were applied in numerous cases in many ways. For example, in Hasan (2013) and Drewniak and Zawiślak (2010), structures of mechanisms e.g., in particular the structures of planetary gears (Drewniak and Zawiślak, 2010) were considered. Linear graphs were used. In Deur *et al*. (2006), the purpose of modeling was an analysis of power flow through a planetary gear. Moreover, in Deur *et al*. (2006), Luo Yutao and Tan Di (2011), Richard and Bouazara (2012) the dynamics of automotive gears was analyzed by means of bond-graphs. In Drewniak and Zawiślak (2010) the authors analyzed kinematics of planetary gears using contour graphs introduced by Marghitu (2005). Here, all the three mentioned graph-based methods will be used simultaneously. The most detailed and wide analysis of references connected with this topic has been published in Zawiślak (2010).

2. Graph-based models of gears

The present paper is a continuation of the considerations discussed in Drewniak and Zawiślak (2010). Besides the contour graphs, other types of graphs are used for modeling of automobile gears, e.g.: mixed graphs (i.e., Hsu graphs - generalized by the current authors) (Zawiślak, 2010) and bond graphs (Brown, 2001; Karnopp *et al*., 2012; Merzouki *et al*., 2013). Modeling is performed for the purpose of kinematical analysis of an automobile gear shown in Fig.1. The descriptions of the figure are as follows: H – brakes, S – multi-plate clutches, Sz – rows of gears, Wk – special clutches, 1,2, …, 6 – gear elements, i.e., wheels and arms/carriers, A, B, C – points, adequate for measurement of radiuses according to the main gear axis.

Despite the restriction of considerations to the exemplary gear, the methods are general and some general, theoretical rules of modeling are described.

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A simultaneous application of three graph-based methods, i.e.: (a) contour graphs; (b) mixed graphs and (c) bond graphs allows their mutual comparisons. It gives an opportunity for a deeper analysis of the outcomes obtained. In general, graph methods - used for modeling of versatile mechanical systems - are performed in the following steps:
(i) abstraction, discretisation – choice of objects (elements) for analysis, at this stage some aspects are neglected, e.g.: lubrication, vibrations, etc.;
(ii) recognition of relations between chosen elements, introduction of descriptions, notation;
(iii) assignment of graphs – i.e., vertices, edges and weights; (iv) transfer of knowledge between the adequate fields of knowledge, i.e., graph theory ↔ mechanics; (v) graph-based analysis - schematic, algorithmic, usually convertible into a computer program e.g. automatic generation of system of equations. The results of analysis are obtained directly or after a special reverse transformation. An engineer receives the equations systems describing, e.g.: the kinematics or dynamics or other results, e.g. parameters of the mechanical system under consideration (in our case, automobile gear is analyzed) or even a family of design solutions.

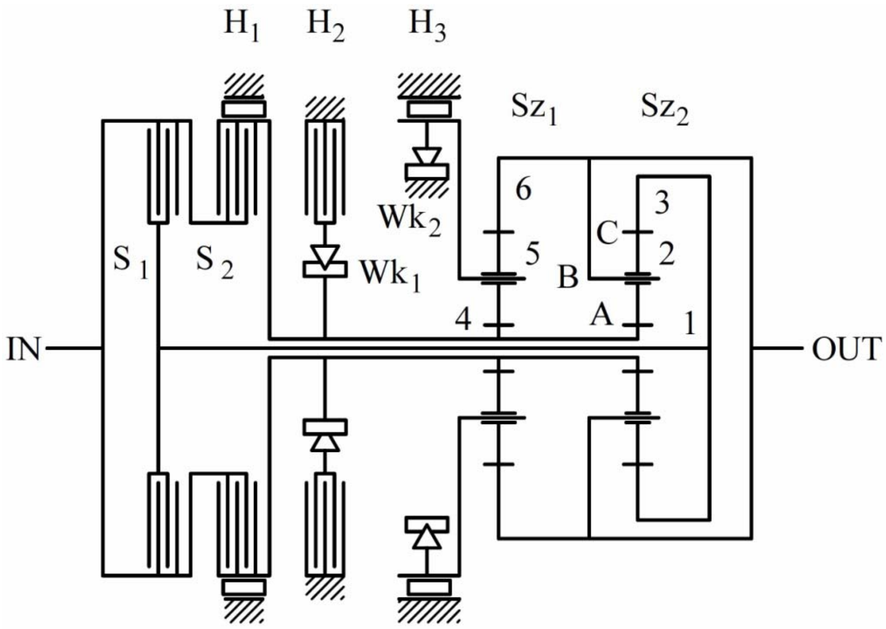

Fig.1. Simplified schema of the automobile planetary gear (based upon (Looman, 1988)).

Table 1. Gear work duty modes (gears/drives).

No	**Notion/ /variable**	**Low gear/1-st drive**	**Second gear/ /second drive**	**Top gear/3-rd drive**	**Reverse gear/ /reverse drive**
	1	2	3	4	5
1	Ratio	*2.479*	*1.479*	*1.0*	*-2.086*
2	Input	*3*	*3*	all elements are clutched together	*4*
3	Output	*6=J*	*J*	all elements are clutched together	*6*
4	Braked/ /stopped	*j*	*1*	-	*j*
5	Temporary redundant	-	*j, 6, 5, 4*	-	*J, 1, 2, 3*

Advantages of these approaches are as follows: simplicity, algorithmic approach, schematic way of activities, deeper understanding of the problem, because transfer of knowledge is necessary. A drawback of the method is that the graph methods are not widely known but just recently the situation has changed. In the present work, kinematics is analyzed but other tasks could also be solved – e.g.: dynamical simulations (Luo Yutao and Tan Di, 2011; Zhonghong *et al*., 2008). In the case of linear graphs, different graphs and different algorithms are needed for each problem but in the case of bond-graph modeling - further analyses arise just based upon the development of a formerly assigned graph, therefore these methods seem to be most universal. The above mentioned methods are utilized for an analysis of the automobile gear presented in Fig.1. The analyses are given underneath where the transformed graphs are assigned to the consecutive gear work modes given in Tab.1. One can observe that the particular modes are connected with the activation of both so called rows (Sz, Fig.1) or one of them. In the gearboxes which perform more drives, more rows are needed.

The particular gears are triggered (activated) due to the adequate arrangements of the control system where some clutches and brakes are activated.

3. Analysis of exemplary automobile gear

In the paper, an exemplary automobile gear is analyzed. Consecutive drives are analyzed via adequate graph transformations. The sequences of ratios are calculated. Each chosen drive was analyzed by means of three graph-based methods discussed above. Despite the fact that the systems of equations are different in each case – the same results were obtained, i.e., identical ratios and rotational velocities. The numbers of teeth (Hasan, 2013) are as follows

$$\begin{aligned} z_1 = z_4 = 35, \\ z_2 = z_5 = 19, \\ z_3 = z_6 = 73. \end{aligned} \tag{3.1}$$

We do not analyze the modulus because it is a variable which is crossed out in our calculations and therefore at this stage can be neglected.

3.1. Mixed graphs of the exemplary automobile gear

Mixed graphs of the automobile gear are presented in Figs 2 a, b and c – for low, second and reverse drive, respectively. The mixed graphs are considered, i.e., a simple graph where a clique is drawn as a shaded polygon (according to Hsu's idea) and the directed path from an input to an output is considered.

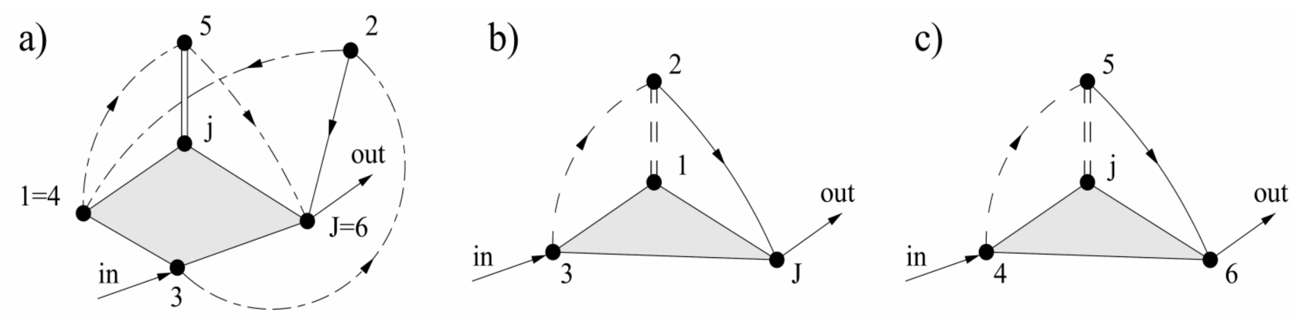

Fig.2. Mixed graphs of gear: (a) low gear, (b) second gear, (c) reverse gear.

The double-line edges symbolize connections where the gear element (encoded by one edge end vertex) is stopped, e.g., arm j in Fig.2a (low gear). The other rules of graph drawing and assignment are collected in Tab.2.

The graphs in Fig.2b and c are transformed (simplified) graphs, i.e., all redundant elements are neglected. The graph for a top gear is omitted because the ratio is equal to 1, so the reverses (rotational velocities) and power are passed directly from an input to an output.

This graph-based method of analysis consists in distinguishing the so called f-cycles containing one stripped edge. Then the codes of these cycles are listed. Finally, a system of equations is generated: based upon a code – an equation is written placing the indices in an algorithmic way and inserting minus or plus in the right-hand part of an equation for external and internal meshing, respectively. We consider the low gear, first. For example, the code for the cycle ((1=4); 5; j) is written in the form (4,5)j where the pair in brackets denotes a stripped edge and a pair of meshing wheels, i.e., 4 and 5. The third element of the code denotes the arm for an adequate planetary wheel, i.e., arm j. Here, only the description 4 was used because just this wheel is in mesh. The formulas (3.2b) were created upon the f-cycles (formulas (3.2a)).

$$\begin{matrix} (4,5)j \\ \\ (5,6)j \\ \\ (2,3)J \\ \\ (1,2)J \\ \\ \\ \end{matrix} \qquad \begin{cases} \omega_4 - \omega_j = -N_{54}\left(\omega_5 - \omega_j\right) \\ \\ \omega_5 - \omega_j = +N_{65}\left(\omega_6 - \omega_j\right) \\ \\ \omega_2 - \omega_J = +N_{32}\left(\omega_3 - \omega_J\right) \\ \\ \omega_1 - \omega_J = -N_{21}\left(\omega_2 - \omega_J\right) \\ \\ \omega_j = 0 \end{cases} \tag{3.2a,b}$$

where: ω denotes rotational velocities and N adequate local ratios expressed by means of the number of teeth, signs +/- are inserted for internal and external meshing, respectively.

The system of Eq.(3.2b) contains one additional equation – for the gear element which is stopped due to an activity of the control system. The solution can be found upon the following rearranged Eq.(3.3), derived from the system (3.2b) written based upon the cycles codes (3.2b). The indices are placed (assigned) in an algorithmic manner.

$$\omega_6\left[\frac{1}{N_{21}}\left(1 + N_{54}N_{65} + N_{21}\right) - 1 + N_{32}\right] = N_{32}\,\omega_3 . \tag{3.3}$$

After final rearrangements, the searched ratio is calculated

$$\frac{\omega_3}{\omega_6} = \left[1 - \frac{1}{N_{21}N_{32}} + \frac{N_{54}N_{65}}{N_{21}N_{32}}\right] = 1 + \frac{z_1}{z_3} + \frac{z_1 \cdot z_6}{z_4 \cdot z_3} = 2.479 . \tag{3.4}$$

In the case of the second gear, according to the data given in Tab.1, the input is connected to element 3 and the output is performed via element J. The adequate mixed graph is shown in Fig.2b. Therefore upon f-cycles (3.5a) we can write the system of Eq.(3.5b)

$$
\begin{array}{ll}
(2,3)J & \\
(1,2)J &
\end{array}
\qquad
\begin{cases}
\omega_2 - \omega_J = +N_{32}\left(\omega_3 - \omega_J\right) \\
\omega_1 - \omega_J = -N_{21}\left(\omega_2 - \omega_J\right) \\
\omega_1 = 0
\end{cases}
\tag{3.5a, b}
$$

Solving the system, we obtain the result

$$\frac{\omega_3}{\omega_J} = \frac{1}{N_{21}N_{32}} + 1 \,. \tag{3.6}$$

Finally, the ratio is as follows

$$\frac{\omega_3}{\omega_J} = 1 + \frac{1}{\frac{z_2}{z_1}\cdot\frac{z_3}{z_2}} = 1 + \frac{z_1}{z_3} = 1 + \frac{35}{73} = 1.479 \,. \tag{3.7}$$

In the case of the reverse gear, the mixed graph is shown in Fig.2c. Based on this graph, one can write the cycles (3.8a) and Eq.(3.8b)

$$
\begin{array}{ll}
(4,5)j & \\
(5,6)j &
\end{array}
\qquad
\begin{cases}
\omega_4 - \omega_j = -N_{54}\left(\omega_5 - \omega_j\right) \\
\omega_5 - \omega_j = +N_{65}\left(\omega_6 - \omega_j\right) \\
\omega_j = 0
\end{cases}
\tag{3.8a,b}
$$

The solution is as follows

$$\frac{\omega_4}{\omega_5} = -\frac{z_5}{z_4}\cdot\frac{z_6}{z_5} = -\frac{z_6}{z_4} = -\frac{73}{35} = -2.086 \,. \tag{3.9}$$

The calculations gave the results which are identical in the formulas and consequently in values to these published in Looman (1988).

3.2. Contour graph of the exemplary automobile gear

The contour graphs of the gear (shown in Fig.1) are presented in Fig.3. The contour graphs are built according to the related methods given in Marghitu (2005). The graphs collected in Fig.2 are assigned to the same drives as previously. For example, the double circle vertex in the contour graph (Fig.2a) is equivalent to the double line in the mixed graph: (*j*, 5) ↔j (case a i.e.: low gear, arm *j* is braked). Some chosen rules and remarks for all three graphs are collected in Tab.2. The contour graphs for the second and reverse gears are presented in Figs 2b and 2c, respectively.

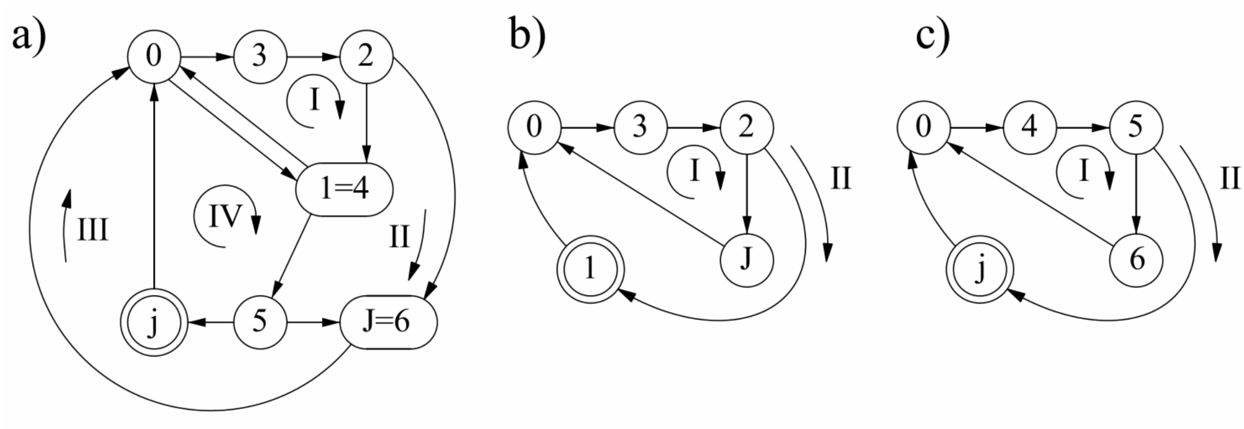

Fig.3. Contour graphs of the considered automobile gear: (a) low gear, (b) second gear, (c) reverse gear.

In the case of the low drive (gear), additional conditions are as follows

$$\omega_{i0} = \omega_i, \qquad \omega_J = \omega_6, \qquad \omega_1 = \omega_4, \qquad \omega_{0j} = 0. \qquad (3.10a,b,c,d)$$

The meaning of these conditions are: (3.10a) general rule that relative velocity in accordance to the support system is the absolute value of the adequate velocity; (3.10b) and (3.10c) special conditions connected with the design of the gear; (3.10d) working condition, i.e., element j is stopped (for the low gear). For example, contour I can be written as (0-3-2-1-0). Every contour (3.11a) starts and ends in the vertex 0 which allows the absolute rotational velocities to return into the system.

$$\begin{array}{ll} (0-3-2-1-0) & \left\{ \begin{array}{l} \omega_{30} + \omega_{23} + \omega_{12} + \omega_{01} = 0 \\ \omega_{23} \cdot r_C + \omega_{12} \cdot r_A = 0 \end{array} \right. \\ (0-3-2-J-0) & \quad \begin{array}{l} \omega_{30} + \omega_{23} + \omega_{J2} + \omega_{06} = 0 \\ \omega_{23} \cdot r_C + \omega_{32} \cdot r_B = 0 \end{array} \\ (0-4-5-6-0) & \quad \begin{array}{l} \omega_{40} + \omega_{54} + \omega_{65} + \omega_{06} = 0 \\ \omega_{54} \cdot r_A + \omega_{65} \cdot r_C = 0 \end{array} \\ (0-4-5-j-0) & \quad \begin{array}{l} \omega_{40} + \omega_{54} + \omega_{j5} + \omega_{0j} = 0 \\ \omega_{54} \cdot r_A + \omega_{j5} \cdot r_B = 0 \end{array} \end{array} \qquad (3.11a,b)$$

where: ω_{ij} - relative rotational velocity of link *i* according to link *j* and *r* radiuses equal to adequate pitch radiuses of geared elements, indices A, B, C are related to the points shown in Fig.1.

After some rearrangements of Eq.(3.11b), the following system (3.12) is obtained, where radiuses are exchanged into adequate numbers of teeth. The variable ω_4 should be excluded.

$$\frac{\omega_3 z_1 - \omega_4 z_1}{2z_2} = \frac{\omega_3 (z_2 + z_1) - \omega_6 (z_2 + z_1)}{z_2},$$
$$\frac{\omega_4 (z_2 + z_1)}{z_2} = \frac{\omega_4 (2z_2 + z_1) - \omega_6 (2z_2 + z_1)}{2z_2}. \tag{3.12}$$

The solution of the system can be calculated upon the derived system. It is worth underling that here - the final calculation formulas are different than previously but the numerical values are the same as in the afore-discussed methods. Further considerations for 2nd and reverse gears gave also similar results.

3.3. Bond graph modeling of an exemplary automobile gear

Bond graphs can also be used for modeling of gears (Brown, 2001; Merzouki *et al*., 2013; Richard and Bouazara, 2012). The bond graph or power flow graph consists of elements (vertices) and bonds (arcs). The power is calculated as a product of two variables, namely, the effort variable and flow variable $(P = e \cdot f)$. We choose torque as the effort variable and angular velocity as the flow variable, respectively, which is a standard approach. Such elements as **1**-nodes and **0**-nodes describe kinematical dependencies of the system components. The **TF** elements (**TF** for transformer) were used for describing the kinematics of an arbitrary pair of meshed wheels (gear ratio or the inverse of gear ratio). The inertial elements **I** were shown in the graph for completeness, but they were neglected in gear ratio calculations.

The bond graph model of the gear (presented in Fig.1) is shown in Fig.4. This bond graph is relevant to the low gear. The core set of equations associated with the graph has the following form

$$\dot{x}_{A1} = \omega_3 \frac{1}{2} m (z_1 + z_2), \qquad \omega_1 = \frac{2\dot{x}_{B1}}{m z_1},$$
$$2\dot{x}_C = \dot{x}_{A1} + \dot{x}_{B1}, \qquad \omega_5 = \frac{z_1 + 2z_2}{z_2}, \tag{3.13}$$
$$\omega_6 = \frac{2\dot{x}_C}{m(z_1 + z_2)}, \qquad \omega_4 = \frac{z_2}{z_1}\omega_5$$

where: notions are the same as previously, *m* - denotes the gear module which will be finally excluded from the final formula and rotational velocities.

Additional dependencies obtained from the bond graph were written down separately

$$\omega_3 = \omega_{IN}, \qquad \omega_4 = \omega_1, \qquad \omega_6 = \omega_{OUT}. \tag{3.14a,b,c}$$

After solving the system of Eqs (3.13) we get the gear ratio for the low gear.

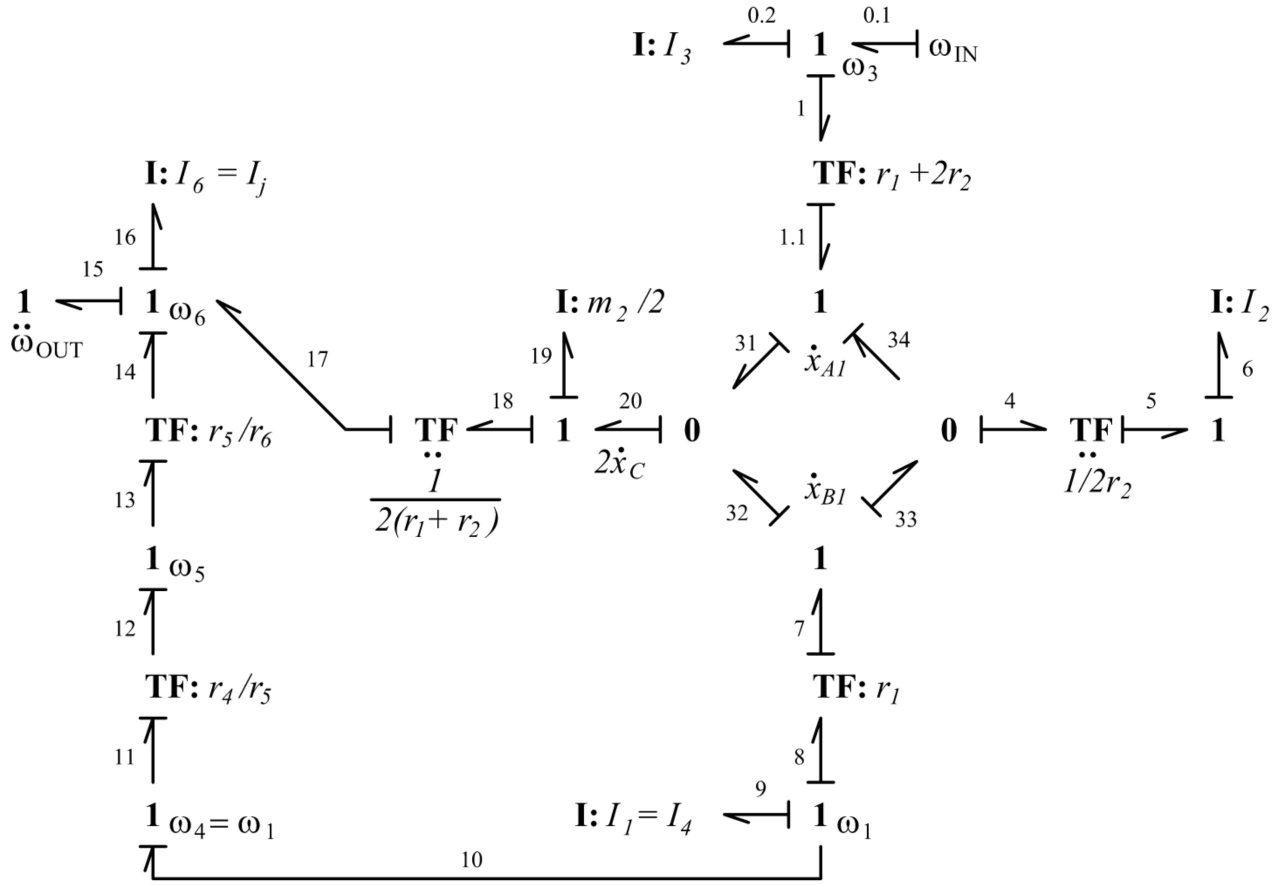

Fig.4. Bond graph of low drive of the gear.

The result is the same as previously. Other drives (gears) can be analyzed by means of similar bond graphs.

4. Final remarks and conclusions

The differences between the models as well as their comparisons are given in Tab.2. The similarities are also highlighted, e.g., sequence of design activities, usage of codes and usage of characteristic subgraphs like e.g., cycles or contours.

The obtained results show the usefulness of the graph-based approach for modeling of automobile gears. The simplicity of analysis of consecutive drives and comparisons of results are interesting features. Versatile analyses and simulations are possible (Luo Yutao and Tan Di, 2011; Kurth, 2012; Rajasri *et al.*, 2014; Richard and Bouazara, 2012; Zanasi and Grossi, 2009). Further considerations on the dynamics will be presented in a separate paper.

Table 2. Comparison of the graph-based methods.

No	Graph type	Mixed graph	Contour graph	Bond graph
	1	2	3	4
1	Rules of drawing	- edges and arcs are considered, - vertices are drawn as dots, descriptions are placed nearby, - double-line edge – for stopped part, - input(s) and output(s) are marked by additional arrows, - clique is shown as a shaded polygon for visual reason only.	- arcs, - vertices are drawn as circles, descriptions are placed inside the circles, - vertex 0 represents bearing system and support system, - double circle vertices for stopped part, - arcs can connect vertices in both directions.	- identification of system, - finding 0-nodes and 1-nodes, - building skeleton graph, - making some transformation according to the bond graphs theory, - derivation of equations according to bond graph rules.
2	Cycles	F-cycle are consider i.e. cycles having one stripped edge	Contours i.e. cycles starting and ending in 0 vertex; every contour has its circulation	Bond graph related subgraphs
3	Coding rules	Code of a cycle $(i, j)k$; where i, j – denotes mating gear elements and k denotes arm (carrier)	Contour code is: $(0\text{-}i\text{-}k\text{-}j\text{-}0)$ – consecutive list of elements which pass rotational movement	Nodes and elements (vertices) describe physical properties of system, since bonds (edges) describe power flow in the gear.
4	System of variables	- absolute rotational velocities, - local ratios expressed via number of teeth	- relative rotational velocities, - pitch radiuses and arm radiuses according to main gear axis.	- absolute velocities and rotational velocities and , - forces and torques,
5	Types of equations	- equivalent to Willis equations where indices are assigned to codes.	- vector equations of static equilibrium for velocities, - accelerations and products of these variables by adequate radiuses, where indices are assigned to codes.	- state system equations, - differential algebraic equations, - generalized Kirch-hoff's equations,
No	Graph type	Mixed graph	Contour graph	Bond graph
6	Special	Path from input(s) to output(s) representing passing of rotational motion and power throughout the gear.	Arcs in both directions between a pair of vertices.	Bond graph theory can unify description of different subsystems: mechanical, mechatronic, electrical, hydraulic, etc.
7	Further tasks performed via graph approach	Synthesis, reverse problem, enumeration of design solutions	Dynamical analyses	

Nomenclature

e – effort variable
f – flow variable
$\boldsymbol{I}$ – generalized inertial element of a bond graph
$(i,j)k$ – so called f-cycle built of elements i, j and k, where i, j denote a pair of mating elements and k denotes the arm (carrier) in basic planetary gear
m – module of geared wheels
Nji – local ratio of teeth numbers of two gear wheels
r_X – radius measured from main axis to the point X, $X = A, B, C$
$\boldsymbol{TF}$ – generalized transformer element of a bond graph
z_i – teeth number, i – description of a gear wheel
$\boldsymbol{0}$ – 0-node, satisfied the equations for adjacent bonds: $f_i = idem$, $\sum e_i = 0$
$\boldsymbol{1}$ – 1-node, satisfied the equations for adjacent bonds: $\sum f_i = 0$, $e_i = idem$
ω_{IN} – input rotational velocity of a gear (from motor)
ω_{ij} – relative rotational velocity of link i with respect to link j
ω_i – absolute rotational velocity of the wheel i
ω_{OUT} – output rotational velocity of a gear

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