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Parameters of the Signal: Their Non-Locality vs. Averaging Character

Abstract

A parameter of the signal segment is regarded as a measurand here. In a set of parameters the support of which consists of more than one element, a sub-set of averaging parameters is defined as those being reflexive, symmetric, bisymmetric, increasing monotonic, cancellable and continuous. These attributes, previously defined for discrete operations (B. Ellis, J. Pfanzagl), are in Section 3 reformulated for functional operations projecting signal segments into real numbers. Their meanings are explained and examples are given. Then, in Section 4, it is analysed to what degree these theoretical attributes are achieved in three kinds of circuits: in a low pass RC filter, in an integrator, and in a sample-and–hold circuit (S/H). The coefficient (12) was proposed for the evaluation of the circuit's ability to provide a symmetric parameter. This attribute, although possessed by the integrator almost perfectly, is not achievable in the filter and S/H circuit. Thus the quasi-instant parameters provided by the S/H cannot be considered as averaged. The period of an effective influence of the input signal on the output voltage, and the delay introduced by the S/H circuit, are defined.

Keywords: measurement subject; signal segment; quasi-instant parameter; averaged parameter; integrator; sample-and-hold circuit.

1. Introduction

Physical observations concern signals, i.e. dependencies of one individual quantity [1] on some number of other individual quantities: $\tilde{y} = f_{phys}(\tilde{x}_1, \tilde{x}_2, .., \tilde{x}_K)$. Exemplary influencing quantities $\tilde{x}_i$ are: time, spatial coordinates, temperature, frequency, amongst many others. If for all quantities $\tilde{y}, \tilde{x}_1, \tilde{x}_2, .., \tilde{x}_K$ appropriate measurement scales are established, then each individual quantity $\tilde{x}_i$ is represented by the appropriate number x_i, and the signal f_{phys} may be identified with some, usually unknown, numerical function $y = f_{num}(x_1, x_2, .. x_K)$. In practice we always deal with a part of the signal for which all its independent variables belong to intervals of limited lengths $(x_{i\,min};\ x_{i\,max})$, $i = 1, .., K$. This part of the signal is here referred to as a signal segment.

If the anticipated range of changes of all the influencing quantities $\tilde{x}_i$ are sufficiently narrow that the corresponding changes of quantity $\tilde{y}$ are negligible, then descriptions of the variability ranges of x_i become a part of the individual quantity $\tilde{y}$ definition. If, however, for at least one influencing quantity, the anticipated range of changes is so large that the corresponding variation of $\tilde{y}$ is significant, e.g., greater than the required resolution of measurement, then a classical model of a measurand as a single specific individual quantity, is no longer maintainable. It namely becomes important which aspect of the variability of the signal $y = f_{num}(x_1, x_2, .. x_K)$ is of interest and should be expressed by a numerical result of the measurement. Pointing out this property of interest consists in defining some parameter of the segment. This definition states a functional projection of the signal's segment, being a vector, into a set of real numbers:

$$\mathcal{P}:\ f_{num}(x_1, x_2, .. x_K)|_{x_i \in (x_{p\,i\,min};\, x_{p\,i\,max}),\ i=1,..,K} \longrightarrow \mathbb{R}\,. \quad (1)$$

The letter "p" in the lower index signifies that the corresponding limit comes from the parameter definition. The interval $(x_{p\,i\,min};\ x_{p\,i\,max})$ fails to have to equal the range $(x_{i\,min};\ x_{i\,max})$ for which the segment of the signal is available, but inclusion should be satisfied: $(x_{p\,i\,min};\ x_{p\,i\,max}) \subset (x_{i\,min};\ x_{i\,max})$. When $(x_{p\,i\,min};\ x_{p\,i\,max}) \neq (x_{i\,min};\ x_{i\,max})$ the problem of parameter representativeness for the segment becomes important.

For signals of only one variable $y = f(x)$ (that are considered for simplicity's sake in the following) signal mean and effective values, the value of the signal's autocorrelation function for a given lag, the signal's global extremes over some interval of x, or even the value of the signal itself for some given value of x, are examples of the signal's parameters.

Thus, the segment of a signal is what actually subjects to any type of measurement. Therefore, in [2] the parameter of the signal segment has been defined as a measurand. This includes, of course, all classical cases where the measurand is stated as a value of a single manifestation of an individual quantity. Physical and technical reasons for the redefinition and widening of the notion of the measurand are also discussed in [2]. Although for macroscopic quantities strictly local or instantaneous results cannot be obtained, there are more and more measuring devices providing high resolution results that refer to non-zero-length time intervals or non-zero-surface spatial points. For such devices it is important to know in which way and how strongly the values of the signal in particular places, or in particular time instants, influence the result of the measurement. On the other hand, there are also measuring techniques that make it possible to obtain an averaged result of a temporal signal in a single comparison procedure [7, 8]. Because the theory of measurement, and especially the notions used within it, should include all the possible ways in which a measurement is (or might be) performed in practice, the notion of the measurand required a generalization.

An important part of the definition (1) is the description of the set $\mathcal{T}$ of independent quantity manifestations for which the parameter is influenced by the signal's values. This set is referred to as a parameter support. If the support of a parameter is formed solely of one element, then such a parameter is called a point parameter, i.e. instantaneous for temporal signals, local for spatial signals, etc. Opposed to them, non-point parameters involve signal values for more than one point. Most of the practically met parameters have supports in the form of an interval or a finite sum of the exclusive intervals. Although non-point parameters are often called "averaged" it seems to be an overuse of this term, since for averaged parameters some special properties are needed.

Awareness of the fact that averaged parameters constitute only a sub-set of all non-point parameters is not new (cf. J. Pfanzagl [4], B. Ellis [3], S.S. Stevens [5]). Formal attributes provided by J. Pfanzagl and by B. Ellis express precisely, although in a slightly different way, what is intuitively understood by averaging. However, both these sets of averaging attributes relate to discrete cases, where a finite set of numbers is mapped into a single number; their average. Acceptance that a measurand is determined by some parameter of the signal's segment, requires the expansion of an idea of averaging for such cases where the support of the parameter is an interval. An appropriate reformulation of averaging attributes for functional mappings constitutes the main topic of this paper. It was also interesting, how these theoretical demands are met in real circuits.

The paper is organized as follows: In Section 2 the attributes of a discrete averaging in the approaches of B. Ellis, J. Pfanzagl, and S.S. Stevens were remembered and compared. Because the concept of the measurand is now expanded to be a parameter of some signal segment, a transposition of the main ideas of discrete averaging into a domain of functional projections was necessary; this is done in Section 3. Then, in Section 4, it is shown that a theoretical requirement of symmetry cannot be satisfied perfectly in the circuits typically used in a physical averaging and is impossible to be achieved in a quasi-instant sampling.

2. Attributes of a discrete averaging

Two sets of attributes of a discrete averaging may be found in references. The one, by B. Ellis ([3], pp. 68-73), concerns operations of two or more arguments and consists of four attributes. The other, by J. Pfanzagl ([4], pp. 82-84, and p. 67), concerns two-arguments' operations and consists of five demands. Although in both approaches the expectations regarding an averaging are basically the same, there is an additional demand in Ellis's approach that any operation to be averaged must contain a special function (called the average generating function) being single-valued, strictly increasing in each argument, commutative, associative, and Archimedean. Thus Ellis partially prejudges a possible structure of any averaging operation. The attributes stated by Pfanzagl (for the notion originally named "middling") seem to be more elementary, as not making any assumption concerning a generating function. Therefore, they are taken here as a starting point.

According to Pfanzagl's definition, a binary operation $\circ$ performed over elements from some metric space $\mathcal{Z}$, to be middling must be: (a) reflexive, i.e. $\bigwedge z \in \mathcal{Z}:\ z \circ z = z$; (b) commutative, i.e., $\bigwedge z_1, z_2 \in \mathcal{Z}: z_1 \circ z_2 = z_2 \circ z_1$; (c) bi-symmetrical, i.e., $\bigwedge z_1, z_2, z_3, z_4 \in \mathcal{Z}:\ (z_1 \circ z_2) \circ (z_3 \circ z_4) = (z_1 \circ z_3) \circ (z_2 \circ z_4)$; (d) cancellable, i.e., $\bigwedge z_1, z_2, z_3 \in \mathcal{Z}$: $(z_1 \circ z_2) = (z_1 \circ z_3) \implies z_2 = z_3$; (e) continuous with regard to both arguments.

The requirement of reflexivity expresses an intuition that an averaging of identical elements should return the same element. The result of averaging should also be independent of the order of arguments, that is described by the demand of commutativity. The majority of the averaging operations fail to be associative (in fact, even a simple arithmetic mean is not associative). The demand of bisymmetry, is weaker than that of associativity. It states that the averaging of more than two elements may be performed as a multi-stage binary operation, in which all possible permutations of the elements in the first stage of averaging should yield the same result. The requirement of cancellability means that if both the result of averaging and also one of its two arguments are known, then the value of the other argument can be determined unequivocally. Considered as a function of two variables, the cancellable binary operation is just a one-to-one function with regard to both variables. For the demand of continuity, it becomes important that the averaged elements come from a space where a measure of a distance is defined. Continuity means that a sufficiently small change of any argument should result in a willingly small variation of the averaged value.

It might seem surprising that among Pfanzagl's attributes there is not an explicit demand for averaging being strictly increasing in each argument. In fact, this property is just a consequence of other assumptions. These are, namely, the assumptions of $\circ$ being reflexive and commutative, and also the assumption that it is a one-to-one and continuous function of a connected and ordered set into an ordered set ([4] pp. 67, 69, 83).

S. Stevens and B. Ellis suggest that the appropriateness of a given procedure as an averaging should depend also on the type of measurement scale on which the quantity $\tilde{y}$ is measured. According to Stevens [6], the type of the scale is determined by a class of permissible transformations ϕ by which the values y_i of individual quantities $\tilde{y}_i$ may be changed while still not losing information about the relations between $\tilde{y}_i$. Ellis demands that any operation $aver(\blacksquare)$ to be a proper averaging should satisfy:

$$aver\big(\phi(y_1), \dots, \phi(y_n)\big) = \phi\big(aver(y_1, \dots, y_n)\big), \qquad (2)$$

i.e., an averaging of the values $\phi(y_i)$ in a new scale should equal the ϕ-transform of the average obtained for the values y_i before the change of the scale.

All the above conditions may be applied directly only to the parameters, the support of which, consists of a limited number of isolated points. Since the parameter supports met in practice usually consist of intervals of finite and non-zero lengths, the reformulation of Pfanzagl's attributes into functional projections is necessary.

3. Transposition to signal segments

In the following, the signal is assumed to be a non-implicitly defined, one variable function $y = f(x)$. Here, x denotes a value of some quantity $\tilde{x}$ measured on at least an interval scale, while y signifies a value of the $\tilde{y}$ quantity measured on a ratio scale. Spatial coordinates and time instances are very typical examples of the signal's independent quantity $\tilde{x}$. Thanks to the above restriction on the scale types, expressions such as $\Delta y/\Delta x$ and $y \cdot \Delta x$, being essential elements of definitions of a derivative of the signal $y = f(x)$ and of its definite integral, respectively, are both physically meaningful.

Reflexivity: The parameter $\mathcal{P}$ assigned to a given segment of the signal $y = f(x)|_{x\in[x_{p\,min};\, x_{p\,max}]}$ will be referred to as reflexive if, for any signal that is constant in that interval, and therein assumes the value y_0, i.e., $f(x) \equiv y_0$ for $x \in [x_{p\,min};\ x_{p\,max}]$, the value of the parameter is: $\mathcal{P}\left[f(x)|_{x\in[x_{p\,min};\, x_{p\,max}]}\right] = y_0$.

Reflexivity is an important property of the parameter. It signifies that for sufficiently short intervals $[x_{p\,min};\ x_{p\,max}]$, over which the signal may be assumed to be almost constant, the value of the reflexive parameter $\mathcal{P}$, as equaling the value y_0 of the signal itself, may be regarded as its quasi-point value. For instance, the mean-square value of the signal: $\frac{1}{(x_{p\,max}-x_{p\,min})}\int_{x_{p\,min}}^{x_{p\,max}} f^2(x)\,\mathrm{d}x$ is not reflexive because the value it assigns to a constant signal equals y_0^2 instead of y_0. In contrast, any parameter of the form:

$$\varphi^{-1}\left\{\frac{1}{(x_{p\,max}-x_{p\,min})}\int_{x_{p\,min}}^{x_{p\,max}} \varphi[f(x)]\,\mathrm{d}x\right\} \qquad (3)$$

where φ is a continuous and strictly increasing function, for which the inverse function φ^{-1} exists, is reflexive.

An important application of reflexivity is the determination of the effective value of temporary signals. If it is known that $\mathcal{P}[f(t)|_{t\in\tau}] = \mathcal{P}[h(t)|_{t\in\tau}]$, and the second segment is a constant signal $h(t)|_{t\in\tau} \equiv h_0$ the value of which is known (e.g., because it is sourced by a calibrator), then we can conclude that $\mathcal{P}[f(t)|_{t\in\tau}] = h_0$. The effective value of the signal fails to be strictly reflexive because squaring in (3) $\varphi(y) = y^2$, not being a one-to-one function, loses information about the sign. Nevertheless, this information is not important for the effective value. The property of reflexivity has been employed in a variety of the so-called ac/dc transfer methods and devices (c.f., e.g. [12-14]).

Symmetry: The parameter $\mathcal{P}$ of the signal segment $f(x)|_{x\in[x_{p\,min};\, x_{p\,max}]}$ is referred to as symmetric if an interchange of any two, mutually exclusive, sub-segments of equal and non-zero lengths $\Delta x > 0$ does not influence the value of that parameter, Fig. 1a. In other words, the locations of the sub-segments of some signal segment should not have an influence on the value of the symmetric parameter. Symmetry is an analog of the discrete property of commutativity.

Parameters that have the structure (3) are symmetric. The mutual interchange of the sub-segments, usually destroys signal continuity, but it neither destroys its integrability nor changes the value of (3). On the other hand, any parameter in the definition of which the signal is multiplied by a weighting (window) function

$A(x)$, other than rectangular, has no chance to be symmetric. It is because the sub-segments of the signal, weighted by different values of the window function $A(x) \cdot f(x)$, would influence the value of the parameter depending on their locations, and thus non-uniformly.

Bisymmetry: If one accepts that the parameter of the signal segment is an adequate model of a measurand, then measurement constitutes a type of functional projection (1), reducing vectors to scalars. Sometimes, a necessity arises in practice, of some parameter evaluation for many sets of x of the same signal, or for different signals. It takes place, e.g., when the shape of the highly noised periodic signal is reconstructed in a digital oscilloscope, or when the mean value of a random, non-ergodic signal is to be measured. Because the majority of averaging operations lack associativity, the idea of the two-stage evaluation of the required parameter seems to be promising. However, the second-stage evaluation concerns real numbers (being the results of the first-stage averaging). Therefore, a formulation of the additional rule becomes necessary, describing how the first-stage results should be combined during the second stage.

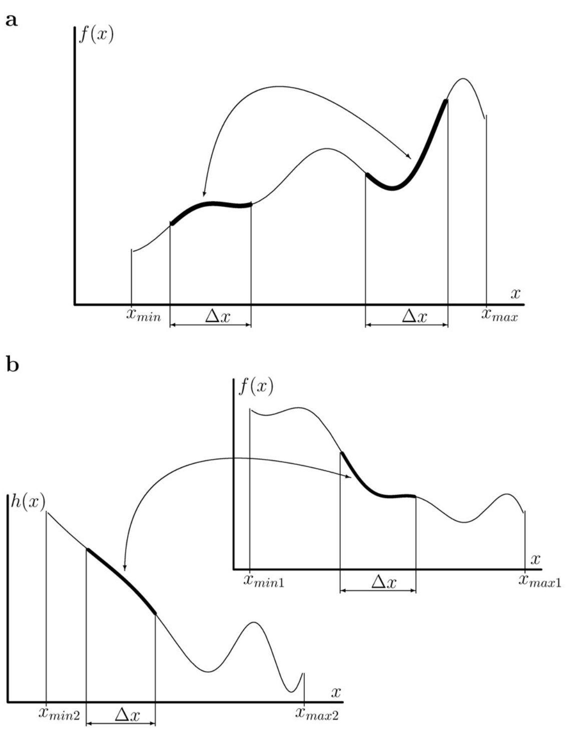

Fig. 1. Symmetry (a) and bisymmetry (b) - the two uniformity of influence aspects of non-zero length sub-segments of the signal on the parameter value

Let us consider two separate segments of the same signal or the segments of two different signals: $f(x)|_{x\in[x_{min1};\, x_{max1}]}$ and $h(x)|_{x\in[x_{min2};\, x_{max2}]}$, Fig. 1b. For these segments, two formulae are given: the first - defines a parameter $\mathcal{P}[\cdot]$ of the signal segment, and the second - combines the values of the first-stage averaging. The parameter $\mathcal{P}$, together with that specific combining formula, will be referred to as bisymmetric if the mutual interchange of any two sub-segments of the above segments, both of equal and non-zero lengths, that additionally satisfy:

$$0 < \Delta x \leq min\{x_{max1} - x_{min1};\, x_{max2} - x_{min2}\}, \tag{4}$$

does not influences the value of the final result, Fig. 1b. The symbol min $\{\cdot\}$ in (4) signifies the smallest value from those listed in the braces.

For the example family (3) of parameters, a corresponding combining formula may have the form:

$$\varphi^{-1}\left\{\frac{(x_{max1}-x_{min1})\varphi[\mathcal{P}^1]+(x_{max2}-x_{min2})\varphi[\mathcal{P}^2]}{(x_{max2}-x_{min2})+(x_{max1}-x_{min1})}\right\} \tag{5}$$

where $\mathcal{P}^1 = \mathcal{P}\left[f(x)|_{x\in[x_{min1};\, x_{max1}]}\right]$ and $\mathcal{P}^2 = \mathcal{P}\left[h(x)|_{x\in[x_{min2};\, x_{max2}]}\right]$ are the values of the first-stage averaging. It is easy to verify that the parameters (3), together with (5), are bisymmetric. On the other hand, just changing the weights standing by $\varphi[\mathcal{P}^1]$ and $\varphi[\mathcal{P}^2]$ in formula (5), and making them unequal to the lengths of the corresponding supports, causes a loss of bisymmetry. Any parameter not being symmetric is unable to be bisymmetric, as well.

Monotonicity: For discrete averaging, its monotonicity was not separately declared. That was because being a strictly increasing operation in relation to both arguments was then a consequence of assumptions mentioned in Section 2. This is mainly due to being a one-to-one and continuous function. On the other hand, a functional projection (1) is not one-to-one since it generally assigns the same number to many signal segments. Moreover, the order relation, being a core concept of the definition of monotonicity, has no natural understanding in the case of signal segments. For our purpose of defining an increasing monotonicity of averaging, a requirement for being "greater than" among segments may be as follows: The segment $f(x)|_{x\in(x_{min};\, x_{max})}$ is to be called greater than the segment $h(x)|_{x\in(x_{min};\, x_{max})}$, if there is:

$$\min_{x\in(x_{min};\, x_{max})} f(x) \;>^{a.e.}\; \max_{x\in(x_{min};\, x_{max})} h(x). \tag{6}$$

Two elements of the above definition should be emphasized: (i) the selection of the minimum and maximum values creates the relation (6) independent of an interchange of any sub-segments, as in Fig. 1a; (ii) The letters "a.e." stand for almost everywhere, i.e., everywhere except, possibly, at a finite number of isolated points. The usage of this weakening condition in (6) eliminates the influence on the segments' order of the values assumed by signals solely in isolated points, without necessity of the bandwidth limitation. Thus, segments containing sharp "steps", being an important theoretical model of the signal's behavior, can still be considered.

Now, a parameter $\mathcal{P}[\cdot]$ can be considered increasing if from the relation (6) it follows that:

$$\mathcal{P}\left[f(x)|_{x\in[x_{min};\, x_{max}]}\right] > \mathcal{P}\left[h(x)|_{x\in[x_{min};\, x_{max}]}\right], \tag{7}$$

i.e., greater values of the parameter are assigned to greater signal segments.

In the parameter class definition (3), the requirement that the function φ should be increasing, guarantees that such parameters are increasing in the above sense.

Cancellability of a parameter $\mathcal{P}[\cdot]$ is a property that becomes significant when two segments $f(x)|_{x\in\tau}$ and $h(x)|_{x\in\tau}$ are considered, where the support τ stands for an interval $\tau = (x_{min};\, x_{max})$. Let the symbols τ_{s1}, τ_{s2} denote proper sub-intervals of τ, both of the same and non-zero length Δx (4), and both entirely although freely included in τ, just like those in Fig.1b. Then, let the values of the parameter $\mathcal{P}$ over the intervals τ, τ_{s1}, and τ_{s2} satisfy the following conditions:
(i) $\mathcal{P}[f(x)|_{x\in\tau}] = \mathcal{P}[h(x)|_{x\in\tau}]$
(ii) $\mathcal{P}\left[f(x)|_{x\in\tau_{s1}}\right] = \mathcal{P}\left[h(x)|_{x\in\tau_{s2}}\right]$.
If from (i) and (ii) it follows that:
(iii) $\mathcal{P}\left[f(x)|_{x\in\tau\setminus\tau_{s1}}\right] = \mathcal{P}\left[h(x)|_{x\in\tau\setminus\tau_{s2}}\right]$
then the parameter $\mathcal{P}[\cdot]$ is referred to as cancellable. Here, $\tau \setminus \tau_{s1}$ stands for a complement of the set τ_{s1} in τ.

The supports' lengths $(x_{max1} - x_{min1})$ and $(x_{max2} - x_{min2})$ of the signals $f(x)$ and $h(x)$, respectively, and the lengths Δx_{s1}, Δx_{s2} of their sub-supports τ_{s1}, τ_{s2} don't have to be equal. However, the relation should be satisfied: $\frac{\Delta x_{s1}}{(x_{max1} - x_{min1})} = \frac{\Delta x_{s2}}{(x_{max2} - x_{min2})}$.

Together with the demand of reflexivity, cancellability of a given parameter means that the knowledge of any two from the following three values: $\mathcal{P}[f(x)|_{x\in\tau}]$, $\mathcal{P}[f(x)|_{x\in\tau_{s1}}]$, $\mathcal{P}[f(x)|_{x\in\tau\setminus\tau_{s1}}]$ signifies the possibility of determining the third one. Unfortunately, for many instruments which are averaging temporary signals, the length of the averaging interval, although known, is not always controllable. Moreover, a temporal location of this interval is usually even not defined. Therefore, the inclusion $\tau_{s1} \subset \tau$ cannot be verified, and we are still unable to recalculate a parameter value for another averaging interval. Acquisition boards are positive exceptions in this regard, but with their quasi-instant sampling and a/d conversion, they are devoted to averaging in a domain of numbers.

Cancellability of a parameter $\mathcal{P}[\cdot]$ means: (a) actual dependence of this parameter value on any sub-segment of the non-zero length included in $[x_{p\,min}; x_{p\,max}]$, (b) uniform influence of all sub-segments on $\mathcal{P}$, and (c) univocality of that influence.

The maximum value over some interval, is an example of the non-cancellable parameter. On the other hand, for all parameters obtained from (3) where φ is a power function of an odd order, cancellability is satisfied.

Continuity: In the case of assigning a numerical parameter to a signal segment (1), some measures of distance must be defined both on the space of signals (being arguments of functional mapping (1)), and on the space of numbers (being its results). Although the latter may be just a simple modulus of the numbers' difference, the former needs some integral definition of a distance between the functions' segments. For this, the root-mean-square norm: $\sqrt{\int_{x_{min}}^{x_{max}} [f_1(x) - f_2(x)]^2\,\mathrm{d}x}$ may be used. The example (3) of a parameter definition consists of three mappings: φ, φ^{-1}, and $\frac{1}{(x_{p\,max} - x_{p\,min})} \int_{x_{p\,min}}^{x_{p\,max}} \ldots\,\mathrm{d}x$. The two former mappings are continuous by the assumption (3). The latter operator is bounded, e.g., if the root-mean-square norm is used. Its boundedness can be verified by the use of the Schwarz-Bunyakovsky inequality. Being linear and bounded, that operator is thus continuous with regard to the same norm. Therefore, any parameter of the form (3) must be continuous as a composition of three continuous functions.

4. How well the theoretical attributes of averaging are reached in real circuits

In this section, it will be verified whether properties of two real circuits, typically used for "averaging", namely a low pass RC filter and an integrator, comply with the theoretical requirements defined above. Then, also a sample-and-hold (S/H) circuit, in the phase of tracking, is analyzed. This circuit is basically used for memorizing the instant values of temporal signals. It is well known, however, that the segment actually influencing a remembered sample cannot have a zero length. Therefore, whether an output voltage of S/H complies with the theoretical requirements of averaging, was analyzed.

The relationship between input and output voltages in all three examples is going to be analyzed in time domain, i.e. the independent quantity x in the above equations describes now time instants t. In the case of a linear and stationary circuit, the output signal $u_{out}(t)$ is described by the following convolution integral:

$$\mathcal{P}[u_{in}(\tau)|_{\tau\in[0;\,t]}] = u_{out}(t) = \int_0^t u_{in}(\tau)\,h(t-\tau)\,\mathrm{d}\tau, \tag{8}$$

where $h(\cdot)$ denotes an impulse response of the circuit. It is assumed that the input signal $u_{in}(\tau)$ starts to influence the circuit at the instant $\tau = 0$, while the state of the circuit's output is observed at $\tau = t$.

If the input signal is constant, then from (8) it results that u_{out} is, though not equal to u_{in}, still it is proportional to it. Thus, the value of $u_{out}(t)$ may be easily made a reflexive parameter of the input signal just by dividing the obtained value of $u_{out}(t)$ by a coefficient $\int_0^t h(t-\tau)\,\mathrm{d}\tau$. Next, the monotonicity and continuity of (8) are guaranteed by the properties of the integral.

In practice, the requirement of symmetry of (8) seems to be the most troublesome. Symmetry means that a mutual interchange of any two sub-segments of the signal should not influence the value of the parameter. In the convolution integral (8) the impulse response $h(\cdots)$ plays the role of the weighting function, after its mirroring and shifting right by t. Unfortunately, the weight $|h(t-\tau)|$ is usually not constant over an interval $\tau \in [0;\,t]$, which excludes the strict symmetry of this type of defined parameter. This problem was investigated in detail for these three particular circuits.

A low pass RC filter: The simplest structure of it (Fig. 2a) may be an approximate model of not only electrical but also mechanical systems. The impulse response of this type of circuit has the form:

$$h(t) = \frac{1}{K_i}\exp\left(\frac{-t}{K_i}\right), \tag{9}$$

where $K_i = RC$ stands for a time constant of the circuit. It results from (8) and (9) that input voltages $u_{in}(\tau)$ influence $u_{out}(t)$ with the weights close to 1 only for time instances immediately preceding the instant of observation t. Strictly speaking, for $\tau \in (t - 0.05K_i;\, t)$ the input signal $u_{in}(\tau)$ influences the output with weights from 0.95 to 1, and in this time interval any sub-segments of $u_{in}(\tau)$ may be mutually interchanged without a significant variation of the output voltage. For earlier τ's, however, the values of the weighting function $h(t-\tau)$ in (8) decrease gradually instead of sharply. Therefore, for a relatively long interval: $t - 3K_i < \tau < t - 0.05K_i$ the influence of the input voltage on u_{out}, still being significant, is variable, and this is why the output voltage $u_{out}(t)$ is a non-symmetric parameter of the input signal segment $u_{in}(\tau)|_{\tau\in[0;\,t]}$. To obtain symmetry, the value of the time constant K_i should be increased, while the interval time in which the input voltage is allowed to influence the output, should be shortened, e.g. by the use of analog switches. This idea is applied in the circuit of the integrator.

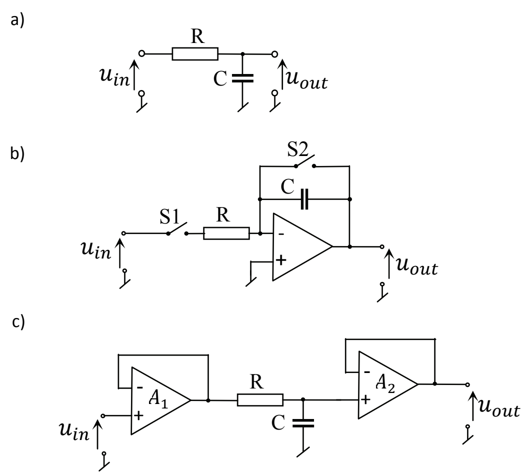

Fig. 2. The output voltages of these circuits concern the input signals of the non-zero length: a) low pass RC filter, b) integrator, c) sample-and-hold circuit

Integrator: Fig. 2b) shows a simplified structure of this circuit. The following analyses concern the phase of integration, when the switch S1 is on, while a discharging switch S2 is off. It was assumed that the integration phase starts at $\tau = 0$, when the capacitor C is completely discharged, i.e. $U_C(0_-) = 0$. The integration completes at $\tau = t$, and consists in switching both the keys off.

In all the analyses that follow, operational amplifiers are assumed to be fully compensated by applying a method of a dominating pole, i.e. revealing a frequency characteristic in the form, [10, 11]:

$$H(\mathrm{j}\omega) = \frac{A_{ud}}{1+\mathrm{j}\omega K_a}, \tag{10}$$

valid at least up to frequencies when $|H(\mathrm{j}\omega)| \le 1$. In Eq. (10) A_{ud} states for a differential gain for low frequencies, while K_a denotes an amplifier time constant. The latter may be determined as $K_a = 1/\omega_u$, where ω_u states for –3dB an open loop upper angular frequency of the amplifier ($\omega = 2\pi f$).

The obtained impulse response of the integrator from Fig. 2b) reads:

$$h(\tau) \cong -\frac{A_{ud}\left[\exp\left(\frac{-\tau}{2K_a}\right)-\exp\left(\frac{-\tau(K_a+A_{ud}K_i)}{K_a K_i}\right)\right]}{K_a+A_{ud}K_i}, \tag{11}$$

where the integrator time constant equals $K_i = RC$. Simplifications applied in Eq. (11) are almost negligible. They were based on the fact that $A_{ud} \gg 1$, while precise operational amplifiers, applied typically in integrator circuits, reveal $A_{ud} \sim 10^5 \div 10^7$.

A shape of the function (11), drawn as a modulus of the weighting function occurring in (8), is presented in Fig. 3. The weighting function is visibly not constant, especially at the end of the integration phase, i.e. over the interval $[t_{max}; t]$. The length of this interval is $K_a \ln(2A_{ud})/A_{ud}$. For parameters of the operational amplifiers typically used in integrators, i.e. for $A_{ud} = 10^6$, $K_a = 1/(2\pi \cdot 1\ \mathrm{Hz}) = 159$ ms, and $K_i = 1$ ms (which corresponds e.g. $R = 100$ kΩ and $C = 10$ nF) this length equals $t - t_{max} \cong 2.31$ μs. This value significantly exceeds turn on/off times of typical analog switches $(\sim 10\ \mathrm{ns})$ and is therefore, usually not negligible.

The fact that the impulse response h is not constant in the entire range of $\tau \in [0; t]$ might be described by the coefficient (cf. Fig. 3): $\frac{h_{max}-h_{min}}{2} / \frac{h_{max}+h_{min}}{2}$. This measure, however, does not takes into account the fact of assuming by h values close to zero over a short final subinterval of $\tau \in [t_{max}; t]$. To measure properly not only a "non-flatness" but also a "non-rectangularity" of the $|h(t-\tau)|$ over the entire interval $\tau \in [0; t]$ the following coefficient seems to be appropriate:

$$\kappa = \frac{\int_0^t [h_{max} - h(t-\tau)]d\tau}{h_{max} \cdot t}. \tag{12}$$

A numerator of this quotient just measures a shadowed area in Fig. 3. For $|h(t-\tau)|$ that is in the shape of a perfect rectangle, i.e. is constant over the interval $\tau \in [0; t]$, and disappears rapidly for $\tau > t$, the coefficient κ would equal zero.

For short integration intervals t, the behavior of $|h(t-\tau)|$ to the right from t_{max} will cause an increase of κ. On the other hand, for the very long integration intervals, a drop of h to the left of t_{max} becomes a dominating reason of κ increase. Therefore, an optimum duration of the integration phase may be expected. The dependence of κ on the integration period is presented in Fig. 4 for three pairs of the amplifier parameters, corresponding to more and more precise elements.

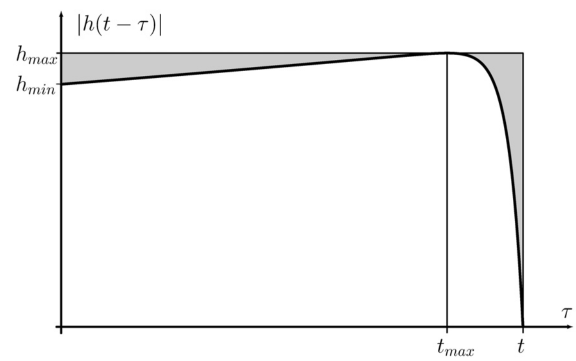

Fig. 3. Impulse response (11) of the integrator as a weighting function in the parameter definition (8). In this diagram values of the parameters in (11) have been chosen that differ from those in the text in order to better expose characteristic elements of the graph

As it is visible from the plots, for integration periods t greater than optimal, κ increases almost linearly and the slower it is the more precise a given amplifier is. For the most precise amplifiers of $A_{ud} = 10^7$, $K_a = 1.59\ s$ (that corresponds to $f_u = 0.1$ Hz) one obtains $\kappa_{min} \cong 0.031\%$ for the integration period of $t \cong 1.006$ ms. Then, an interval $(t_{max};\ t)$, of significant variability of $|h(t-\tau)|$, has the length $t - t_{max} \cong 2.68$ μs, which amounts only to 0.266% of the entire period of integration. In addition, Fig. 4 shows that even for ordinary operational amplifiers, revealing $A_{ud} = 10^5$, and $K_a = 1/(2\pi \cdot 10\ \mathrm{Hz}) = 15.9$ ms, the values of κ less than 5% are easy obtainable. This means that the value of the weighting function $|h(t-\tau)|$ in (8), although not being perfectly constant, is sufficiently constant to guarantee an approximate symmetry of the parameter (8) over almost the entire interval of integration $[0; t]$.

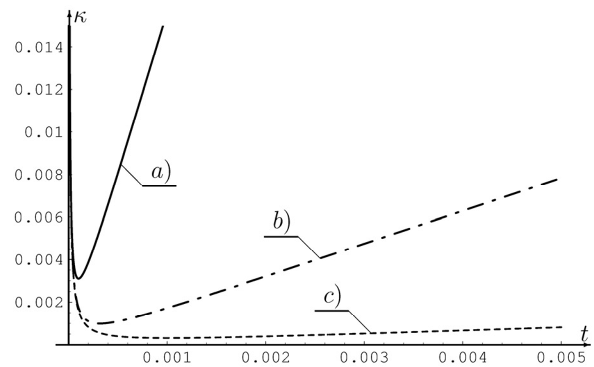

Fig. 4. The dependence of the weighting function $|h(t-\tau)|$ "rectangularity" coefficient κ (12) on the length t of the integration interval: a) $A_{ud} = 10^5$, $K_a = 1/(2\pi \cdot 10\ \mathrm{Hz}) = 15.9$ ms, b) $A_{ud} = 10^6$, $K_a = 1/(2\pi \cdot 1\ \mathrm{Hz}) = 159$ ms, c) $A_{ud} = 10^7$, $K_a = 1/(2\pi \cdot 0.1\ \mathrm{Hz}) = 1.59$ s

In the above analyses, many important properties both of operational amplifiers and of switches were not taken into account. For instance, the influence of the input offset voltage and the bias currents of the amplifier (both being integrated together with the input voltage), the influence of a limited slew rate of the amplifier, etc., were omitted. Most of them have already been considered in available textbooks, cf., e.g., [9]. Here, however, the problem of symmetry of the parameter defined by the output voltage of an integrator was solely of interest.

A sample-and-hold circuit: The purpose of this sub-section is the analysis of the impulse response of the S/H circuit during its tracking phase. It is interesting whether this response $|h(t-\tau)|$, that plays the role of a weighting function in (8), is sufficiently constant to consider output voltage as a symmetric parameter of the input signal segment. Although there are a considerable variety of possible structures of S/H circuits [10], the one presented in Fig. 2c) seems to be best suited for the construction of fast sampling circuits, i.e. such the output voltage of which constitutes a parameter (8) of a relatively short support. Unfortunately, due to its not having an external feedback path, the chosen structure is not most precise.

Both amplifiers in Fig. 2c) are assumed to have a transmittance function in the form (10) up to frequency at which $|H(\mathrm{j}\omega)| \leq 1$, at least. Both have to be fast and accurate, and additionally bias currents of the second amplifier should be as low as possible. The latter demand means that in the input stage of this amplifier FETs should be used.

In Fig. 2c, R stands for the sum of the output resistance of the first amplifier and the on-resistance of an analog switch, while C stands for a holding capacitance. In numerical calculations these parameters were assumed to equal: $R = 50\ \Omega$, and $C = 0.5$ nF.

The following structure was obtained for an impulse response of the circuit from Fig. 2c:

$$h(t) = A_{ud1} \cdot A_{ud2} \left[\frac{\exp\left(\frac{-t}{K}\right)}{C_1} + \frac{\exp\left(\frac{-t}{K_2}\right)}{C_2} + \frac{\exp\left(\frac{-t}{K_3}\right)}{C_3} \right], \qquad (13)$$

where:

$$K = RC,\ K_2 \cong \frac{K_{a2}}{A_{ud2}},\ K_3 \cong \frac{K_{a1}}{A_{ud1}},$$

$$C_1 \cong \frac{(K\,A_{ud1} - K_{a1})(K\,A_{ud2} - K_{a2})}{K},\ C_2 \cong \frac{(K\,A_{ud2} - K_{a2})(K_{a1}\,A_{ud2} - K_{a2}\,A_{ud1})}{K_{a2}},$$

$$C_3 \cong -\frac{(K\,A_{ud1} - K_{a1})(K_{a1}\,A_{ud2} - K_{a2}\,A_{ud1})}{K_{a1}}.$$

In the above equations the symbols A_{ud1}, and A_{ud2} signify low frequency gains of the respective amplifiers, while K_{a1}, and K_{a2} stand for their time constants. The value of $h(t)$ in Eq. (13) is usually a small difference of very large components. Therefore, any simplification of each of these components must be made with great caution. The errors caused in (13) by the simplifications made in its coefficient calculations does not exceed 0.006%. This has been verified for the following, realistic values of the circuit parameters:

$$\begin{gathered} K = 25 \text{ ns, e.g. for } R = 50\ \Omega \text{ and } C = 0.5 \text{ nF}, \\ A_{ud1} = 3 \cdot 10^5, \quad K_{a1} = 1/(2\pi \cdot 320 \text{ Hz}) = 1 \text{ ms}, \\ A_{ud2} = 3 \cdot 10^4, \quad K_{a2} = 1/(2\pi \cdot 1270 \text{ Hz}) = 0.25 \text{ ms}. \end{gathered} \qquad (14)$$

In Fig. 5 the shape of the impulse response (13) is presented, already as a weighting function $|h(t-\tau)|$ in (8). This weighting function assumes non-zero values for all τ's preceding an observation instant t. Therefore, for the determination of an interval length t_{eff} for which the input signal meaningfully influences the output voltage, some formal criterion must be defined. Here, t_{eff} was chosen such that: $\frac{S_1}{S_1+S_2} = 0.95$ (please, cf. Fig. 5). Generally, t_{eff} should be the shortest interval meeting the above equation. Because, however, the values of $|h(t-\tau)|$ disappear sharply in the vicinity of the instant t, the latter value was chosen as limiting t_{eff} from top. For the parameter values (14) the length thus defined equals $t_{eff} \cong 88.6$ ns. Unfortunately, as it is visible from Fig. 5, a time period around t_{max}, over which $|h(t-\tau)|$ may be considered to be approximately constant, constitutes only a negligible part of t_{eff}. Therefore, the output voltage of the S/H circuit, although not an instant value of the input signal, is also unable to be regarded as a symmetric parameter of it, and thus it also cannot be an averaged parameter. If, however, the input signal is sufficiently constant at least over the period t_{eff}, then the output voltage becomes proportional to the input voltage thanks to the reflexivity of all the parameters like (8).

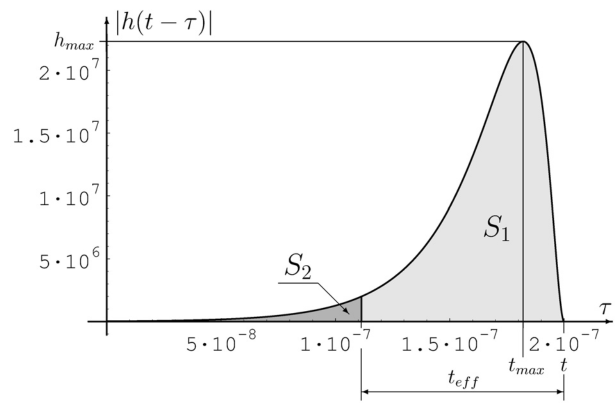

Fig. 5. Weighting function of the S/H circuit during the tracking phase evaluated for the parameter values (14) and $t = 200$ ns

As it is noticeable in Fig. 5, the time instant t_{max}, when the input signal exerts the greatest influence on the output, precedes the instant t of the output observation. The difference $t - t_{max}$ may be regarded as a delay introduced by a sampling circuit. Its exemplary value, obtained for the parameter values (14), amounts 17.9 ns.

A structure of the S/H circuit with an external feedback path has also been analyzed. In this type of structure the output of the second amplifier is connected to the inverting inputs of both the amplifiers. Although not best fitted for the fastest applications, this structure is usually more accurate than the one presented in Fig. 2c. The equation for $h(t)$ of the S/H circuit with external feedback, though being highly complicated has, after simplification, a structure close to that of Eq. (13). Unfortunately, for typical values of the amplifier parameters, the weighting function $|h(t-\tau)|$ is also significantly variable in the entire interval in which the input signal influences the output. Thus, also for that structure, the output voltage cannot be considered to be an averaged parameter of the input signal segment.

5. Conclusions

The attributes of the averaging operation, postulated by B. Ellis and J. Pfanzagl, seem to be intuitive and important both from theoretical and practical points of view. As originally defined for discrete operations they needed redefinition for functional mappings. Here the properties of: reflexivity, symmetry, bisymmetry, monotonicity, cancellability, and continuity were defined for parameters related to signal segments. Thus, from a relatively wide class of non-point parameters, a much narrower category of averaged parameters was separated, possessing all the above six attributes. In particular, the exemplary parameters' family (3) was given for the signals describing a dependence of some additive quantity on a quantity for which an interval scale was established. That family, for $\varphi(y) = y^k$, and k being an odd natural number, defines parameters for which all the requirements of averaging are satisfied. A widely used, simple integral average is also covered in this case.

Then, the arrival of the defined attributes was verified for three circuits: for a low pass RC filter, for an integrator, and also for a S/H circuit during its tracking phase. A positive meeting of the requirements of reflexivity, monotonicity and continuity was a consequence of these circuits linearity. On the other hand, a requirement of symmetry was found troublesome. Playing a role of a weighting function in the convolution integral (8), the impulse response of the circuit should be possibly "rectangular", i.e. constant over an averaging interval and disappearing rapidly outside it. The coefficient κ (12) was proposed as a measure of this "rectangularity". For the circuit of the integrator, the values of

κ, obtainable for real conditions, are quite small, signifying that the integrator output voltage is an almost symmetrical parameter of the input signal. As opposed to it, output voltages of both the low pass RC filter and the S/H circuit usually fail to be symmetrical. For the S/H circuit the period of an effective influence on the output voltage, and also the introduced delay, were defined.

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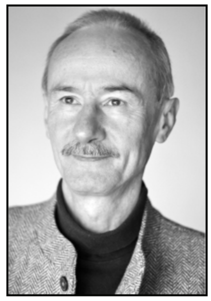

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