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Coupled wave-equation and eddy-current model for modelling and measuring propagating stress-waves*

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Abstract: The coupling of the propagating stress wave with the eddy current model is presented. The applied stress produces magnetization in the sample that can be measured outside the sample by measuring the resulting magnetic flux density. The stress and flux density measurements are made on a mechanically excited steel bar. The problem is modelled with the finite element method for both the propagating wave and the eddy current. Three aspects are considered: eddy current model using magnetization from the measurements, coupled wave and eddy current models, and coupled different dimensions in the wave model. The measured stress can be reproduced from the measured flux density by modelling. The coupled models work both for stress and flux couplings as well as for the different dimensionality couplings.

Key words: coupled models, eddy-currents, finite element method, magnetostriction, stress-wave

1. Introduction

Magnetostriction as a phenomenon couples the magnetic and mechanical behaviour of iron-based materials [1]. In the inverse magnetostriction, a change in the mechanical state of the matter produces a change in its magnetic state. These magnetic changes can be measured through contactless methods. Measuring the changes of the magnetic state of matter in real-time thus makes it possible to measure the stress-levels indirectly. This is an advantage compared to the traditional stress measurement techniques based on the strain gauges.

In long steel shafts, there is a need for the measurement of rapidly varying stress levels, i.e. measurement of rapidly moving stress-wave [2]. The propagating stress-wave produces

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changes in magnetization in the shaft. These changes in magnetization are measured by a coil around the shaft.

This paper presents measurements of propagating stress waves in long shaft and simultaneously measured magnetic flux density around the shaft. The idea is to understand how the magnetic flux density is produced due to the magnetoelastic coupling and the inverse magnetostriction. Using a simple model for the inverse magnetostriction, the form of the propagating stress wave is reproduced from the magnetization curve.

Finite element model is used to model the coupled elastomechanic problem. With Finite element method the propagating stress wave is used as a source for the magnetization. The eddy currents must be taken into account because without eddy-currents the model would ignore the reaction field and lead to erroneous results.

The first study uses Comsol multiphysics for modelling the coupled magnetic and elastic models. The magnetic model is transient and takes into account the eddy-currents. The elastic model assumes a linear relation between the propagating stress-wave and the magnetization.

In further studies the Elmer finite element solver is used to model coupling of the wave-equation with the magnetic eddy-current problem. The wave-equation is naturally a simplification of the true nature of the propagating stress-wave. However, assuming isotropic material, the wave-equation can be used to model the propagating stress wave [3].

The problem of propagating stress-wave is three dimensional in nature. For 3D coupled finite element problems, the number of degrees of freedom is relatively high. For this reason, Elmer is used for modelling the coupling between different dimensions as well. The propagating stress-wave is modelled partly by a 2D model and in the parts, where the 3D eddy-current model is needed, the wave-equation is 3D as well. For this purpose the finite element wave-equation is modelled in a coupled 2D-3D approach.

2. Material

2.1. Magnetostriction and inverse-magnetostriction

Magnetostriction refers to a set of magnetoelastic phenomena in which stress and strain are coupled with magnetization. The phenomenon has been known for a long time [4, 5]. In [4] an exhaustive overview is given. In [5] the physical basis for magnetoelastic coupling is presented. The book of Bozorth [1] gives a good introduction to the magnetostriction for ferromagnetic materials. In [6] a detailed study of magnetostrictive phenomena has been presented from a theoretical point of view.

The set of magnetostrictive phenomena is very wide [7]. Joule magnetostriction refers to phenomenon where the strain and the magnetic field are in the same direction. The effect occurs at constant volume and thus also negative transverse strain occurs. Villari effect refers to the change in magnetization due to applied stress. The Villari effect is thus the basis for static force and stress electromagnetic-based sensors. However, the stress induced magnetization is typically considered to arise in pre-magnetized samples. Thus at zero magnetization, the stress induced magnetization is theoretically considered to be zero. In the ΔE effect the elasticity of magnetostrictive materials is composed of conventional stress-strain elasticity arising from inter-atomic forces and magnetoelastic part due to the rotation of magnetic moments. The change in elastic modulus is $\Delta E = E_s - E_0$, where E_0 is the minimum elastic modulus and E_s is the elastic modulus at magnetic saturation. Circular magnetic field around a shaft of magnetostrictive material causes twisting of the shaft. This effect is called Wiedeman effect. Its inverse, twisting of shaft that causes circular magnetic field around the shaft, is called Matteuci effect. This effect is used, e.g., in torque sensors. The volume of magnetostrictive material does not change under magnetic field in normal cases. In certain extreme situations also the volume can change. This effect is called the magneto-volume effect or Barret effect. Its inverse, change of magnetization due to change in volume is called Nagaoka-Honda effect.

Starting from energetic considerations, the governing equations for magnetostrictive behaviour are [8]:

$$dS = \frac{1}{T} C^{H\sigma} dT + \sum_{i=1}^{3} \eta_i^{\sigma} dH_i + \sum_{k=1}^{6} \alpha_k^H d\sigma_k, \tag{1}$$

$$dB = \eta_j^{\sigma} dT + \sum_{i=1}^{3} \nu_{ij}^{\sigma T} dH_i + \sum_{k=1}^{6} g_{jk}^T d\sigma_k, \tag{2}$$

$$d\varepsilon = \alpha_l^H dT + \sum_{i=1}^{3} g_{il}^T dH_i + \sum_{k=1}^{6} s_{kl}^{TH} d\sigma_k, \tag{3}$$

where S in the entropy, T the temperature, H the magnetic field strength, σ the stress, B the magnetic flux density, and ε is the strain. The constants are in tensor form: $C^{H\sigma}$ is the heat capacity, η_i^{σ} and η_j^{σ} refer to the magnetochaloric effect α_k^H and α_l^H are the thermal-expansion coefficients, $\nu_{ij}^{\sigma T}$ refers to the components of permeability at constant stress and temperature, g_{jk}^T and g_{il}^T refer to the magnetostriction effect and s_{kl}^{TH} is the compliance. However, the practical usage of these equations is difficult due to non-linearities.

The linearized equations taking into account only the magnetic and elastic interaction, i.e. neglecting the thermal effects, are as follows [9]:

$$\varepsilon = \frac{\sigma}{E_Y^H} + d_{33} H \tag{4}$$

and

$$B = d_{33}^{*} \sigma + \mu^{\sigma} H, \tag{5}$$

where E_Y^H is the Young modulus at constant applied magnetic field, H is the applied magnetic field strength, B is the magnetic induction, μ^{σ} is the permeability at constant stress, and d_{33} and d_{33}^{*} are two magnetomechanical coefficients.

The magnetoelastic coupling constants are problematic from the modelling point of view. In [10] a method presents how the constants can be measured. However, due to non-linearity and hysteretic behaviour, deeper understanding of the magneto-elastic processes is needed.

The magnetostriction strain is typically modelled as having a quadratic dependence on the magnetic flux density [11]. The magnetostriction in terms of magnetization can be thus presented as [12, 13]:

$$\lambda = \frac{3}{2}\frac{\lambda_s}{M_s^2}M(t,x)^2, \tag{6}$$

where M_s denotes the saturation magnetization, λ_s the saturation magnetostriction, and λ is the magnetostriction that indicates the relative change in length of the material from the ordered, but unaligned state, to the state in which domains are aligned. $M(t, x)$is the magnetization, which depends on time and space.

The saturation magnetostriction depends on the material. Giant magnetostrictive materials are the most useful from the transducer point of view. Terfenol-D is the most used one. Terfenol-D is used both in actuator and sensor applications [14].

For Terfenol-D the nominal quasistatic magnetostrictive strain, i.e. λ, is $1640\cdot10^{-6}$ and for iron $20\cdot10^{-6}$ [13]. Naturally the orientation of the crystal has some effect, i.e. the constants are different for different directions [1, 12]. For iron the constants are typically much lower and strongly temperature dependent. From the measurement application point of view, this makes the use of iron more difficult.

Inverse magnetostrictive effect refers to phenomenon where applied stress in ferromagnetic material will change its state of magnetization [15, 16]. The duality of the phenomenon, i.e. magnetization produces change in mechanics and vice-versa, makes all the materials that have magnetostrictive properties, possible both for actuator and sensor applications. The strength of these phenomena reduces their practical use in many materials.

2.2. Applications

Magnetostriction has been used for several specific sensor and actuator applications. In [17, 18] Calkins et al. studied the magnetostriction for actuator applications and discussed on the most suitable materials. In [14] they reviewed magnetostriction for sensor applications.

The inverse magnetostriction has been used for current sensing [19] and static mechanical stress sensing [20].

In [21] force sensor was developed for the automobile brake system. The sensor was based on the change of permeability under stress, i.e. inverse magnetostriction. In [22] a torque sensor was developed for automotive applications too. A Magneto-elastic ring was press fitted around the shaft. In these studies all the sensors were pre-magnetized.

2.3. Modelling and finite element analysis

From the modelling point of view the hysteretic behaviour is very demanding. In [12] the hysteretic behaviour was modelled for transducers. In [23] the hysteretic models were further developed for Jiles-Atherton model.

The finite element modelling of magnetostriction is demanding because the parameters are typically not always known and because of hysteretic behaviour. In [24] the variational formulation in terms of magnetic vector potential and displacement was used to solve coupled problem for magnetostrictive materials. The authors analyzed both 2D and 3D cases. In [25] the constitutive linearized equations were used for finite element modelling of magnetostrictive material. In [11] 3D finite element modelling was used. The authors included direct and inverse effects through strongly coupled models.

For actuator applications finite element analysis is used to model the problem e.g. in references [26-28]. For sensor applications the FEM is used e.g. in reference [29].

2.4. Measurement of propagating stress wave

The propagating stress wave produces magnetization due to inverse magnetostriction [16]. If the changes in magnetization are rapid, the changing magnetic field produces eddy currents in the sample. The changes in the magnetization of the sample can be measured outside the sample by a measurement coil. The samples are typically pre-magnetized.

The propagating stress wave is of interest e.g. in percussive machines [2, 3, 30]. In [31] a detailed analysis on the usage of magnetostriction in measuring propagating stress-wave starting from the physical properties of iron is presented. In [30] an overview of stress waves propagation in rods and measurements of the wave-propagation with strain gauge are presented. The authors used steel as material (st 37). The sensor principle was presented and the authors proposed to use two coils one for excitation and another one as pick up coil. As a result they found a linear correlation between the impact speed of the percussion and the signal-voltage of magnetoelastic sensor. In [3] the authors developed the model further for measurements of longitudinal waves in rods. The starting point for the mechanical wave modelling was the wave equation. In [32] a method was patented for time-dependent gradient of a shock wave in a ferromagnetic element subjected to a percussion load.

3. Method

3.1. Wave-equation and eddy-current model

The propagating pressure wave follows the wave equation,

$$\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2} - \nabla \cdot \frac{1}{\rho}(\nabla p) = 0, \tag{7}$$

where p is the pressure, ρ the mass density, and c the speed of the wave. The source term in right hand side is zero for the only source in these models is the pressure on the boundary. The propagating wave is assumed lossless, viscous effects are neglected, and a linearized isotropic equation of state is used [33, 34]. For high pressure impact wave, even one-dimensional wave equation may be used to model the propagation [3]. This study does not consider true tensorial form of the stress wave. This assumption is based on reference [3] and the accordance of results and simulation. The stress wave travels in a symmetrical rod with velocity

$$v = \sqrt{\frac{E}{\rho}}, \tag{8}$$

where E is the Young modulus [3].

In this study, the stress, σ , and magnetization, **M**, are linearly related

$$\mathbf{M}_s = \alpha\sigma, \tag{9}$$

where α is a scaling factor. This is a simplification of linear constitutive model, that is restricted to small signal excitation levels [35]. Both the stress and the magnetization have reversible and irreversible parts but here the changes are considered to be so small that only the reversible part is taken into account.

For general three-dimensional eddy current problem [36] the equation with magnetic vector potential, **A,** is,

$$\nabla \times (\nu\nabla \times \mathbf{A}) - \mathbf{M}_s + \sigma\frac{\partial \mathbf{A}}{\partial t} = \mathbf{J}_s, \tag{10}$$

where ν is the reluctivity, $\mathbf{M}_s$ is the magnetization source, σ is the conductivity, and $\mathbf{J}_s$ is the source current.

3.2. Finite element analysis

The finite element models are carried out with Comsol Multipysics ® and Elmer. Three separate cases are studied:

1) *Measured stress and axisymmetric eddy current model*

The stress wave can be modelled as one-dimensional for simple cases. The x-dimension of the mechanical model may be larger than the eddy current model to model e.g. a long shaft. The eddy current model can be axisymmetric for e.g. cylindrical rod.

In this study a time stepping magneto-dynamic approach was adopted in view of catching the time dependence of the eddy currents. An axisymmetric finite element model of 3 m long and 10 mm diameter bar was built with Comsol multiphysics. The source for the eddy current model was the magnetization from the equation (3) with $\alpha = 160$ A/mMPa.

2) *Coupled wave equation and eddy current model in 3D*

A sample model for coupled wave-equation and eddy current model is presented in Figure 1. In the inner part of the model there is a rectangular steel bar surrounded by air. The parts are more explained in the Results section.

In this model the surface tension is neglected. The material is considered isotropic. From the application point of view the principal stress level is the most interesting quantity. With these assumptions the use of simple scalar valued wave equation is reasonable.

The eddy current model needs to be either two- or three- dimensional. Three dimensional models are needed if the geometry is not axisymmetric – e.g. for rectangular bar.

In this study the propagating wave and eddy current model are presented with 3D models. The coupling is one directional, i.e. the magnetization depends only on the stress level. The modelling is done with Elmer.

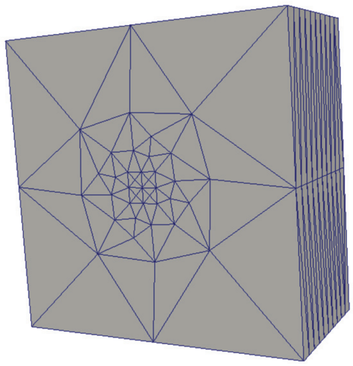

Fig. 1. The mesh of the finite element model for wave equation and eddy current model. Mesh generated with Gmsh

Edge elements are used for the eddy current problem. The wave equation is modelled with nodal elements. The coupling from stress to magnetization is done by simple linear relatioship (Eq. 9).

3) *Coupled dimensions for propagating stress wave*

The stress wave can be modelled as one dimensional or two dimensional problem for simple cases – as stated in [3]. Thus modelling part of the problem as 1D or 2D model, the number of degrees of freedom can be reduced. For this purpose the 2D wave equation is coupled with 3D wave equation. The coupling can be done with scalar valued functions with Elmer for the moment. The full stress model needs tensor valued variables. However, because the impact wave is typically very well modelled with simple wave equation, it is worthwhile to show the coupling procedure for scalar equation.

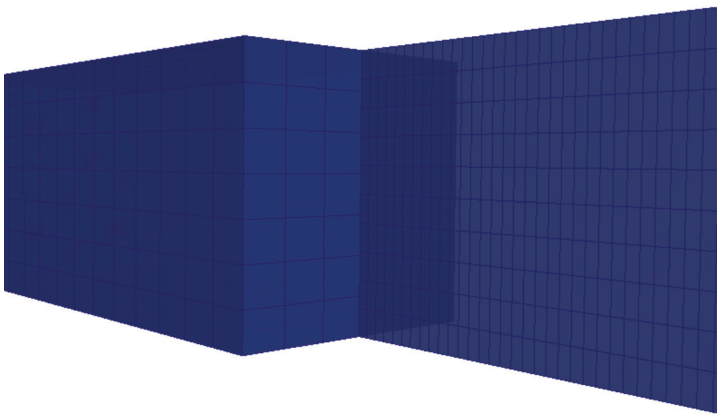

Fig. 2. The mesh for coupled 2D-3D model. Meshes are generated with Gmsh and ElmerGrid

The coupling is possible for scalar valued functions, like the pressure. The coupling is based on Mortar boundary method with the exception that the other boundary is of lower dimensionality. The 3D coupling boundary is mapped to two-dimensional line. When the boundaries have the same dimensionality, the original 2D boundary can be used as source for the 3D boundary (that is mapped to be 2D boundary) and periodic mortar boundary condition can be applied. We call boundary condition periodic because values from other side are directly coupled to other side. Here, however, the boundaries are attached physically. Mortar condition is used for non-conformal meshes.

Part of the coupled 2D-3D model is shown in Figure 2. As can be seen from the figure, the meshes can be non-conformal too as in the general case of Mortar boundaries.

4. Results

4.1. Measured stress and axisymmetric eddy current model

The finite element model of the sample 3 m × 0.01 m is shown in Figure 3. The stress wave comes from the bottom of the steel bar and travels through it producing magnetic flux around the bar as shown in the figure.

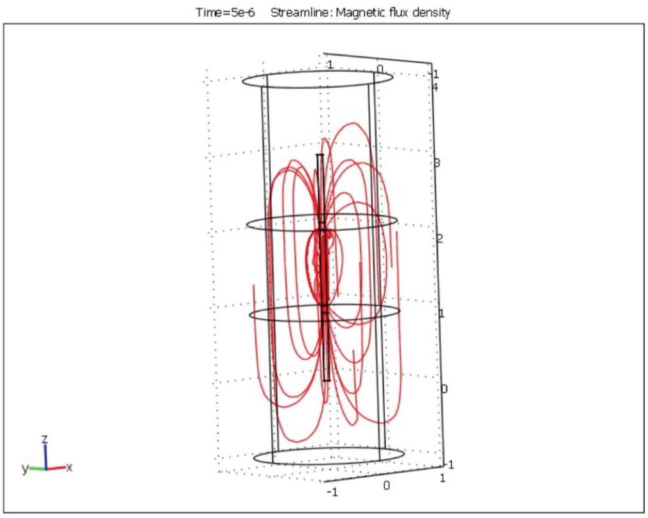

Fig. 3. The magnetic flux density lines around the sample (3 m × 0.01 m). The source is the magnetization that follows the form of stress shock wave. Modelling with Comsol multiphysics ®

An example of the measurement of the propagating stress wave and average flux density is shown in the Figure 4. In the figure the computed flux density from the axisymmetric eddy current model is shown as well as the estimated stress intensity (the estimation is explained in [37]). The sample is considered isotropic. The pressure wave refers to applied percussive impact on the other end of the sample; inside the material the pressure wave causes stress to change according to material laws. Basically, the variables pressure and stress are the same.

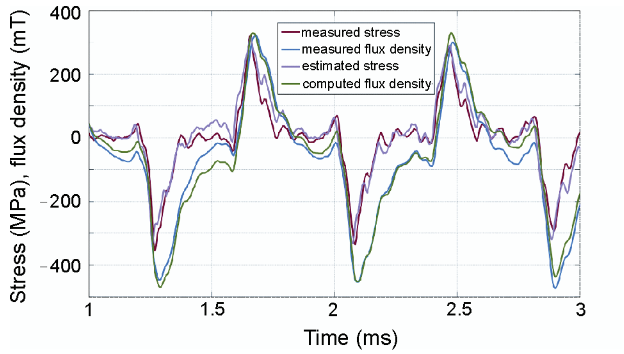

Fig. 4. The measured stress wave, the measured flux density, the computed flux density, and the estimated stress

4.2. Coupled wave equation and eddy current model in 3D

Here simple coupling is made with Elmer to show the coupling of wave equation and eddy current model. The idea is to use the coupling for more complicated geometries were e.g. the different dimensions can be coupled.

The model consists of rectangular steel bar (0.2 m × 0.2 m × 1 m) with air (1 m × 1 m × 1 m) around it. The pressure is given as boundary condition in the other end of the steel bar with $p = 400$ MPa·cos(2π400t).The corresponding mesh is shown in Figure 1. The result of the magnetic flux density is shown if Figure 5.

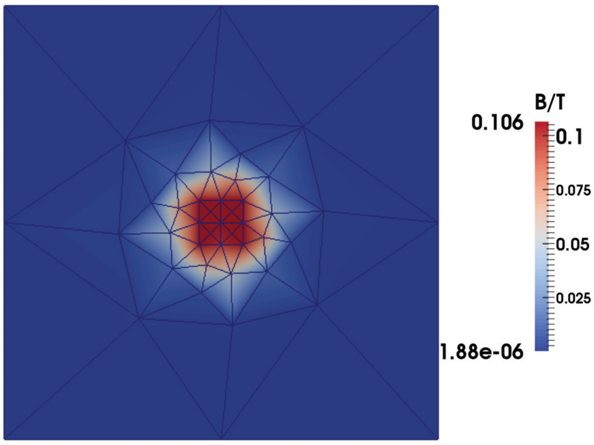

Fig. 5: The magnetic flux density around the steel bar. The source is the magnetization computed from the pressure that follows the wave equation solution. Axial view, modelling with Elmer

4.3. Coupled dimensions for propagating stress wave

Figure 6 shows the coupled 2D-3D model for wave equation. The propagating wave travels smoothly in the coupling boundary. The pressure is continuous. However, the limitation is that the coupled variable must be scalar valued. In the coupling boundary the meshes can be non-conformal.

In Figure 6 only the steel parts are shown, which are needed for the wave equation solution. Adding air around the steel bar in 3D, the eddy current solution may be shown as well as is shown in section 4.2.

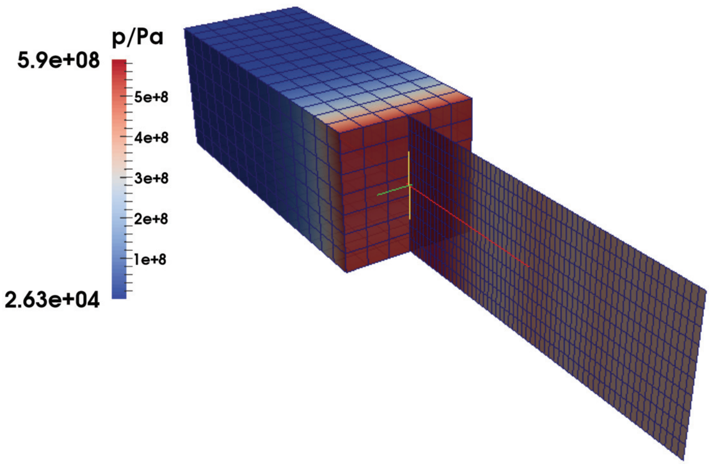

Fig. 6. The coupled model for 3D and 2D wave equation. The percussive shock is applied to the 2D side of the model. The speed of the travelling wave is 5000 m/s and the frequency of the applied pressure is 400 Hz

5. Conclusion

The simple linear relationship between stress and magnetization seem to give reasonable results in the sense that the measured flux densities can be reproduced with eddy current model. This is the direct problem from the measured stress to the calculated magnetic flux density. To model the propagating stress wave, a simplified scalar-valued wave equation for the pressure is presented and used for coupled models with eddy current. This model can be used with coupled different dimensionalities as well. This part deals with scalar quantities and further investigations are needed to implement more comprehensive models with e.g. tensor valued stress variables. However, the coupling of different dimensionalities may still be applied, if the coupling can be done in regions where the propagating stress wave is more like a plane wave. The open software finite element solver, Elmer, can be freely modified and thus the implementation of such models is under work.

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